Low-Wage Employment Subsidies in a Labor-Turnover Model of the 'Natural Rate'

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Abstract

This paper models two kinds of wage subsidy in a model of the natural rate having a continuum of workers ranked by their productivity—a flat wage subsidy and a graduated wage subsidy, each financed by a proportional payroll tax. In the small open economy case, with the graduation as specified, we show that both schemes expand employment throughout the distribution; for those whose productivity is sufficiently far below the mean, take-home pay is unambiguously up, though the tax financing lowers take-home pay at the mean and above. For any particular class of workers paid the same amount of the wage subsidy under the two plans, the graduated plan expands employment more. In the closed economy case, employment is increased for workers whose productivity levels are below or equal to the mean but the interest rate is pulled up, and that may cause employment to fall at productivity levels sufficiently far above the mean. (*JEL* E24, H22)

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There is considerable agreement that the extraordinarily low commercial productivity of active-age persons in the lower reaches of the distribution relative to median productivity is the number one social problem of our time. In creating a huge wage gap it makes the less productive incapable of supporting a family or in some cases themselves (in a way meeting community standards of decency at any rate) and having access to mainstream community life. In reducing the wage incentives that private enterprise can afford to offer low-wage workers relative to their other resources and attractions, it worsens unemployment and nonparticipation. Both sets of effects operate in turn, especially in areas where there is a high concentration of these effects, to increase dependency on welfare and property crime, spread drug use and violence, widen illegitimacy and blight the upbringing of children (Freeman, 1996; Murray, 1984; Phelps, 1994b, 1997; Wilson, 1996).

There is far less agreement on what, if anything, would be useful to do about it. An important line of thinking, however, looks to wage subsidies of one kind or another. The pioneers were Arthur Pigou (1933) and Nicholas Kaldor (1936), who sought the conditions for employment subsidies to be self-financing. Targeted hiring subsidies were championed by Daniel Hammermesh (1978), Michael Hurd and John Pencavel (1981) and by Robert Haveman and John Palmer (1982). The employment-expanding effects of a constant employment subsidy were studied by Richard Jackman and Richard Layard (1986). One of us argued informally for a graduated employment subsidy to raise low-end wage rates (Phelps, 1994a) and to reduce unemployment (Phelps, 1994b) as a counterweight to the welfare system. A hiring subsidy targeted at the long-term unemployed has been championed by Dennis Snower (1994). Wage subsidies were urged to counter the effects of payroll taxes by Jacques Dréze and Edmond Malinvaud (1994). Christopher Pissarides (1996) has studied the effects of such tax relief.

These analyses focus on the subsidies' near-term effects. None of the papers expressly argues that there would be a permanent effect on unemployment. Some of the authors may have thought the effect was only temporary but a way to buy valuable time. To study the long-term effects, however, requires an intertemporal model in which workers accumulate wealth and firms invest in capital of one or more kinds according to expectations of the future and interest rates.

As a comparative exercise, the first section undertakes a neoclassical analysis of the effects in the steady state of a flat (constant) subsidy, financed by a proportional payroll tax on the equilibrium level of manhours supplied. We show that wealth decumulation serves ultimately to eliminate the employment decline first brought by the tax and wealth accumulation operates to eliminate all the employment gains brought by the subsidy. The employment is ultimately neutral although the take-home wage is increased for low-wage workers.

We then shift to the theory of the natural rate of unemployment. Using our labor-turnover model, with its incentive wage, we study two employment subsidies: a flat (constant) subsidy and a graduated subsidy that decreases with the wage rate and vanishes asymptotically at the top—each program financed by a flat-rate payroll tax (as if no reflow of budgetary savings and revenue gains resulted). In this model (Phelps, 1968, 1994c; Hoon and Phelps, 1992), quitting by employees poses an incentive problem for the firm, since it must invest in the firm-specific training of workers to make them functioning employees and such an investment is lost whenever an employee quits. The problem prompts firms to drive up the going wage. This leads in turn to involuntary unemployment in labor-market equilibrium. Our 1992 paper posited worker-savers in overlapping cohorts to obtain a general-equilibrium framework with which to endogenize the rate of interest or the accumulation of net foreign assets. The present paper introduces a continuum of workers differentiated by productivity in each cohort.

The gist of our findings can be indicated. Due to incentive-wage considerations, the two schemes permanently expand employment in the long run. The proportional payroll tax used to finance the subsidy is neutral for employment. With employment unchanged, the payroll tax lowers take-home pay in the same proportion for every type of worker but nonwage income is also reduced by the same proportion. As a result, the incentive-wage condition is invariant to the proportional payroll tax in the long run. The subsidy, however, is nonneutral. If, before, a penny increase in hourly labor compensation by the firm had a marginal benefit equal to marginal cost at the original employment rate, it must now have a marginal benefit less than the marginal cost as that additional penny now has a smaller impact on quitting. Hence firms lower the incentive pay and as a result employment is expanded throughout the distribution in the long run.

For low-wage workers, there is an added boost to employment in the short run. Given net wealth and the interest rate, the higher take-home pay induces a decline in the propensity to quit. The result is a rightward shift of the zero-profit curve and an additional rightward shift of the incentive-wage curve on top of the wedge caused by the subsidy. In the long run, wealth accumulation leads to a proportionate rise of nonwage income at given employment, thus shifting back the zero-profit curve to its original position. The incentive-wage curve also shifts leftward but only by enough to leave a wedge caused by the subsidy. The net result, then, for low-wage workers is that the expansionary effect on employment is even larger in the short run than in the long run.

