Development of Adaptive Modeling Techniques for Nonlinear Hysteretic Systems

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Abstract

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Abstract

Adaptive estimation procedures have gained significant attention by the research community to perform real-time identification of nonlinear hysteretic structural systems under arbitrary dynamic excitations. Such techniques promise to provide real-time, robust tracking of system response as well as the ability to track time-variation within the system being modeled. An overview of some of the authors' previous work in this area is presented, along with a discussion of some of the emerging issues being tackled with regard to this class of problems. The trade-offs between parametric based modeling and nonparametric modeling of nonlinear hysteretic dynamic system behavior are discussed. Particular attention is given to 1) the effects of over- and under-parametrization on parameter convergence and system output tracking performance, 2) identifiability in multi-degree-of-freedom structural systems, 3) trade-offs in setting user-defined parameters for adaptive laws, and 4) the effects of noise on measurement integration. Both simulation and experimental results indicating the performance of the parametric and nonparametric methods are presented and their implications are discussed in the context of adaptive structures and structural health monitoring.

1 Introduction and Background

1.1 Motivation

Problems involving the identification of structural systems exhibiting inelastic restoring forces with hereditary characteristics are widely encountered in the applied mechanics field. Representative examples include buildings under strong earthquake excitations, aerospace structures incorporating joints, and computer disk drives. Due to the hysteretic nature of the restoring force in such situations, the nonlinear force cannot be expressed in the form of an algebraic function involving the instantaneous values of the state variables of the system. Consequently, much effort has been devoted by numerous investigators to develop models of hysteretic restoring forces and techniques to identify such systems. Noteworthy contributions in this area have been made by Caughey (1960 [10], 1963 [11]), Jennings (1964 [24]), Iwan (1966 [19]), Bouc (1967 [8]), Karnopp and Scharton (1966 [25]), Iwan and Lutes (1968 [20]), Kobori *et al.* (1976 [26]), Wen (1976 [50]), Masri and Caughey (1979 [32]), Baber and Wen (1981 [4], 1982 [5]), Grossmayer (1981 [14]), Spanos (1981 [44]), Toussi and Yao (1983 [48]), Andronikou and Bekey (1984 [1]), Park *et al.* (1985 [35]), Spencer and Bergman (1985 [45]), Sues *et al.* (1985 [46], 1988 [47]), Powell and Chen (1986 [38]), Iwan and Cifuentes (1986 [21]), Vinogradow and Pivovarov (1986 [49]), Jayakumar and Beck (1987 [23]), Peng and Iwan (1987 [36], 1992 [37]), Roberts (1987 [39]), Wen and Ang (1987 [52]), Yar and Hammond (1987a [55], 1987b [56]), Worden and Tomlinson (1988 [54]), Capecchi (1990 [9]), Roberts and Spanos (1990 [40]), Masri *et al.*(1991 [33]), Loh and Chung (1993 [31]), Benedettini *et al.* (1995 [7]), Chassiakos *et al.* (1995 [12]), Iwan and Huang (1996 [22]), and Ni *et al.* (1999 [34]).

One of the challenges in actively controlling the nonlinear dynamic response of structural systems undergoing hysteretic deformations is the need for rapid identification of the nonlinear restoring force so that the information can be utilized by on-line control algorithms for determining the proper actuator forces needed to ensure stable response control of the oscillating flexible structure. Consequently, the availability of a method for the on-line identification of hysteretic restoring forces is crucial for the practical implementation of structural control concepts (Housner *et al.*1994 (1WCSC [17]), Housner *et al.*, 1997 [16], Kobori *et al.*, 2WCSC 1998), whether in the context of adaptive structures or for structural condition assessment.

This paper presents an overview of work by the authors on the development of two broad classes of nonlinear system identification approaches (one parametric and the other nonparametric) that are suitable for on-line applications involving the monitoring and control of systems exhibiting hysteretic behavior that cannot be adequately treated as an equivalent linear system. The paper explains the basic concept and features of each approach, and it illustrates their respective strengths and limitations through several realistic examples composed of simulation studies as well as tests on physical structures undergoing time-varying hysteretic deformations.

1.2 Scope

The most basic system under consideration is the single-degree-of-freedom (SDOF) system whose forced vibration is governed by:

$$m\ddot{x}(t) + r(x(t), \dot{x}(t), r(t)) = u(t) \tag{1}$$

where $x(t)$ is the displacement of mass m, $r(x(t), \dot{x}(t), r(t))$ is the nonlinear restoring force and $u(t)$ is the system's external excitation. A simple diagram showing this system is presented in Fig. 1(a).

Several studies tackled the modeling problem as a "force-state mapping" problem. In other words, models were developed to determine how the states – (typically displacement x and velocity $\dot{x}$) – were mapped to the nonlinear restoring force r. The surface which defined r in the x and $\dot{x}$ space were fitted using basis functions of the states. For example, Masri and Caughey ([32]) used Chebyshev polynomials as the basis to yield models for the nonlinear force-state mapping problem which were *equivalent nonlinear* models in the case of hysteretic problems. The term *equivalent nonlinear* is important, because in reality the hysteresis restoring force is not simply a function of the states x and $\dot{x}$, but also of the past restoring force r. (This was written generically as $r(x(t), \dot{x}(t), r(t))$ in Eq. (1)). In other words, for hysteresis there is not a unique surface in the x and $\dot{x}$ space which defines the restoring force.

One of the more widely used models for hysteretic nonlinearities, because it can capture many commonly observed types of hysteretic behavior, is the Bouc-Wen model (Wen, 1980).

$$\dot{r} = (1/\eta)\left[A\dot{x} - \nu(\beta|\dot{x}||r|^{n-1}r - \gamma\dot{x}|r|^{n})\right] \tag{2}$$

This model will be considered in more detail later in the paper for parametric modeling. A sample of the achievable forms of nonlinear hysteretic behavior for this model is shown in Fig. (2). The different hysteretic

