Unrestricted Variables in LP

Variables

x_1 – **loaves of bread baked**

x_2 – **oz. flour bought or sold (positive if bought, negative if sold)**

Objective:

$$\max 30x_1 - 4x_2$$

Constraints:

5 packages of yeast

$$x_1 \leq 5$$

need enough flour to bake x_1 loaves

$$5x_1 \leq 30 + x_2$$

Nonnegativity: x_1 **is nonegative,** x_2 **is unrestricted.**

Can't just run simplex with an unrestricted variable, beacause it won't set x_2 to a negative value.

Dealing with the unrestricted variable

Idea:
Let $x_2^{'}$ be the amount of flour bought
Let $x_2^{''}$ be the amount of flour sold

Now we can replace x_2 with

$$x_2 = x_2^{'} - x_2^{''}$$

$$x_2^{'}, x_2^{''} \geq 0$$

Interpreting the solution: **What if both $x_2^{'}$ and $x_2^{''}$ are positive in solution?**

It can't happen with simplex!

LP

$$
\begin{aligned}
\textbf{maximize} \quad & 30x_1 - 4x_2' + 4x_2'' && (1) \\
\textbf{subject to} \quad & \\
& 5x_1 - x_2' + x_2'' \leq 30 && (2) \\
& x_1 \leq 5 && (3) \\
& x_1 , x_2' , x_2'' \geq 0 && (4)
\end{aligned}
$$

Put into standard form:

$$
\begin{aligned}
z &= 30x_1 - 4x_2' + x_2'' && (5) \\
s_1 &= 30 - 5x_1 + x_2' - x_2'' && (6) \\
s_2 &= 5 - x_1 && (7)
\end{aligned}
$$

Choose x_1 to enter

Choose s_2 to exit

Iteration 1

$$
\begin{aligned}
z &= & & 30x_1 &-& 4x_2' &+& x_2'' \quad (8)\\
s_1 &= 30 &-& 5x_1 &+& x_2' &-& x_2'' \quad (9)\\
s_2 &= 5 &-& x_1 & & & & \quad (10)
\end{aligned}
$$

Choose x_1 to enter
Choose s_2 to exit

$$
\begin{aligned}
z &= 150 &-& 30s_2 &-& 4x_2' &+& x_2'' \quad (11)\\
s_1 &= 5 &+& 5s_2 &+& x_2' &-& x_2'' \quad (12)\\
x_1 &= 5 &-& s_2 & & & & \quad (13)
\end{aligned}
$$

Choose x_2'' to enter
Choose s_1 to exit

Iteration 2

$$\begin{aligned}
z &= 150 - 30s_2 - 4x_2^{'} + x_2^{''} && (14)\\
s_1 &= 5 + 5s_2 + x_2^{'} - x_2^{''} && (15)\\
x_1 &= 5 - s_2 && (16)
\end{aligned}$$

Choose $x_2^{''}$ to enter
Choose s_1 to exit

$$\begin{aligned}
z &= 170 - 10s_2 - 4s_1 && (17)\\
x_2^{''} &= 5 + 5s_2 + x_2^{'} - s_1 && (18)\\
x_1 &= 5 - s_2 && (19)
\end{aligned}$$

So we set $x_1 = 5$ and $x_2 = x_2^{'} - x_2^{''} = 0 - 5 = -5$. We bake 5 loaves and sell 5 oz. of flour. Total profit 170

General Rule

1. For each unrestricted variable x_i, introduce 2 variables, $x_i^{'}$ and $x_i^{''}$.

2. Use the equation $x_i = x_i^{'} - x_i^{''}$ to replace x_i in the LP.

3. Add the non-negativity constraint, $x_i^{'} \geq 0$ and $x_i^{''} \geq 0$

4. Solve the modified LP.

5. In the final LP solution

(a) if $x_i^{'} = x_i^{''} = 0$ then set $x_i = 0$.

(b) if $x_i^{'} > 0$ and $x_i^{''} = 0$, then set $x_i = x_i^{'}$.

(c) if $x_i^{'} = 0$ and $x_i^{''} > 0$, then set $x_i = -x_i^{''}$.