The long-run question in the closed-economy case is the subsidies' effect on the rate of interest and the effect in turn on wages and employment. Here we find that if the zero-profit curve is elastic, aggregate wealth supply is increased, but it increases by less than the increase in asset demand. The result is a rise in the rate of interest. However, for workers whose productivity levels are below or equal to the mean, employment is expanded; at productivities far enough below the mean, take-home wages will rise.

It is also found that the graduated scheme, besides having (for the same subsidy rate at the bottom) a lighter budgetary burden than the constant subsidy, has an extra downward impact on hourly labor cost, as firms moderate wage rates above the bottom to win a larger subsidy, with the result that employment receives an extra boost. Such an effect raises the fear that some middle-wage workers would see their wage reduced on balance. We show, however, that unless the subsidy tapers off too fast no such wage effect occurs. Finally, we show that the gross hiring rate is increased the most for low-wage workers.

The paper is organized as follows. Section I analyzes the effects of a flat (constant) subsidy in a neoclassical model. Section II presents the basic features of the labor-turnover model with a continuum of workers exhibiting constant marginal training cost. Section III studies the incidence of the subsidies in the steady-state, general-equilibrium model of the small open economy while Section IV analyzes the closed economy case. Section V briefly discusses the case of rising marginal training cost in the small open economy case. Section VI concludes.

I. Neoclassical Theory

We follow the treatment by Olivier Blanchard (1985) of finitely-lived agents with no bequest in a one-sector setup. (See George Kanaginis and Phelps, 1994 and Phelps, 1994c, chapter 16.) In each cohort, the workers form a continuum when ranked by their respective potential productivity levels. The productivity, or ability, of worker input at location i in this continuum is measured by a labor-augmenting, hence Harrod-neutral, parameter denoted Λ_i. There is a known and unvarying distribution of Λ_i in the working population, which we normalize to one. The proportion of workers with productivity level Λ_i or less is $G(\Lambda_i)$ and the density function is $g(\Lambda_i) = G'(\Lambda_i)$. We call a worker with productivity level Λ_i a type-i worker.

Each agent of type i derives utility from consumption and leisure, which we assume are additively separable and take the log form. He has a finite life and faces an instantaneous probability of death θ that is constant throughout life. Solving the agent's problem, and denoting aggregate variables by capital letters, we obtain $C_i = (\theta + \rho)[H_i + W_i]$ and $(\bar{L} - L_i)/C_i = 1/v_i^h$, where C_i is consumption, L_i is labor supply, H_i is human wealth, and W_i is nonhuman wealth per member of the type-i workforce. Here ρ is the time preference parameter, $\bar{L}$ is total time available, and v_i^h is the real hourly household wage received by a type-i worker, which is related to the hourly labor cost to the firm of a type-i worker, v_i^f, by $v_i^f \equiv (1 + \tau)v_i^h - s_i$, τ being the proportional payroll tax rate. Under the flat subsidy scheme, s_i equals s^{F}, a constant. Under a graduated subsidy scheme, s_i is a decreasing function of the wage paid by the firm to each type-i worker, denoted $s_i = S(v_i^f)$, and tapers off asymptotically. We impose throughout the conditions $S'(v_i^f) < 0$ and $|S'(v_i^f)| < 1$.

In the small open economy, the path of the domestic interest rate conforms to the exogenously given world interest rate, r^*: $r = r^*$, r^* a constant > 0. The level of net external assets adjusts endogenously to bring about this condition. The steady-state H_i equals $v_i^h L_i/(r^* + \theta)$ and nonwage income of a type-i worker is given by $y_i^w \equiv (r^* + \theta)W_i$, θW_i being actuarial dividend. In the steady state, setting $\dot{C}_i = 0$, we also have $r^* = \rho + [\theta(\theta + \rho)W_i/C_i]$. This can be rewritten, after some substitutions, as

$$r^* = \rho + \frac{\theta}{1 + (\frac{v_i^h \bar{L}}{y_i^w})(\frac{L_i}{\bar{L}})}. \tag{1}$$

The steady-state labor-supply relation in manhours can also be expressed as

$$\frac{L_i}{\bar{L}} = \frac{1 - [\frac{\theta+\rho}{r^*+\theta}](\frac{v_i^h \bar{L}}{y_i^w})^{-1}}{1 + [\frac{\theta+\rho}{r^*+\theta}]}. \tag{2}$$

Turning to the production side, let the production function be written as $Y = [\int_{\underline{\Lambda}}^{\infty} \Lambda_i L_i g(\Lambda_i) d\Lambda_i] f(K/\int_{\underline{\Lambda}}^{\infty} \Lambda_i L_i g(\Lambda_i) d\Lambda_i)$, where $\underline{\Lambda}$ is the minimum productivity level, and K is capital stock. Firms' optimal choice of labor and the capital-labor ratio, $k \equiv (K/\int_{\underline{\Lambda}}^{\infty} \Lambda_i L_i g(\Lambda_i) d\Lambda_i)$, imply

$$\frac{v_i^f}{\Lambda_i} = f(k) - kf'(k); \tag{3}$$

$$r^* = f'(k). \tag{4}$$

The given world interest rate, r^*, pins down the optimal capital-labor ratio, k. Consequently, the wage paid by the firm, v_i^f, is pinned down, being directly proportional to Λ_i. Observe that the wage-to-nonwage income ratio in (1) is an implicit function of r^* at each L_i:

$$\frac{v_i^h \bar{L}}{y_i^w} = \Upsilon(r^* - \rho, \frac{L_i}{\bar{L}});\ \Upsilon_1 < 0,\ \Upsilon_2 < 0. \tag{5}$$