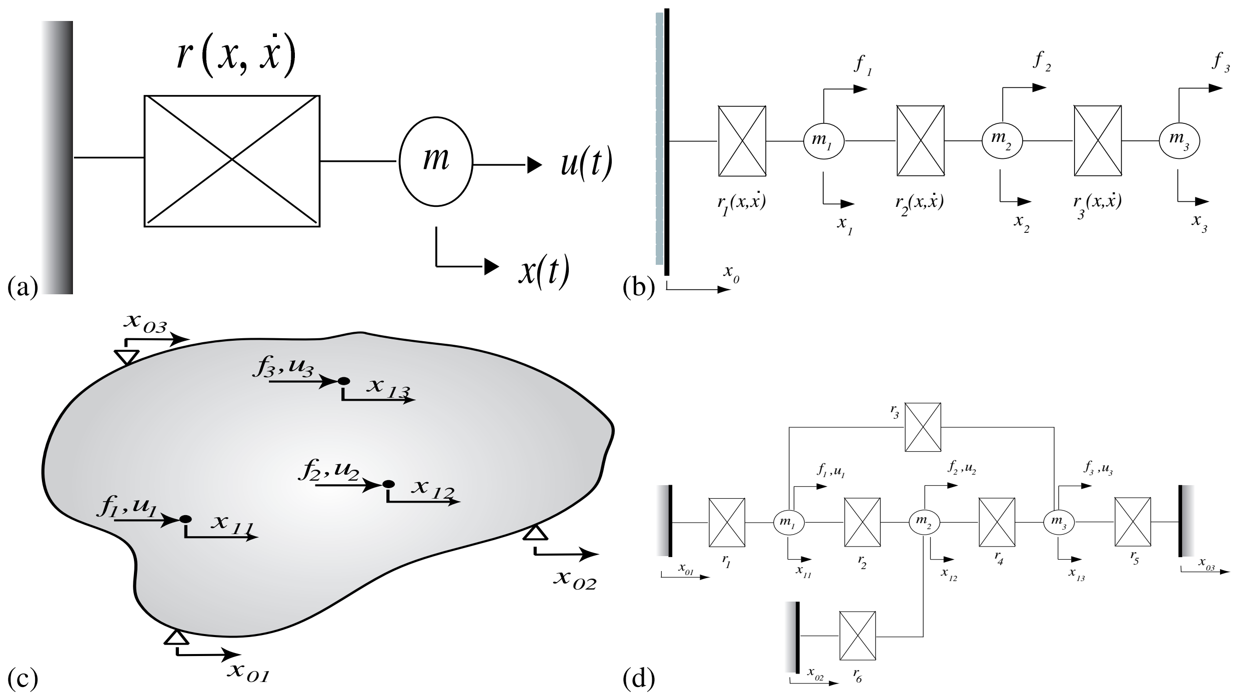


Figure 1: Model of hysteretic systems. a) SDOF system, b) chain-like MDOF system, c) general continuous MDOF system with multiple supports and force excitations, and d) discrete reduced order representation of general system shown in c).

loops correspond to different combinations of critical parameters in the model. When discussing the force-state mapping problem, one of the immediately noticeable benefits of the Bouc-Wen's model form in Eq. (2) is that the derivative of the restoring force can be defined by two states; the velocity $\dot{x}$ and the restoring force r itself.

One of the main motivations for exploring adaptive techniques, in the context of active control applications, comes from the recognition that since structures behave in unexpected forms and often exhibit nonlinear hysteretic behavior when excited by strong-ground motions, the implementation of conventional fixed controller strategies may prove to be naive. Often the governing response properties only exhibit themselves for the first time when subjected to strong shaking. As a result of this, active control strategies should incorporate flexible adaptive identification schemes which can quickly capture and emulate the essential response signature of a structural system and react accordingly. Of course, another key feature of adaptive techniques is that they can model time-varying behavior, for example, structural deterioration is often observed during the course of strong excitation.

Adaptive identification schemes can be employed in either the form of a parametric or nonparametric model. Parametric adaptive identification schemes have been investigated in the context of strong non-stationary excitations (Smyth *et al.*[42], Sato and Qi[41]). However, this work is limited by the chosen parametric model to identifying certain classes of nonlinearities. In this paper, the parametric modeling will be reviewed, and the motivation for moving to nonparametric techniques in the context of active control will be discussed. Recently the authors have developed a new type of adaptive artificial neural network

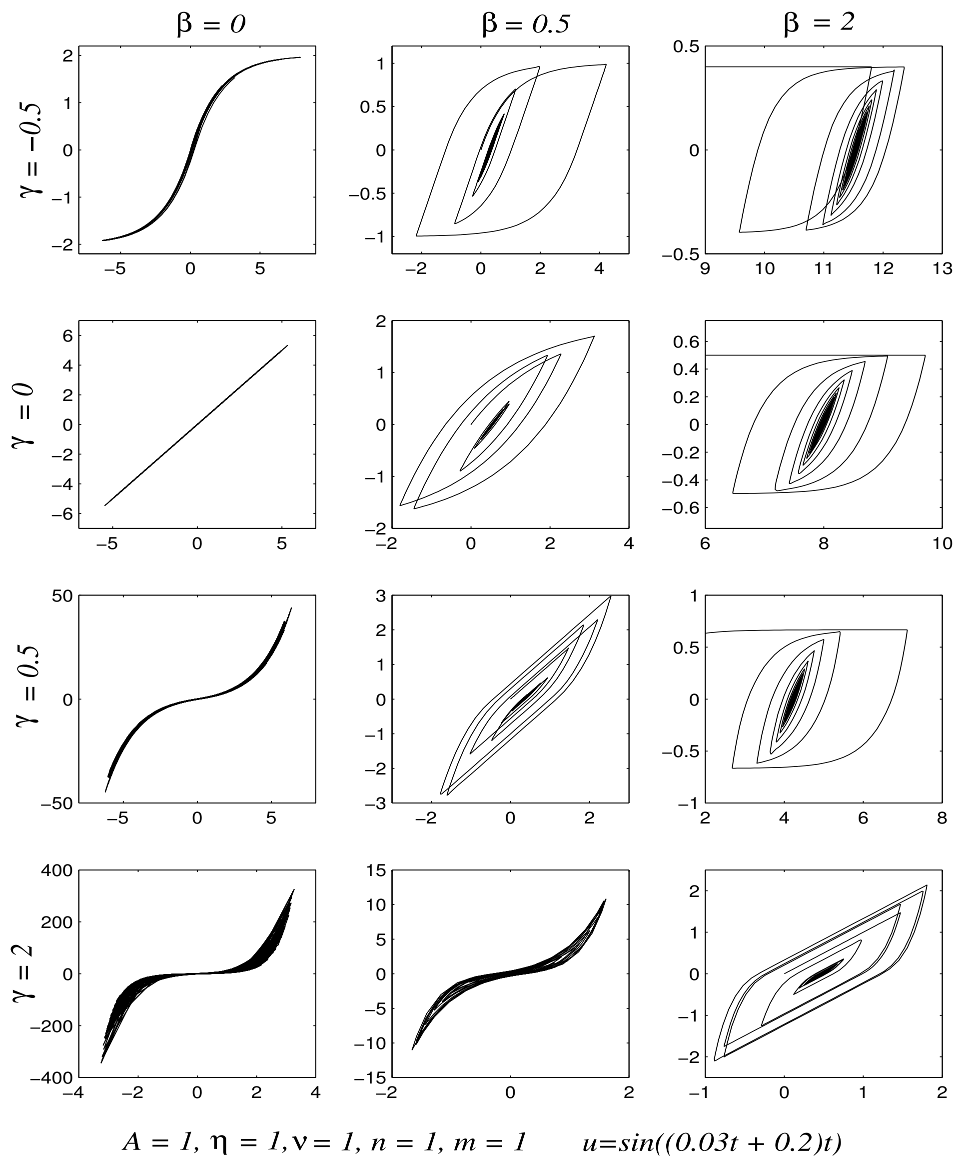


Figure 2: Comparison of the restoring force vs. displacement phase-plane plots for Bouc-Wen's models with different γ and β combinations. Note that each column of plots has a fixed β value, and each row, a fixed γ value.

identification technique for the real-time hysteretic modeling problem (Kosmatopoulos *et al.,* [28]). This approach can cope with a much broader family of unknown nonlinear response behaviors. Some details of the approach will be reviewed, and the general methodology will be discussed with a view toward future development in this area of research.

2 Problem Formulation

The fundamental problem which will be considered here is the prediction of the restoring force of nonlinear hysteretic structural elements, and the adaptive estimation of either a nonparametric or parametric model which describes the elements' dynamic behavior. The prediction and parameter estimation is conducted based on the system's measured dynamic response (usually acceleration measurements).

The structural topologies which will be considered range in complexity from the simple SDOF system shown in Fig. 1(a), to the very general continuous system shown in Fig. 1(c). For the nonlinear SDOF system shown in Fig. 1(a), the equation of motion was given in Eq. (1). The identification problem may be formulated in several ways, depending upon which system parameters are known. For example, it will be shown that if the mass of this system is considered unknown *a priori*, it can still be identified as one of the system parameters under some conditions.

