

Homework #3: Problems on the Normal Distribution

Problem 1. The plant manager of a manufacturing facility is concerned about drug use among plant workers and plans to implement random drug testing. One of the tests to be applied measures the level of factor- X in blood samples. Among recent users of cocaine, the level of factor- X is normally distributed with a mean of 10.0 and a standard deviation of 1.3. Among non-users, the level is normally distributed with a mean of 6.75 and a standard deviation of 1.5.

The employer plans to send a warning letter to all employees with a factor- X level of x or greater, with x to be determined.

- a. Find the value of x that will ensure that 90% of recent cocaine users will be sent a warning letter.
- b. If your answer to part (a) is adopted, what proportion of non-users will also be sent warning letters? [If you could not answer part (a), use $x = 9$.]
- c. Daniel Bernoulli is a worker in the plant and a recent cocaine user. What is the probability that he will not be sent a warning letter, based on his factor- X level, assuming that x in part (a) is chosen correctly.

Problem 2. The Uris Index has a return of X_1 and the Hermes Index has a return of X_2 , both over the next year. Suppose these random returns have the following features:

- o X_1 has a mean of 0.15 and a standard deviation of 0.04.
 - o X_2 has a mean of 0.19 and a standard deviation of 0.08.
 - o The correlation between the two returns is 0.40.
 - o The returns are normally distributed.
- a. A pension fund manager will invest a fraction p of the fund's wealth in X_1 and the remainder in X_2 . Let Y be the pension fund's return. How should p be chosen to achieve an expected return $E[Y]$ of 0.16?
 - b. What is the resulting standard deviation of Y , i.e., of the pension fund's return? (If you could not answer (a), use $p = 1/2$.)
 - c. The pension fund manager's performance will be evaluated relative to the performance of the Uris Index; specifically, if $Y \geq 1.3X_1$, that is, the pension fund's return exceeds the Uris Index return by 30% or more, the fund manager will receive an enormous bonus. If your answer to (a) is adopted, what is the probability that the manager will get the enormous bonus? (If you could not answer (a), use $p = 1/2$.)

Problem 3. From a study of its catalog customer base, the *J. Uriis* clothing company has determined that catalog orders placed by men are normally distributed with a mean of \$62.00 and a standard deviation of \$12.00, whereas those placed by women are normally distributed with a mean of \$84.00 and a standard deviation of \$16.00.

During March and April, *J. Uriis* wants to send its special swimsuit catalog to women placing new orders. Rather than try to determine whether an individual order was placed by a man or a woman, the company will simply send the swimsuit catalog to all customers ordering more than x dollars of merchandise, with x to be determined.

a. Find the value of x that will ensure that 80% of women placing orders will be sent the swimsuit catalog.

b. If your answer to part (a) is adopted, what proportion of men placing orders will also receive the swimsuit catalog? [If you could not answer part (a), use $x = 82$.]

Problem 4. Suppose X is normally distributed with mean 1 and standard deviation 0.25, and Y is also normal with mean 1.5 and standard deviation 0.4. Suppose that X and Y have correlation coefficient 0.6. Find the following probabilities:

(a) $P(X \geq 1.3)$

(b) $P(X + Y \geq 2.5)$

(c) $P(X + Y \geq 3)$

(d) $P(Y - X \leq 0)$

(e) $P(Y \leq X)$