Using this in (2), we obtain a reduced-form labor supply relation in the steady state:

$$\frac{L_i}{\bar{L}} = \frac{1 - [\frac{\theta+\rho}{r^*+\theta}][\Upsilon(r^* - \rho, \frac{L_i}{\bar{L}})]^{-1}}{1 + [\frac{\theta+\rho}{r^*+\theta}]}. \tag{6}$$

This equation uniquely determines the labor supply in manhours and is independent of the tax and subsidy rates. It is also independent of Λ_i.

To understand this result, we notice that the labor demand curve in the $(L_i/\bar{L}, v_i^f)$ plane is infinitely elastic. With wealth and hence y_i^w given, the labor supply schedule is upward sloping. Under a balanced-budget policy, the flat subsidy case yields a convenient expression for the tax rate, namely, $\tau = s^{\text{F}}/v_{\text{mean}}^h$, where $v_{\text{mean}}^h \equiv \int_{\underline{\Lambda}}^{\infty} v_i^h g(\Lambda_i) d\Lambda_i$. For an employee whose $\Lambda_i < \Lambda_{\text{mean}}$, the tax liability (τv_i^h) is therefore less than the subsidy (s^{F}). Hence, at given y_i^w, a low-wage worker increases his equilibrium labor supply. Wealth accumulation then brings his y_i^w up until the original v_i^h/y_i^w is restored. On the other hand, for an employee whose $\Lambda_i > \Lambda_{\text{mean}}$, his v_i^h is reduced. Such a high-wage worker decumulates wealth until once again the original v_i^h/y_i^w is restored. Thus, in the long run, the tax-subsidy scheme is neutral for employment for all workers throughout the distribution. A similiar argument holds for the graduated subsidy scheme.

In the closed-economy case, the essential task is to endogenize the rate of interest. One approach to the problem is to work toward a diagram involving an asset demand curve and a wealth supply schedule, the intersection giving us the general-equilibrium rate of interest. Using the following two conditions:

$$r = \rho + \frac{\theta}{1 + (\frac{v_i^h \bar{L}}{y_i^w})}, \tag{7}$$

$$\frac{L_i}{\bar{L}} = \frac{1 - [\frac{\theta+\rho}{r+\theta}](\frac{v_i^h \bar{L}}{y_i^w})^{-1}}{1 + [\frac{\theta+\rho}{r+\theta}]}, \tag{8}$$

we prove in the Appendix that we can write $L_i/\bar{L}$ as a decreasing function of r, given ρ and θ, that is,

$$\frac{L_i}{\bar{L}} = \psi(r; \rho, \theta);\ \psi'(r) < 0. \tag{9}$$

Using the firm's optimal condition $r = f'(k)$ and (9), the aggregate asset demand given by

$$A = k \int_{\underline{\Lambda}}^{\infty} \Lambda_i \bar{L} \psi(r) g(\Lambda_i) d\Lambda_i \tag{10}$$

is decreasing in r.

The average supply of wealth per member of the type-i workforce is obtained by substituting $y_i^w \equiv (r + \theta)W_i$ in (7):

$$W_i = [\frac{(v_i^h \bar{L})(\frac{L_i}{\bar{L}})}{r + \theta}][\frac{r - \rho}{\theta + \rho - r}]. \tag{11}$$

Excluding the case where $r - \rho > \theta$, we have a well-defined steady state with the righthand side of (11) being unambiguously positive. Observe that the first bracketed term in (11) is simply human wealth per type-i worker, and for a given after-tax real wage $(v_i^h L_i)$, human wealth, H_i, is decreasing in r. On this account, W_i falls as r rises. On the other hand, a rise of r has a positive effect on W_i on account of the second bracketed term, W_i/H_i. The total supply of wealth per worker is given by $W \equiv \int_{\underline{\Lambda}}^{\infty} W_i g(\Lambda_i) d\Lambda_i$. Using (11), we obtain

$$W = [\frac{r - \rho}{(\theta + \rho - r)(r + \theta)}] \int_{\underline{\Lambda}}^{\infty} v_i^h L_i g(\Lambda_i) d\Lambda_i.$$

Under a balanced-budget, we get

$$W = [\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}] \int_{\underline{\Lambda}}^{\infty} v_i^f L_i g(\Lambda_i) d\Lambda_i. \tag{12}$$

Using $(v^f/\Lambda_i) = f(k) - kf'(k)$ and (9), and noting that k is a decreasing function of r, we obtain an expression of total wealth supply as a function of the rate of interest:

$$W = [\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}] \int_{\underline{\Lambda}}^{\infty} [f(k) - kf'(k)]\psi(r)\bar{L}\Lambda_i g(\Lambda_i) d\Lambda_i. \tag{13}$$