As mentioned in the introduction, identification approaches can be divided into two categories: parametric and nonparametric. Parametric identification is the most desirable, because if successful, the parameters in a model for the restoring force will have some physical meaning. A simple example of this would be the stiffness k or damping c parameters in a linear restoring force problem. As will be shown later, while parametric approaches have this obvious benefit, they suffer from the fact that, to obtain each of these parameters which have physical meaning, the corresponding restoring force signal must be measurable, and an appropriate phenomenological form of the model must be selected. This often heavy requirement that signals be measurable has led researchers (out of necessity) to turn to nonparametric models. While such models may be able to model response behavior accurately and with considerable flexibility, their parameters usually have little or no physical meaning.

2.1 Parametric Modeling

As previously mentioned, the Bouc-Wen model was chosen for its ability to capture, in a continuous function, a range of shapes of hysteretic loops which resemble the properties of a wide class of real nonlinear hysteretic systems (Vinogradov and Pivovarov[49]). The shape of the hysteretic loop is governed by the combination of the parameters $\eta,\ A,\ \nu,\ \beta,\ \gamma$ and n, and it can be made to assume a wide range of qualitative features spanning the range from purely polynomial-like nonlinearity to a fully elastoplastic system.

The parametric modeling of a nonlinear element can be made quite flexible by incorporating additional terms into the model. For example (Smyth *et al.*[42]), the Bouc-Wen model may be complemented by a linear damping parameter c and a cubic term parameter d. In the SDOF *Mass Known* case (i.e., it is assumed that the value of m is available) the variable of interest r could be related to an auxiliary variable z by

$$z = r = u - m\ddot{x}$$

$$\begin{aligned} z &= kx + c\dot{x} + dx^3 \\ &\quad - \int_0^t (1/\eta)\left[\nu(\beta|\dot{x}||r|^{n-1}r - \gamma\dot{x}|r|^n)\right] dt \end{aligned} \tag{3}$$

The signal to be predicted, z in Eq. (3), can be rewritten in a more generic form as a *linear* combination of the product of the unknown parameter clusters (in vector $\boldsymbol{\theta}$) and the corresponding *nonlinear* observed signal combinations (in vector $\boldsymbol{\phi}$):

$$z = \boldsymbol{\theta}^T\boldsymbol{\phi} \tag{4}$$

It should be noted that if the power n which appears in the model is an unknown parameter to be identified,it will be difficult to identify directly since it appears nonlinearly in the equation. The way this problem is circumvented is by making a short series of terms with powers of $n = 0, 1, 2, \ldots$, and then identifying the corresponding coefficient clusters. If the cluster is zero, then one can assume that that particular power term does not appear in the model of the system. It was found that terms of up to $n = 3$ are more than adequate for most applications encountered in the applied mechanics field. The parametric model with its series terms would then look something like:

$$\dot{z} = sz = k(sx) + c(s\dot{x}) + d(sx^3) - (1/\eta)\sum_{n=1}^{n=N} a_n\nu(\beta|\dot{x}||r|^{n-1}r - \gamma\dot{x}|r|^n) \tag{5}$$

where s is the Laplace operator, and N is a user-specified model-order parameter.

After some manipulation, and defining the following terms

$$\bar{z} = \frac{sz}{(s+\alpha)} \tag{6}$$

$$\boldsymbol{\theta}^* = [k, c, d, -(1/\eta)a_1\nu\beta, (1/\eta)a_1\nu\gamma, -(1/\eta)a_2\nu\beta, \ldots]^T \tag{7}$$

and

$$\bar{\boldsymbol{\phi}} = \left[\frac{sx}{(s+\alpha)}, \frac{s\dot{x}}{(s+\alpha)}, \frac{sx^3}{(s+\alpha)}, \frac{|\dot{x}||r|^0 r}{(s+\alpha)}, \frac{\dot{x}|r|}{(s+\alpha)}, \frac{|\dot{x}||r|r}{(s+\alpha)}, \frac{\dot{x}|r|^2}{(s+\alpha)}, \ldots\right]^T \tag{8}$$

one may re-write Eq. (5) as a parametric model which is linear in $\boldsymbol{\theta}^*$ as follows

$$\bar{z} = \boldsymbol{\theta}^{*T}\bar{\boldsymbol{\phi}} \tag{9}$$

This result is noteworthy because it means that the estimation of the desired parameters contained in the $\boldsymbol{\theta}^*$ vector can be done using the filtered signals $\bar{z}$ and $\bar{\boldsymbol{\phi}}$ and be expressed in the form of a linearly parameterized estimator. Any low-pass filtering of signals $\ddot{x}$, r and u to remove measurement noise should be done independently before inserting the signals into the parametric model. Note, that the parameter α should be carefully chosen and should in general be quite small (Smyth *et al.* [42]).

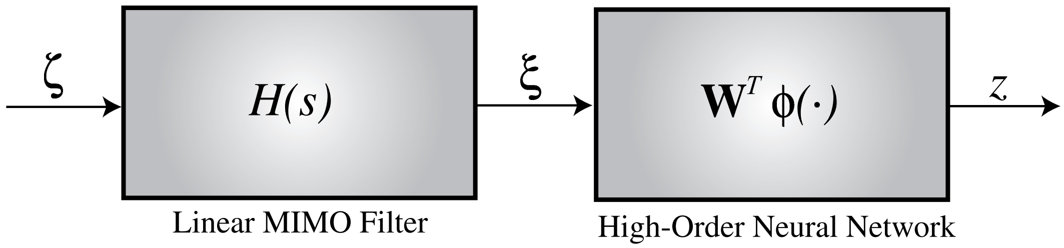


Figure 3: Block diagram of the Volterra Wiener Neural Network

2.2 Nonparametric Modeling

An alternative parameterization whose parameters do not have any physical meaning is also explored. The nonparametric model which the authors developed in a recent study is the Volterra/Wiener Neural Network (VWNN) (Kosmatopoulos [27], Kosmatopoulos *et al.* [28]) which is linear-in-the-weights, and can hence be written just like Eq. (4). Such a parameterization requires very little *a priori* information about the system properties, and will potentially require fewer measurement quantities to be available. These are two very significant advantages over physical model based identification techniques, because a system may not behave within the class of models initially assumed. The VWNN model allows the system nonlinearities to be extremely general and completely unknown at the outset. When the system becomes more complex than the SDOF system shown in Fig. 1(a), with many interconnected elements, such as the general system shown in Fig. 1(c), parametric modeling approaches to this problem require more signals to be measurable than is realistic in civil applications. In the case of nonparametric identification, the internodal restoring forces are not estimated because of insufficient sensor information, and therefore, the internodal elements parameters remain unknown. Rather, the resultant force which each node experiences due to the elements to which it may be connected is estimated.