What is the shape of the supply of wealth? There are two opposing forces. In the general equilibrium, an increase of r lowers the real wage as well as the supply of manhours; and, as remarked above, it lowers the present value of these expected earnings. So human wealth is reduced. However, the second braketed term in (11) works to increase desired supply of wealth as r rises. At r sufficiently low that W_i is at or near zero, the former effects are outweighed by the latter though at sufficiently high r the opposite may occur. Hence the per worker supply of wealth schedule is upward-sloping initially but at very high r may bend backward. In the same plane, per worker demand for the domestic assets is downward-sloping. We will suppose that the equilibrium r is unique or that only the lowest equilibrium r is empirically relevant. (See Figure 1.) The important thing to observe from (10) and (13) is that the pair of equations are independent of the tax-subsidy parameters. Hence the balanced-budget tax-subsidy policy is neutral for the rate of interest, and, consequently, also neutral for employment.[1] Nevertheless, for low-wage workers, their take-home pay is increased.

II. Basic Features of the Economy in Modern-Equilibrium Theory

The preceding neoclassical theory has difficulty explaining why, under plausible assumptions, the policy shift and other aggregate shocks experienced in recent decades should cause large changes in equilibrium labor input

[1] Another way to see that the policy is neutral for the rate of interest is to use the requirement that aggregate supply be equal to aggregate demand. Equating the aggregate demand to aggregate supply in the equation, $(r-\rho)\int_{\underline{\Lambda}}^{\infty} C_i g(\Lambda_i) d\Lambda_i = \theta(\theta+\rho)\int_{\underline{\Lambda}}^{\infty} W_i g(\Lambda_i) d\Lambda_i$, we obtain, $r = \rho + [\theta(\theta+\rho)k/f(k)]$, which, noting that k is decreasing in r, determines the general-equilibrium r independently of the tax-subsidy parameters.

and national income. The theory does not allow for *unemployment*; rather changes in labor input are attributable entirely to variations in the work week.

To study the effects of the tax-subsidy schemes on the equilibrium path of unemployment, we need to draw on modern-equilibrium theory, which sees unemployment as structural in nature and traces its vicissitudes to changes in the structure of the economy (Phelps, 1994c). At the center of this theory is the relationship between the firm and the employee arising from their incentives in the modern setting of asymmetric information. The economics of incentive (or efficiency) wages plays a key role in generating involuntary unemployment and shaping its equilibrium path.

There are many identical firms. For convenience we may think of them in fixed number (normalized to one) and equal in size. Consider the representative firm j. Its problem is to choose the wage and hiring-training policies that maximize

$$\int_0^\infty \int_{\underline{\Lambda}}^\infty N_{jit}\{\Lambda_i[1-\beta h_{jit}] - v^f_{jit}\}g(\Lambda_i)e^{-\int_0^t r_\nu d\nu} d\Lambda_i dt,$$

which is the present value of the stream of real quasirents, subject to

$$\dot{N}_{jit} = N_{jit}[h_{jit} - \zeta(\frac{z^{he}_{it}}{v^h_{jit}}, \frac{y^w_{it}}{v^h_{jit}}) - \theta]$$

and given N_{ji0}. Note that s_i is implicit in v^h_{ji} and v^f_{ji}, given τ. (Since to simplify we will initially work with constant marginal training cost, we also assume that h_{jit} is bounded, $0 \leq h_{jit} \leq \bar{h}$.) Here, N_{jit} is the stock of type-i employees at the representative firm j taken as a ratio to the type-i workforce (equivalently, the rate of employment among type-i workers), βh_{jit} is the fraction of their working time type-i employees devote to training new hires, h_{jit} is the gross hiring rate of new type-i recruits, ζ similiarly measured is the quit rate, and z^{he}_{it} is a proxy for the expected value of real wage earnings of a type-i worker employed at firm j if he quits.[2]

[2]The quit rate function has the following first derivatives: $\zeta_1 > 0$ and $\zeta_2 > 0$. By virtue of the firm's second-order condition for maximization, $\zeta_{11} > 0$ and $\zeta_{22} > 0$. We also make the assumption that an increase in the nonwage income raises a worker's marginal propensity to quit with respect to wage prospects elsewhere, that is, $\zeta_{12} > 0$.

We may write the current-value Hamiltonian as

$$\int_{\underline{\Lambda}}^{\infty}\{\Lambda_i[1-\beta h_{jit}]-v_{jit}^f+q_{jit}[h_{jit}-\zeta(z_{it}^{he}/v_{jit}^h, y_{it}^w/v_{jit}^h)-\theta]\}N_{jit}g(\Lambda_i)d\Lambda_i,$$

where q_{jit} is the co-state variable.[3] It measures the shadow value of a type-i worker after training by the employer. First-order necessary conditions (which are also sufficient under our assumptions) are given by

$$\left.\begin{array}{ll} h_{jit}=\bar{h} & \text{if } q_{jit}>\Lambda_i\beta; \\ h_{jit}=0 & \text{if } q_{jit}<\Lambda_i\beta; \\ h_{jit}\in[0,\bar{h}] & \text{if } q_{jit}=\Lambda_i\beta; \end{array}\right\} \tag{14}$$

$$N_{jit}\{-1+q_{jit}[(\frac{z_{it}^{he}}{v_{jit}^{h2}})\zeta_1+(\frac{y_{it}^w}{v_{jit}^{h2}})\zeta_2]\frac{dv_{jit}^h}{dv_{jit}^f}\}=0; \tag{15}$$

$$\dot{q}_{jit}-r_t q_{jit}=-\{\Lambda_i-v_{jit}^f-q_{jit}[\zeta(\frac{z_{it}^{he}}{v_{jit}^h},\frac{y_{it}^w}{v_{jit}^h})+\theta]\}; \tag{16}$$

$$\lim_{t\to\infty}\exp^{-\int_0^t r_\nu d\nu} q_{jit}N_{jit}g(\Lambda_i)=0. \tag{17}$$

The equations represented by (14) characterize the optimal number of new hires. In the case arising in the steady-state analysis below, the shadow value of a trained worker is equal to the marginal training cost in output terms. Equation (15) gives the optimal real wage-turnover cost trade-off, equating the marginal cost of raising v_i^f to the marginal benefit. Equation (16) relates the shadow value of functional employees to the total marginal benefit of having one more employee. The transversality condition is in (17). These equations summarize the conditions that have to be satisfied for the typical firm.

To move to the equilibrium conditions, we use the Salop-Calvo approximation for z_{it}^{he}, namely, $z_{it}^{he}=N_{it}^e v_{it}^{he}$. (Using the exit rate from the unemployment pool would not differ in the steady state.) On any equilibrium (correct-expectations) path with identical firms, $v_{jit}^h=v_{it}^h=v_{it}^{he}$ and $N_{jit}=1-u_{it}\equiv N_{it}=N_{it}^e$. Hence we obtain a subsystem of equations in

[3]The flow of output at firm j is then given by $\int_{\underline{\Lambda}}^{\infty}\Lambda_i[1-\beta h_{jit}]N_{jit}g(\Lambda_i)d\Lambda_i$.

the equilibrium path of the economy. For any exogenously given path of the instantaneous real interest rates, this subsystem is

$$\dot{q}_{it} = q_{it}[\zeta(N_{it}, \frac{y^w_{it}}{v^h_{it}}) + \theta + r_t] - [\Lambda_i - v^f_{it}]; \tag{18}$$

$$\dot{N}_{it} = N_{it}[h_{it} - \zeta(N_{it}, \frac{y^w_{it}}{v^h_{it}}) - \theta]; \tag{19}$$

$$N_{it}\{-1 + q_{it}[(\frac{N_{it}}{v^h_{it}})\zeta_1 + (\frac{y^w_{it}}{v^{h2}_{it}})\zeta_2]\frac{dv^h_{it}}{dv^f_{it}}\} = 0. \tag{20}$$

III. Open-Economy Incidence of Tax-Subsidy Schemes

In steady state, $\dot{N}_{it} = 0$. This and (19) give the steady-state employment (SSE) condition that hires balance quits and mortality:

$$h_i = \zeta(N_{it}, \frac{y^w_{it}}{v^h_{it}}) + \theta. \tag{21}$$

This implies that $q_i = \Lambda_i\beta$.

With $\dot{q}_{it} = 0$ in (18) and $q_i = \Lambda_i\beta$, the zero-profit (ZP) condition that quasirents cover interest and depreciation on training becomes

$$\frac{v^f_i}{\Lambda_i} = 1 - \beta[\zeta(N_i, \frac{y^w_i}{v^h_i}) + \theta + r^*], \tag{22}$$

where r^* is substituted for the domestic interest rate. Since quitting is increasing in N_i and y^w_i, the zero-profit wage must be decreasing in those variables.

Assuming that the employment rate is always strictly positive, we obtain from (20) the incentive-wage (IW) condition that gives the incentive-pay level minimizing cost. The cost per employee of paying a penny more in annual wages is one. The cost saving, or benefit, per employee of doing so is the opportunity cost of replacing each defector, $\beta\Lambda_i$, times the number of annual quits per employee that would be saved. Equating these two gives

$$1 = \beta\Lambda_i[N_i\zeta_1 + (\frac{y^w_i}{v^h_i})\zeta_2][(\frac{1}{v^h_i})(\frac{dv^h_i}{dv^f_i})]. \tag{23}$$

The flat (constant) subsidy case gives

$$\begin{aligned} 1 &= \beta\Lambda_i[N_i\zeta_1 + (\frac{y_i^w}{v_i^h})\zeta_2][\frac{\frac{1}{1+\tau}}{\frac{v_i^f + s^{\mathrm{F}}}{1+\tau}}] \\ &= \beta\Lambda_i[N_i\zeta_1 + (\frac{y_i^w}{v_i^h})\zeta_2][\frac{1}{v_i^f + s^{\mathrm{F}}}], \end{aligned} \tag{24}$$

since $dv_i^h/dv_i^f \equiv 1/(1+\tau)$ and $v_i^h \equiv (v_i^f + s^{\mathrm{F}})/(1+\tau)$, while the graduated subsidy case gives

$$1 = \beta\Lambda_i[N_i\zeta_1 + (\frac{y_i^w}{v_i^h})\zeta_2][\frac{1+S'(v_i^f)}{v_i^f + S(v_i^f)}]. \tag{25}$$

Notice that (25) can be satisfied as an equality only if $|S'(v_i^f)| < 1$. If $|S'(v_i^f)| > 1$, each firm would find it profitable to drive the wage all the way down in order to gain a higher subsidy.