The details of the VWNN can be found in the authors' recent work Kosmatopoulos *et al.*([28]), however for this overview paper it is sufficient to focus on its basic structure. The VWNN has the same very general function approximation properties of the Volterra/Wiener series expansion. Its neural network architecture is designed so that it can be trained adaptively without highly costly back-propagation or other computationally heavy training methods. In short, this architecture is achieved through the use of two basic modules; 1) a dynamic module consisting of a bank of stable transfer functions whose parameters are user-defined, and 2) a high-order neural network (HONN) which combines outputs of the transfer functions in polynomial cross-terms and whose parameters need to be trained. A schematic representation of the two modules is shown in Fig. (3). Notice that the notation of the second module is reminiscent of the parametric notation $\boldsymbol{\theta}^T\boldsymbol{\phi}$. Similarly, this single-layer network has unknown parameters contained in a matrix $\mathbf{W}$, and the observation vector $\boldsymbol{\phi}$ is actually a vector composed of a nonlinear activation function applied to the output signals of the dynamic module $H(s)$. The $H(s)$ bank of stable transfer functions serves as a *memory* operator allowing the overall model to be able to handle hysteresis. The ζ vector is simply the vector of raw (directly or indirectly) measured quantities used as input to the model, and z, as before is the objective function one wishes to track.

2.3 Adaptive Laws

The adaptive law which tracks the measured restoring force and correspondingly updates the parameter vector $\boldsymbol{\theta}$ is driven by the error between the predicted force and the measured force at the previous time-step. This is mathematically written as

$$\dot{\boldsymbol{\theta}} = \mathbf{P}\epsilon\bar{\boldsymbol{\phi}} \tag{10}$$

where the normalized tracking error $\epsilon = (\bar{z} - \hat{z})/\lambda^2$, and $\hat{z}$ =estimate of $\bar{z}$ using the current estimate $\boldsymbol{\theta}$ of the exact $\boldsymbol{\theta}^*$. $\mathbf{P}$ is what is called the gain matrix, and this can be a constant (positive semi-definite) matrix in the case of the gradient method or it can be time-varying, e.g., in the case of the least-squares adaptive law.

The authors have found that both least-squares based, and gradient projection algorithms obtain very good performance in the context of civil structural elements. Recently, additional effort has been made to refine the rate of adaptation in *optimal* ways (Lin *et al.*[29]), without *a priori* information about the level of excitation or type of excitation, and the corresponding response.

Although the authors have worked with several adaptive laws including the gradient method [13], from experience, it was found ([42]) that the least-squares adaptive law with a forgetting factor had several desirable properties of tracking performance, parameter convergence, and requiring relatively few user-defined parameters of its own. There are typically trade-offs between adaptability for tracking purposes, and smooth parameter convergence; i.e., high gains can yield excellent tracking, but relatively unstable (and overly sensitive) parameter convergence. Ideally, because the goal is adaptive identification, it is important to minimize the number of user-defined parameters in an adaptive law, because one wishes to make the law as autonomous as possible. In its most basic form, the least-squares law with forgetting factor simply has two design coefficients which must be chosen before use. The other necessary feature of all adaptive laws is their computational efficiency. The least-squares adaptive law (which can be related to the Kalman filter), has an adaptive gain matrix (often termed the covariance matrix in Kalman filter applications) which depends only on the measurements in the observation vector $\boldsymbol{\phi}$. No matrix inversion is required because the algorithm can be written in a recursive form.

For the VWNN identification model, a gradient adaptive law with projection was used simply because it was most convenient in these early stages of development of the VWNN to formulate the necessary convergence proofs with a gradient based adaptive law.

It should be noted that, in contrast to off-line or "batch"-mode identification of linear or nonlinear systems, the "output-error" approach is not realistically possible within the constraints of computational efficiency required by online applications. For the "output-error" approach one would determine error based upon a simulated response (from the beginning of the dynamic event) with the parameter values under consideration (e.g., Beck [6]) . In contrast, the "equation-error" approach which is adopted here, differs in that to determine error for given parameters, the measured states, for example displacement or velocity which may appear in a model are those which are measured, and are not re-simulated with the parameter values.

3 Applications

Several applications of parametric and nonparametric identification are presented in the context of civil structures during strong excitations.

3.1 Applications of Parametric Identification

The authors' results from Chassiakos *et al.* ([13]) of an adaptive identification using the parametric Bouc-Wen model with $N = 2$ in Eq. (5), is shown in Fig. 4 for actual experimental data from a cyclic test of a steel beam-column connection. The time-variation of the system's stiffness can be clearly seen in the progressive decrease in the identified θ_0 parameter. In addition to the experimental data from the steel beam column connection, an additional adaptive identification was performed on data collected from a shake-table test of a reinforced concrete beam-column connection (Smyth *et al.* [42]). The tracking and parameter convergence is shown in Fig. 5. Clearly the nonlinearity is quite different from the previous example, and a distinct 'dead-space' nonlinearity (due to concrete cracking) can be seen. Notice also, that in this case most of the parameters have not converged, even though restoring force tracking is quite good. (In this case θ_0 was the mass, and can clearly be seen to converge rapidly to its true value.)

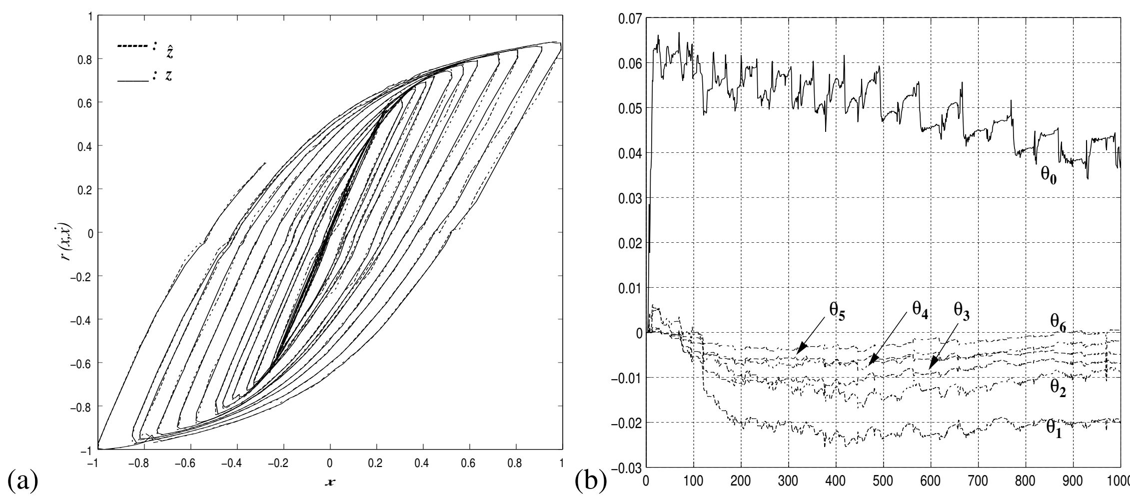


Figure 4: Adaptive identification of structural steel sub-assembly undergoing cyclic testing. (a) Phase plane plot of restoring force prediction vs. exact measured force; (b) evolution of the estimated parameters.

Beyond the experimental data cases which clearly demonstrate the real-world potential of the method, careful studies were performed using simulated data generated from a Bouc-Wen model with known parameters.

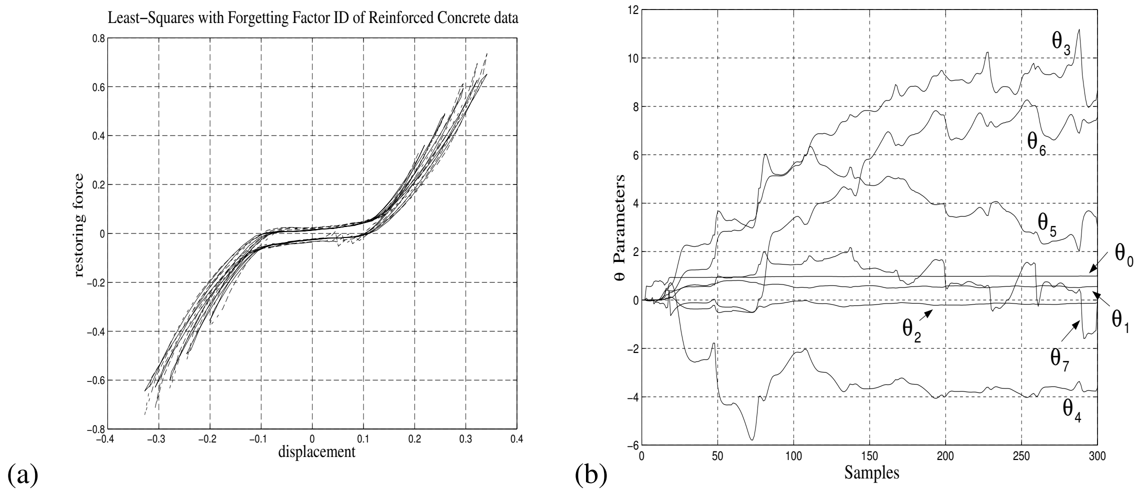
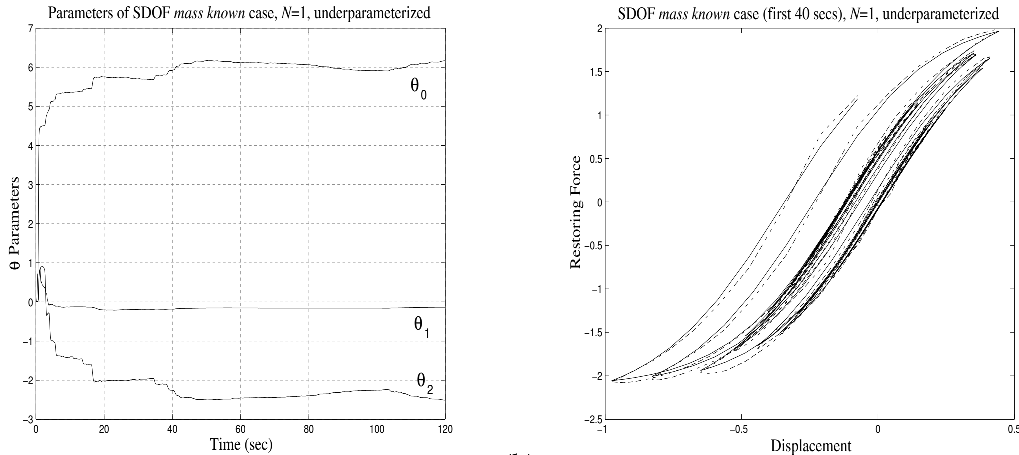


Figure 5: Adaptive identification of structural reinforced concrete sub-assembly undergoing cyclic testing. (a) Phase plane plot of restoring force prediction vs. exact measured force; (b) evolution of the estimated parameters.

3.1.1 Over- and Under-Parameterization

An important issue to consider, is that to model a system's response, the model needs to be complex enough to cope with the type of system behavior exhibited. While this may seem obvious, it is useful to know what happens when one chooses models which are either too simplistic (under-parameterized) or too complex (over-parameterized) in the context of adaptive identification. The first concern is perhaps the former case when one has a model which is under-parameterized. A simulation was conducted in Chassiakos *et al.* [13] where a Bouc-Wen's model was simulated with a value of $n = 2$, however the model (Eq. (5)- Eq. (9)) used to identify the system was only expanded up to $n = 1$ in the series. The tracking performance and the parameter convergence are shown in Fig. (6). It is interesting to note in this case that, although tracking of the response is good, the one parameter ($\theta_0 = k$) which actually is in the simulated model, is converging to the wrong value (the exact value is 5 while the estimated value is ≈ 6). This occurs as the estimation model compensates for its incorrect parameterization. In this case, the three estimation parameters $\theta_0 = (k)$, $\theta_1 = (-(1/\eta)a_1\nu\beta)$, and $\theta_2 = ((1/\eta)a_1\nu\gamma)$ appear to be converging to fixed values, indicating that even with the incorrect parameterization, good tracking can be closely mimicked for this range of response.

For over-parameterization, one is less concerned that one will be able to track the system output, because of course the real system will be simpler than the capabilities of the model. It is interesting to note, however, that assuming certain criteria about the richness of the excitation and the response (as will be discussed later) that the unnecessary parameters will converge to zero. If those criteria are not satisfied, then the parameters may not go to their true values, but will typically still yield a model which performs well from an output standpoint.



(a) (b)

Figure 6: (a) On-line estimates of the identified hysteretic system parameters. The parameterization excluded the m, c and d terms, and is of order $N = 1$, i.e. under-parameterized for this example. (b) Phase-plane plots of restoring force z vs. the response displacement x. The solid line represents the exact response, and the dashed line is the tracking obtained by algorithm with $N = 1$ parameterization. Only the first 40 seconds of the simulation are shown for added resolution.

3.1.2 Simulation studies of MDOF Chain-Like Systems

In addition to the experimental results from SDOF systems, the methodology may also be used for the identification of MDOF nonlinear structural systems, assuming that sufficient sensor data is available. MDOF chain-like structures (a typical simplification made to analyze multi-story buildings) can be decomposed into a series of SDOF elements whose response parameters can be identified as outlined above. A simulation study was conducted on a support-excited, chain-like MDOF system of the form shown in Fig. 1(b). In the context of civil structures, this would be analogous to a building experiencing earthquake loading. Each inter-story element was intentionally given a different type of structural nonlinearity to demonstrate the range of behavior which the identification method can handle. The 1^{st} story element is a pure Bouc-Wen element, the 2^{nd} story is a purely linear element, and the 3^{rd} is a hardening Duffing oscillator. The phase-plane plot of these elements subjected to a random base excitation is shown in Fig. 7(a). In 7(b) the rapid convergence to the correct $\boldsymbol{\theta}$ values can be seen. The exact parameter values which correspond to the identification parameterization are listed as follows:

$$\begin{array}{lll} \text{Element \#1:} & \text{Bouc-Wen Type Nonl.} & \theta_0^* = 5, \theta_1^* = -0.01, \theta_2^* = 0, \theta_3^* = -0.1, \theta_4^* = -0.5 \\ \text{Element \#2:} & \text{Linear} & \theta_0^* = 3, \theta_1^* = 0.1, \theta_2^* = 0, \theta_3^* = 0, \theta_4^* = 0 \\ \text{Element \#3:} & \text{Duffing Oscillator} & \theta_0^* = 0.125, \theta_1^* = -0.05, \theta_2^* = 0.1, \theta_3^* = 0, \theta_4^* = 0 \end{array} \quad (11)$$

$$\text{where} \qquad \theta^* = [k, c, d, -(1/\eta)a_1\nu\beta, (1/\eta)a_1\nu\gamma, -(1/\eta)a_2\nu\beta, \ldots]^T \quad (12)$$

3.1.3 Time-varying model identification

The strength of adaptive identification approaches, beyond obtaining good tracking performance for control purposes, is also the ability to detect changes in structural parameters. This was demonstrated by the authors