The third general-equilibrium condition arises from the firms' assets. The assets are the investments in their employees, the ownership claims to which—the equity shares—generate nonwage income and have an equilibrium value. As before, we use the Blanchard-Yaari setup to generate, in steady state, the equation:

$$r^* = \rho + \frac{\theta}{1 + (\frac{v_i^h}{y_i^w})N_i}. \tag{26}$$

This condition makes the nonwage-income-to-wage ratio an implicit function of the unemployment rate and of the interest rate:

$$\frac{y_i^w}{v_i^h} = \Omega(r^* - \rho, N_i),\ \Omega_1 > 0, \Omega_2 > 0. \tag{27}$$

Long-Run Effects of the Flat Subsidy

Substituting (27) into (22) and (24) gives the reduced-form system in the flat-subsidy case:

$$\frac{v_i^f}{\Lambda_i} = 1 - \beta[\zeta(N_i, \Omega(r^* - \rho, N_i)) + \theta + r^*], \tag{28}$$

$$\frac{v_i^f}{\Lambda_i} + \frac{s^{\mathrm{F}}}{\Lambda_i} = \beta[N_i\zeta_1(N_i, \Omega(r^* - \rho, N_i)) + \Omega(r^* - \rho, N_i)\zeta_2(N_i, \Omega(r^* - \rho, N_i))]. \tag{29}$$

Suppose that initially the ad valorem payroll tax rate is zero and the subsidy is also zero. Equation (28) can be represented as a downward-sloping zero-profit schedule while (29) can be depicted as an upward-sloping wage curve in the Marshallian plane shown in Figure 2. Examining (26), and recalling that in the absence of the tax-subsidy scheme $v_i^h \equiv v_i^f$, notice that we can also draw a family of hyperbolas in Figure 1 with each hyperbola lying north-east corresponding to a higher level of y_i^w. Note also that when the ZP curve cuts the hyperbola from below, as we have drawn in Figure 2, the labor-cost elasticity of labor demand is implied to exceed one. (In that case, as we shall see, the proportionate increase of N_i effected by the subsidy exceeds the proportionate decrease of v_i^f/Λ_i that the increased N_i induces so that, on balance, the product $(v_i^f/\Lambda_i)N_i$ is up.) The algebraic slope of the zero-profit curve is given by $-\beta[\zeta_1 + \zeta_2\Omega_2]$, which, in the absence of any other factors leading to diminishing returns to labor, depends only on the sensitivity of the quit function to the economy-wide rate of employment (or unemployment). The zero-profit curve slopes downward both because a lower rate of unemployment implies a tighter labor market, which induces higher quits, and because it implies a higher nonwage income-to-wage ratio, which also raises the propensity to quit. For the United States over the period 1931-62, Eagly (1965) obtains an estimate of the elasticity of the quit rate with respect to the unemployment rate that is equal to -0.634.[4] If we accept that, in the equilibrium steady-state scenario we are considering, the quit rate does not vary much with movements in the employment rate, the zero-profit curve will be somewhat flat, that is, the labor-cost elasticity of the zero-profit curve will be high. We also notice that the same diagram (Figure 2) represents the equilibrium for every type-i worker. The employment rate,

[4] Looking at the effects of wage differentials on quits, Alan Krueger and Lawrence Summers (1988, p.280) find that "at the mean the elasticity of quits with respect to the wage premium is $-.07/.26 = -.27$." They reason that taken together, "these results imply that a 10 per cent increase in the wage differential brings about a .3 per cent increase in output through reduced quits alone. This suggests that although turnover does adversely affect output, reductions in turnover alone are not sufficient to justify wage premiums of the magnitude actually observed unless fixed costs of hiring are very high or labor's share in output is very low."

N_i, real effective wage, v_i^f/Λ_i, and the nonwage income taken as a ratio to productivity level, y_i^w/Λ_i, are the same for every type-i worker so the real wage, v_i^f, is twice as high for a worker who is twice as productive as another worker.[5] The nonwage income, y_i^w, corresponding to the hyperbola passing through E_0 is also twice as high for a worker who is twice as productive as another worker.

Consider now the long-run employment effects of a flat (constant) subsidy. The derivative of N_i with respect to s^{F} is calculated to be

$$\frac{dN_i}{ds^{\mathrm{F}}} = \frac{\Lambda_i^{-1}}{\beta[2(\zeta_1 + \zeta_2\Omega_2) + N_i(\zeta_{11} + \zeta_{12}\Omega_2) + \Omega(\zeta_{21} + \zeta_{22}\Omega_2)]} > 0 \qquad (30)$$

for every type i. The argument that this inequality is unambiguously positive is the following: Assume that there was no change in unemployment so that we were at an unchanged point (N_i, v_i^f) on the zero-profit curve so firms have returned to the original point that they were at before. The proportional payroll tax, taken by itself, has two effects. First, a penny increase in v_i^f increases v_i^h by only a fraction of a penny, namely, $1/(1+\tau)$. This lowers the marginal benefit of a penny increase in v_i^f. Second, the proportional payroll tax lowers v_i^h and, under correct expectations, v_i^{he}, in the same proportion for every type i. With the employment rate unchanged, y_i^w would also be reduced by the same proportion. Then each additional penny received by an employee now has a greater impact on v_i^h taken as a ratio to expected real wage earnings elsewhere and taken as a ratio to nonwage income so that the salutary effect on quitting is increased. Through this channel the marginal benefit of a penny increase in v_i^f is increased. If, instead of financing the subsidy, the proceeds from the payroll tax were, say, thrown into the sea, the two effects would exactly cancel out, leaving employment unaffected. There is, however, a third effect arising from the presence of the constant subsidy. In the presence of the subsidy, an additional penny received by an employee has a smaller impact on v_i^h/v_i^{he} and v_i^h/y_i^w so that the salutary effect on quitting is