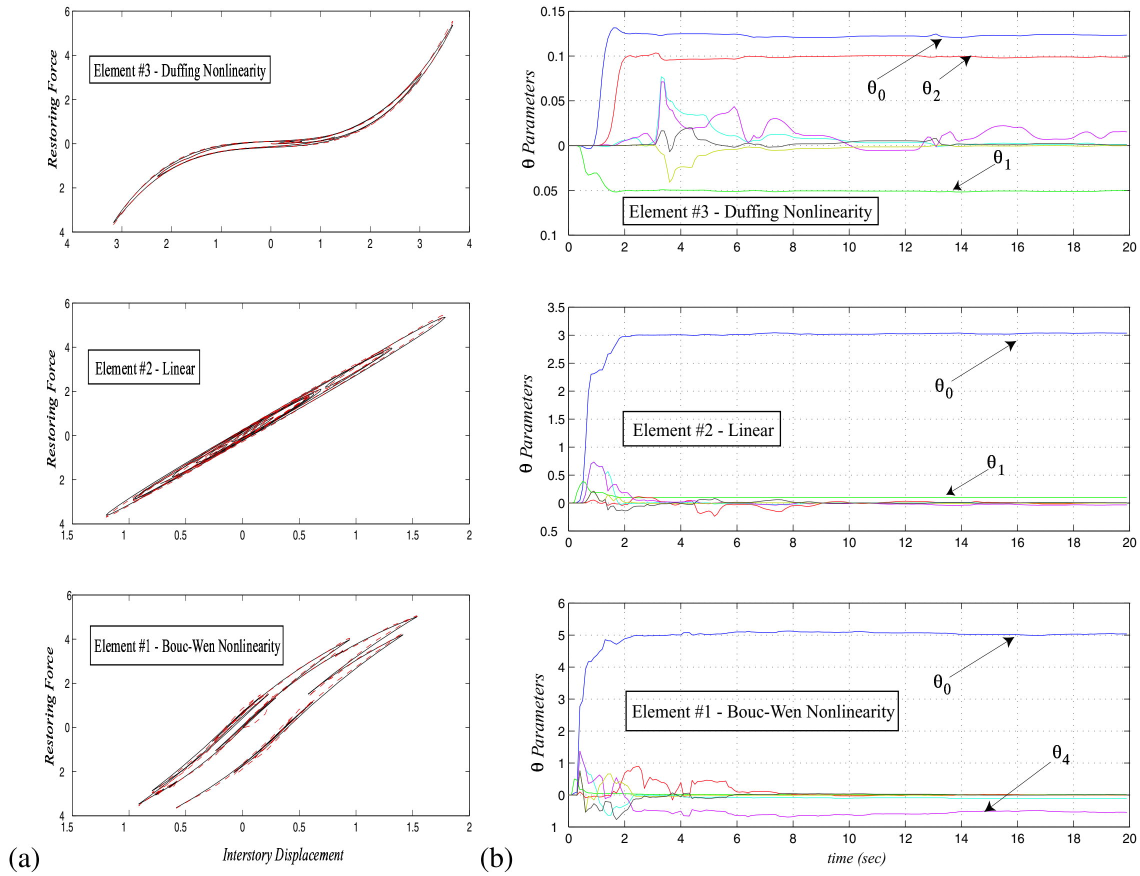


Figure 7: 3DOF chain-like system and the on-line estimates of hysteretic parameters for each inter-story element for the base-excited case.

for the experimental data set in Fig. (4) which exhibited gradual stiffness degradation (Chassiakos *et al.*, [13]).

The chain-like MDOF system example which is presented above, was repeated, however this time with the assumption that the first story element undergoes a sudden stiffness drop after 5 seconds. Specifically the element's stiffness parameter drops from 5 to 3. Figure 8(a) shows the parameter convergence, and Fig. 8(b) shows the phase plane plot of the exact and estimated restoring force versus inter-story displacement. Notice that the θ_0 value drops at $t = 5$ seconds, however not perfectly to the value of 3. This has to do with several factors. The first is the current value of the adaptive gains in the covariance matrix $\mathbf{P}$ which relate to θ_0. These would often tend to decrease after a simulation begins. The forgetting factor is used to limit this *covariance wind-up* effect. If the forgetting factor is not large enough to cope with such a sudden parameter shift then, in general, the algorithm will converge to the new value more slowly than if the identification scheme had been initially activated (i.e., with initial $\mathbf{P}_0$) at $t = 5$ seconds. This result is by no means meant to represent the best performance obtainable by this method, rather it is presented to illustrate some of the issues in algorithm autonomy which still need to be addressed. Lin *et al.*([29]) made some progress with regard to this type of problem by incorporating a variable forgetting factor in the adaptive identification of these types of time-varying systems.

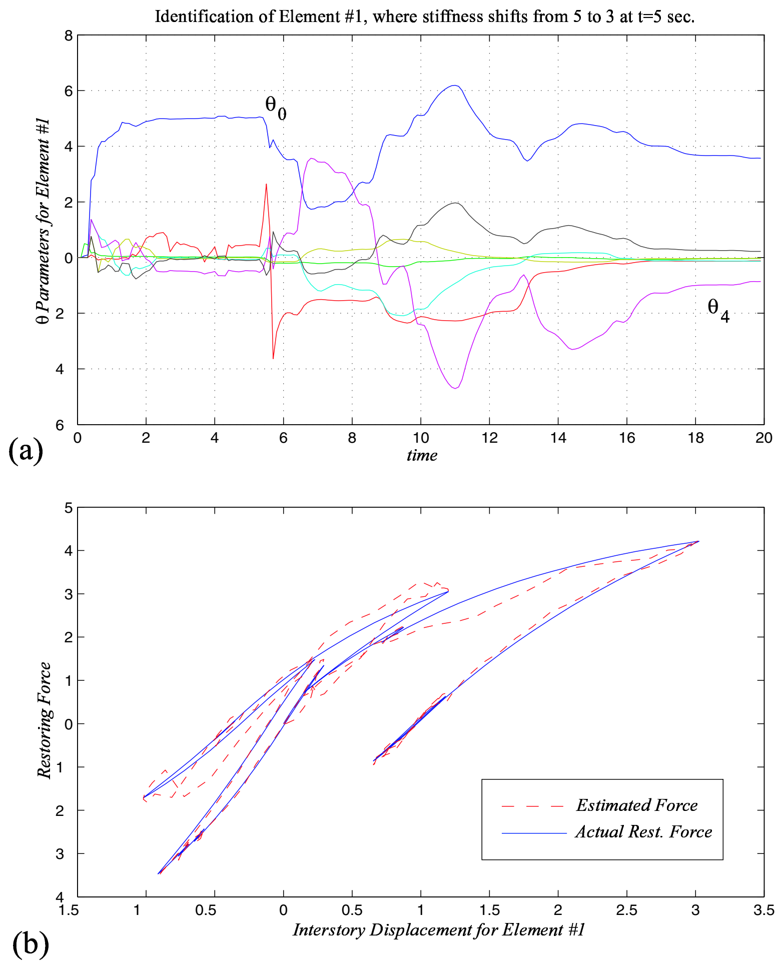


Figure 8: Identification results for the 3DOF system's first story element under base-excitation, and with a time varying stiffness parameter. At $t = 5$ sec. θ_0^* for the element drops from 5 to 3.

3.1.4 Identifiability and Parametric Modeling

The limitations of the parametric approach can be shown for system identification purposes, when either the model is phenomenalogically different from the assumed class of models, or when insufficient measurements are available.

As mentioned previously, there are limitations to using the parametric identification approach for certain system topologies. This is best seen by the following simple example. Given the SDOF system configuration shown in Fig. (9), and the response of the mass to some force excitation, one may wish to identify the properties of both of the elements. If one were to attempt to solve this problem with the method presented here, one would write:

$$\begin{aligned}
\dot{z} = sz = \quad & m(s\ddot{x}) + k_1(sx_1) + c_1(s\dot{x}_1) + d_1(sx_1^3) \\
& - (1/\eta_1) \sum_{n=1}^{n=N} a_{1n}\nu_1(\beta_1|\dot{x}_1||r_1|^{n-1}r_1 - \gamma_1\dot{x}_1|r_1|^n) \\
& + k_2(sx_2) + c_2(s\dot{x}_2) + d_2(sx_2^3) \\
& - (1/\eta_2) \sum_{n=1}^{n=N} a_{2n}\nu_2(\beta_2|\dot{x}_2||r_2|^{n-1}r_2 - \gamma_2\dot{x}_2|r_2|^n)
\end{aligned} \tag{13}$$

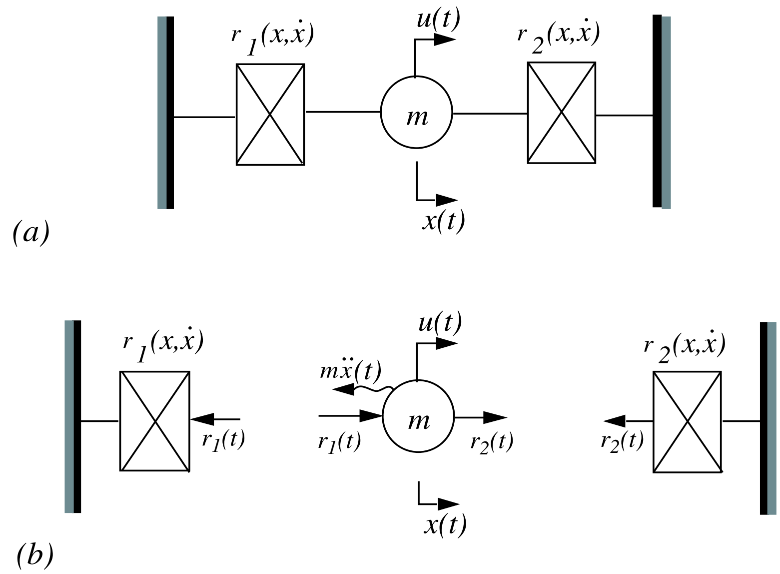


Figure 9: (a) Simple SDOF system which exhibits limitations of on-line hysteretic identification method, (b) free-body diagram of dynamic system.

For example, in the *Mass Unknown* case, $z = u$ and the mass m becomes a parameter to be identified in the $\boldsymbol{\theta}$ vector. Here, all the necessary signals are available for measurement except the element restoring forces r_1 and r_2 themselves. Without the restoring force signal information, the $\bar{\phi}$ signal vector cannot be generated, and therefore the identification scheme cannot be used. Even if the mass were known, one would only be able to indirectly measure $r_1 + r_2$, i.e., the resultant force. The single element SDOF case and the chain-like MDOF examples were solvable because the system topologies permitted an indirect means of obtaining each hysteretic elements' restoring force. Of course, if sensors are installed into a structural system to measure element restoring forces, then such a system would be solvable by this online identification technique. Alternatively, the system's behavior could be parameterized, not in terms involving restoring force response, but in terms of its motion responses only. This may yield satisfactory tracking, but may be of limited use for anything other than control applications.

3.2 Application of Nonparametric Identification

The adaptive neural network modeling technique can be applied to nonlinear systems with increased complexity in the inter-connections of the nonlinear elements. An example of such a complex interconnected system is shown in Fig. 1(d) which is a reduced-order approximation of the extremely general continuous system in Fig. 1(c). In this paper, however, for comparison with the previous MDOF example, the VWNN approach is applied to the simulated 3DOF chain-like system, which is a simplistic representation of a three story building. In this case, all of the interstory elements are hysteretic. Figure 10 shows the real-time adaptation of the algorithm to yield accurate estimates of the interstory element restoring forces. In this case the network was not trained at all before the simulated event, therefore the model is completely unknown *a priori*. Despite this, the network adapts within a few cycles.

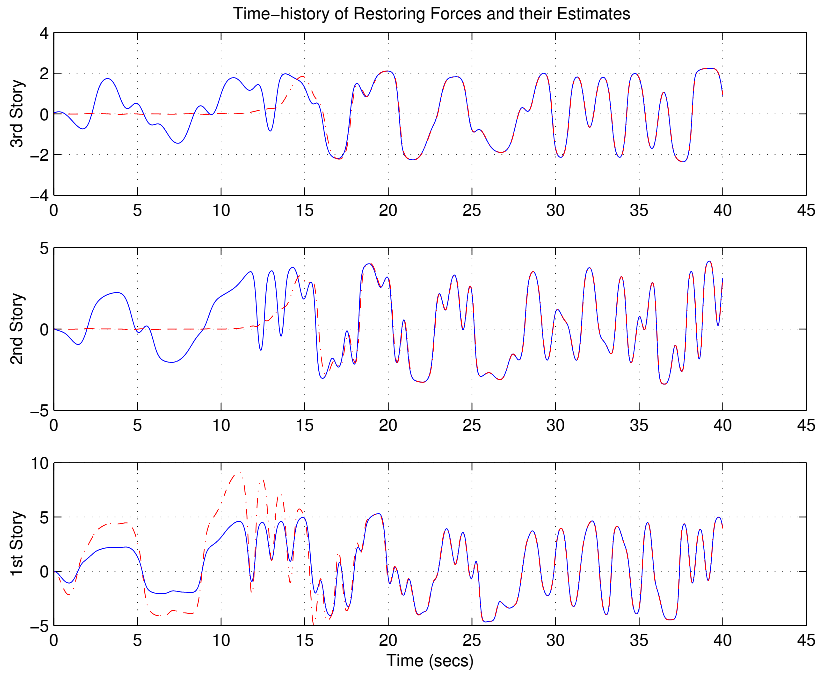


Figure 10: Time History of actual (solid curve) and estimated (dashed curve) restoring forces when adaptation is on for the VWNN adaptive identification approach.

While tracking performance is important, another consideration as mentioned before is the convergence of the model parameters. Figure (11) shows the convergence properties of a subset of the many parameters to be trained in the network. While there are many parameters (and this is a disadvantage), one important advantageous property of this network architecture is clearly demonstrated: that is, that the parameters converge. This is not necessarily to be expected because most neural networks will give non-unique solutions for nonlinear systems. Because this network however is basically a single-layer network, with some pre-defined processing on the front-end, it has very similar convergence properties of regular parametric adaptive models that are linear-in-the-weights. These common conditions for convergence are that the system be *Persistently Excited*. Loosely speaking, *Persistence of Excitation* (*P.E.*) exists if the components of ϕ (or $\bar{\phi}$) are linearly independent, in other words, each component has distinctive signature information (Ioannou & Sun, 1996). This *P.E.* property is related to the *Sufficient Richness* conditions of the system excitation. White noise excitation is an ideally rich input, which, parameterization permitting, will yield *P.E.* conditions.