[5] The equalization of unemployment rate result depends on the assumption that the marginal training cost in *manhours*, β, is the same across all types of workers. If we have $\beta_i > \beta_j$, then it can be shown that the unemployment rate for type-i workers will be higher than that for type-j workers. Note that this assumption is consistent with $\Lambda_i\beta_i < \Lambda_j\beta_j$, that is, although the marginal training cost for type-i workers is higher when measured in manhours, it could be lower when measured in terms of output on account of its lower productivity.

reduced. In the general equilibrium involving correct expectations and long-run capital market equilibrium, the incentive-wage condition can be written as

$$1 = \beta\Lambda_i[N_i\zeta_1 + \Omega(r^* - \rho, N_i)\zeta_2][\frac{\frac{1}{1+\tau}}{\frac{v_i^f + s^{\text{F}}}{1+\tau}}].$$

We can see from the righthand side of this equation that the two effects arising from the presence of $(1+\tau)$ exactly cancel out. This implies that in the long run, after wealth has fully adjusted, the payroll tax is neutral for employment. It follows that if a penny increase in v_i^f had a marginal benefit equal to marginal cost at the original employment rate, it must now have a marginal benefit less than the marginal cost. Hence firms cut their v_i^f and employment is expanded as a result.

We can see that, with the same dollar amount of wage subsidy given to each type-i worker, a less productive worker enjoys a higher subsidy relative to his productivity level. In Figure 3 we show that the employment effect is larger for a less productive worker as his wage curve is shifted further down than that of a more productive worker.

Consider now the long-run wage effects of the flat subsidy. We note that under a balanced-budget policy, the following relationship holds:

$$\int_{\underline{\Lambda}}^{\infty} s^{\text{F}} N_i g(\Lambda_i) d\Lambda_i = \int_{\underline{\Lambda}}^{\infty} \tau v_i^h N_i g(\Lambda_i) d\Lambda_i.$$

As noted earlier, around an equilibrium with no tax-subsidy, N_i, is equal for every type i. It follows that the budget constraint can be simplified to $\tau = s^{\text{F}}/v^h_{\text{mean}}$, where $v^h_{\text{mean}} \equiv \int_{\underline{\Lambda}}^{\infty} v_i^h g(\Lambda_i) d\Lambda_i$. Using this, and noting that around a zero-tax-subsidy equilibrium, $(v_i^h/v^h_{\text{mean}}) = (\Lambda_i/\Lambda_{\text{mean}})$, it is straightforward to show that

$$\left.\frac{dv_i^h}{ds^{\text{F}}}\right|_{\tau=0} = \frac{\eta_{\text{ZP}}}{\eta_{\text{ZP}} + \eta_{\text{IW}}} - \frac{\Lambda_i}{\Lambda_{\text{mean}}}, \tag{31}$$

where η_{ZP} and η_{IW} are the elasticities of the zero-profit and wage curves, respectively. For a worker whose Λ_i is sufficiently low, say $\Lambda_i \to \underline{\Lambda} \to 0$, the derivative of v_i^h with respect to s^{F} is unambiguously positive. But employment is expanded everywhere.

Short-Run Effects of the Flat Subsidy

Consider now briefly the short run in which wealth and y_i^w are given. Here the subsidy provides an additional boost to employment. With net wealth and interest unchanged, the increased take-home pay leads a worker to value his job more highly. This has the effect, at any employment rate, of raising the firm's real demand wage as the propensity to quit is reduced. Around a zero-tax-subsidy equilibrium, the vertical shift of the iso-y_i^w ZP curve is given by

$$\left.\frac{dv_i^f}{ds^{\mathrm{F}}}\right|_{\mathrm{ZP}} = \frac{\beta\Lambda_i\zeta_2(\frac{y_i^w}{v_i^{h2}})[1-\frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}]}{1-\beta\Lambda_i\zeta_2(\frac{y_i^w}{v_i^{h2}})},$$

which is positive for any worker whose productivity is below the mean.[6] The decreased propensity to quit on account of the reduced nonwage income relative to wage ratio also has the effect of shifting down the incentive-wage curve that is on top of the shift due to the wedge caused by the subsidy. The vertical shift of the iso-y_i^w IW curve is given by

$$\left.\frac{dv_i^f}{ds^{\mathrm{F}}}\right|_{\mathrm{IW}} = \frac{-1-\beta\Lambda_i(\frac{y_i^w}{v_i^{h2}})[\zeta_2+N_i\zeta_{12}+(\frac{y_i^w}{v_i^h})\zeta_{22}][1-\frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}]}{1+\beta\Lambda_i(\frac{y_i^w}{v_i^{h2}})[\zeta_2+N_i\zeta_{12}+(\frac{y_i^w}{v_i^h})\zeta_{22}]},$$

which is unambiguously negative for a worker whose productivity level is below the mean. From (26), we see that at given N_i, the nonwage income, y_i^w, is increased by the same proportion as the rise in v_i^h for the low-wage worker. Hence, in the long run, wealth accumulation ultimately shifts back the ZP curve to its original position and the IW curve also shifts up as wealth catches up to the increased take-home pay. However, a wedge remains, implying that employment is expanded throughout the distribution in the long run as shown earlier. For low-wage workers, there is an additional boost to employment in the short run.