4 Discussion

4.1 Integration of Measured Data and *a priori* Assumptions

The adaptive identification schemes discussed here depend on measured data from the structural system response. Generally, only one of these signals is measured (usually acceleration), and the other two are obtained by integration and/or differentiation schemes. This is an old problem in the identification of both linear and nonlinear systems, and has been considered by others. The problem is particularly important in

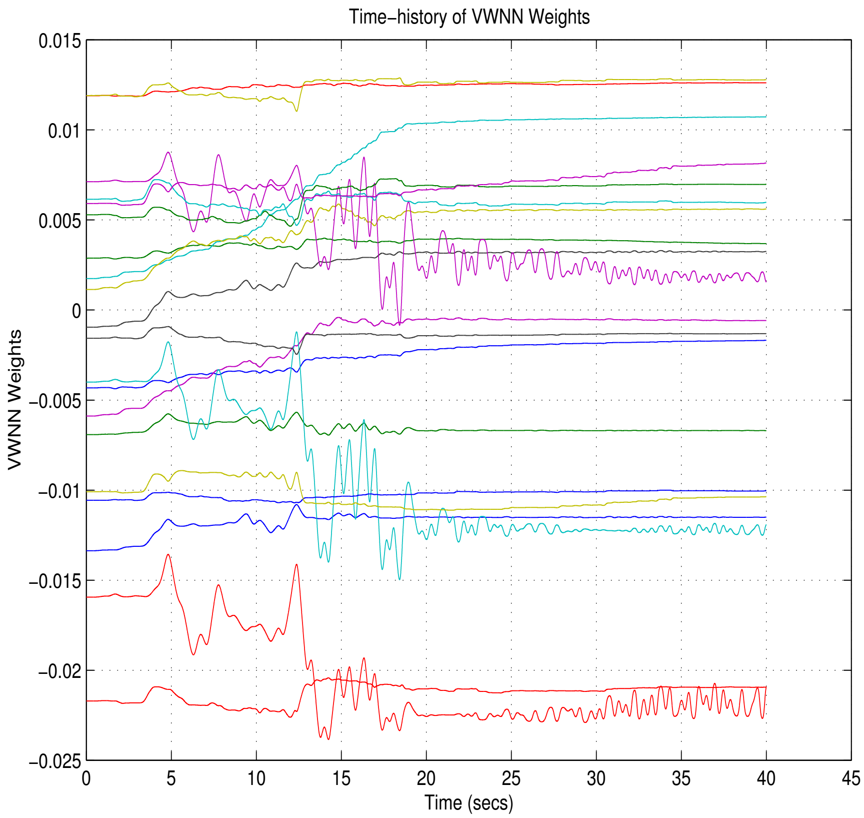


Figure 11: Time History of a subset of the VWNN adjustable parameters.

the context of nonlinear identification, because the nonlinear restoring forces are often modeled as nonlinear functions of velocity and displacement.

Relatively sophisticated methods exist which will yield an accurate displacement and velocity record from an original noise-contaminated acceleration record; however such methods cannot be employed in an efficient, causal regime required by on-line applications. For example, no forwards/backwards filtering can be used to negate effects of phase distortion. No frequency domain filtering can be conducted, because the entire response record must be available. Unfortunately, precisely those discrete time-domain integration schemes which are accurate for integrating the true (uncontaminated) part of a signal, also amplify the noise component in the signal (particularly low-frequency noise). The converse is also true.

To show the effect of distorted measurements on the identification problem, consider the following simple example of a SDOF linear system:

$$m\ddot{x}_k + c\dot{x}_k + kx_k = u_k \tag{14}$$

where the subscript k denotes the discrete sample at $t = t_k$. With measured and integrated acceleration data one will actually be performing identification on the following system:

$$m(\ddot{x}_k + \eta_k) + c(\dot{x}_k + G(z)\eta_k) + k(x_k + G^2(z)\eta_k) = u_k \tag{15}$$

$$m\ddot{x}_k + c\dot{x}_k + kx_k = u_k - (m\eta_k + cG(z)\eta_k + kG^2(z)\eta_k) \tag{16}$$

where $G(z)$ is the discrete transfer function of the integration rule which is used. Notice that if the parameters to be identified are c and k, then they will actually be identified from Eq. (16) rather than from the true system in Eq. (14). The difference between these two equations is the bracketed term on the right of Eq. (16) which could be considered to be like an unknown exogenous input. Clearly, without knowing the noise η, one will not be able to identify the true parameters c and k because one is in effect working with the wrong system equation.

In mechanical engineering applications, often one has the luxury of exciting a structure with a desirably banded input signal, thus not aggrevating some of the low-frequency noise problems discussed above. Unfortunately, in civil engineering applications one has little control over the excitation spectral properties. This integration problem is largely ignored in the literature. However, a few notable exceptions are by Worden [53], Hamming [15], Audenino & Belingardi [2] , Xistris & Kumar [57], Smyth and Pei [43], and others. The issue was explored in Lin *et al.*([30]) for adaptive identification of hysteretic systems, where increasing parameter drift was observed with increased measurement noise level. In that study a third order predictor-corrector integration scheme was used.

In addition to difficulties arising from integrating noise contaminated acceleration measurements, an additional complication often arises when an erroneous *a priori* mass estimate is assumed. The reason for this, is that in nonlinear systems, often the only way of "measuring" the restoring force is indirectly by $r(t) = u(t) - m\ddot{x}(t)$. In this case, the accuracy of the restoring force is highly dependent on the mass estimate m. It has been shown in Smyth *et al.* (1999)([42]), that small errors in the mass estimate can greatly skew the identified parameters. In that study, a 3 story building with nonlinear inter-story elements was studied, and the effects on identification results of erroneous assumed mass were investigated. A Monte Carlo simulation was conducted where the identification was performed with an uncorrelated Gaussian error distribution on each individual mass. The identification model used was a 7 parameter model based on the Bouc-Wen hysteretic model. As the error approached about 5%, clear skewness was exhibited in several identified parameter distributions.

4.2 Trade-offs in setting user-defined parameters in adaptive laws

As illustrated in section 3.1.3, the choice of certain user-defined variables within the adaptive law, e.g., the forgetting factor might have yielded better performance in adapting to sudden changes in the structural model parameters. This forgetting factor, for example, depending upon its value, can place more or less emphasis on recent errors corresponding to the current parameter estimate $\boldsymbol{\theta}$. When the user chooses the necessary design variables within the adaptive law, it is typically not known exactly what the level of system response will be, how drastic and sudden parameter changes will be, and what the measurement noise levels will be. If these were known *a priori* one might be able to choose an optimal adaptive law. Unfortunately this is not generally possible; however, some additional autonomy can be built into the adaptive law, so that it adjusts its own critical variables according to certain criteria. As previously mentioned, a good example of this is the variable forgetting factor implemented in Lin *et al.*([29]), where a balance was struck between smoothness of parameter convergence, and model output tracking.

5 Conclusions

The authors' work in the area of adaptive modeling of nonlinear hysteretic systems has been reviewed in general terms, with an emphasis on fundamental concepts which dictate needed developments in this important area of research. The advantages and disadvantages of parametric and nonparametric modeling approaches are presented and their complementary features discussed. The need for additional sensor information, such as direct velocity or displacement data in addition to the usual acceleration measurements, would go a long way towards helping with real-world data pre-processing problems such as the integration problem discussed earlier. While nonparametric schemes appear to be extremely promising, the *model unknown* approach adopted with the VWNN implementation may be a more severe restriction (in that the method assumes that virtually no information is available concerning the nature of the system being modeled) than real-world applications dictate. If the user had an idea of the class of nonlinearities to be expected in advance of a severe dynamic event, then they could choose a pared down version of what would otherwise be an over-complicated (or stated more technically, an over-parameterized) model. To achieve this, better understanding is required of how nonparametric models are able to match certain types of nonlinearities. In some sense, this means bringer them closer to parametric models, where each parameter has a recognizable meaning.

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