Long-Run Effects of the Graduated Subsidy

Now the graduated subsidy: Equation (29) is replaced by

$$\begin{aligned}\frac{v_i^f+S(v_i^f)}{\Lambda_i[1+S'(v_i^f)]} &= \beta[N_i\zeta_1(N_i,\Omega(r^*-\rho,N_i)) \\ &\quad +\Omega(r^*-\rho,N_i)\zeta_2(N_i,\Omega(r^*-\rho,N_i))].\end{aligned}\qquad(32)$$

[6] Around a zero-tax-subsidy equilibrium, $1-\beta\Lambda_i\zeta_2(\frac{y_i^w}{v_i^{h2}}) = \beta\Lambda_iN_i/v_i^h > 0$.

Around a zero-tax-subsidy equilibrium, the response of N_i to a small change in $s^* \equiv S(v_i^{f*})$ is then calculated to be

$$\left.\frac{dN_i}{ds^*}\right|_{\tau=0} = \frac{\Lambda_i^{-1} + \Lambda_i^{-1}\{\frac{[v_i^{f*}S''/(1+S')]\tilde{\eta}_{\text{IW}}}{(1-[v_i^{f*}S''/(1+S')])\tilde{\eta}_{\text{IW}}+\eta_{\text{ZP}}}\}}{(1+S')\beta[(\zeta_1+\zeta_2\Omega_2)+N_i(\zeta_{11}+\zeta_{12}\Omega_2)+\Omega(\zeta_{21}+\zeta_{22}\Omega_2)]+\beta[\zeta_1+\zeta_2\Omega_2]}, \tag{33}$$

where

$$\tilde{\eta}_{\text{IW}} = \frac{\Lambda_i\beta[(\zeta_1+\zeta_2\Omega_2)+N_i(\zeta_{11}+\zeta_{12}\Omega_2)+\Omega(\zeta_{21}+\zeta_{22}\Omega_2)]N_i}{\frac{v_i^f}{1+S'}} > 0.$$

Expressing $\eta_{\text{IW}} \equiv \{(1+S') - [v_i^f S''/(1+S')]\}\tilde{\eta}_{\text{IW}}$, the condition that the wage curve be positively sloped in the (N_i, v_i^f) plane is that $S'' < (1+S')^2/v_i^f$. Given the restriction that $|S'(v_i^f)| < 1$, a sufficient condition for a graduated subsidy scheme paying $s^* = s^{\text{F}}$ to give an extra boost to employment is therefore that $0 < S'' < (1+S')^2/v_i^f$. With graduation, there are two effects that are at work when compared to the constant subsidy case as shown in Figure 4. Firstly, with graduation firms are induced to moderate wage rates above the bottom in order to gain a larger subsidy. For $s^* = s^{\text{F}}$, Figure 4 shows that the wage curve is shifted further down under a graduated scheme. Secondly, graduation changes the slope of the wage curve. Whereas a constant subsidy scheme has no effect on the slope of the wage curve (there being a parallel shift), the new wage curve becomes steeper at higher wages with a graduated scheme. The restriction on S'' is sufficient to ensure that the "shift" as well as the "slope" effects of graduation leave a bigger boost to employment compared to the constant subsidy case. Note also that by designing a subsidy plan such that the subsidy asymptotically reaches zero as v_i^f is increased, we ensure that employment is raised throughout the distribution although the expansionary effect is smaller at higher v_i^f.

Consider now the long-run wage effects. We can show that around a zero-tax-subsidy equilibrium, the following derivative holds:

$$\left.\frac{dv_i^h}{ds^*}\right|_{\tau=0} = \{\frac{\eta_{\text{ZP}} - [\frac{v_i^{f*}S''}{1+S'}]\tilde{\eta}_{\text{IW}}}{(1-[\frac{v_i^{f*}S''}{1+S'}])\tilde{\eta}_{\text{IW}}+\eta_{\text{ZP}}}\} - [\frac{\Lambda_i}{\Lambda_{\text{mean}}}][\frac{d\mathcal{S}}{ds^*}], \tag{34}$$

where $\mathcal{S} \equiv \int_{\underline{\Lambda}}^{\infty} S(v_i^f)g(\Lambda_i)d\Lambda_i$ and $d\mathcal{S}/ds^* > 0$. If we further restrict the value of S'' such that $0 < S'' < (1+S')^2/v_i^f - (\eta_{\text{IW}}/\eta_{\text{ZP}})(1+S')/v_i^f$, the first curly brace term in (34) is unambiguously positive. Notice from (33) that employment is increasing in S''. If we strike a balance in our choice of S'' with regard to the extra expansionary employment effect on the one hand and the wage effect on the other hand, we can obtain a higher take-home wage for a worker whose Λ_i is sufficiently low along with higher employment.

Long-Run Effects of a Hiring Subsidy

Before concluding our analysis of the small open economy, let us examine the effects of a hiring subsidy in our model. Suppose that an ad valorem payroll tax is used to finance a flat hiring subsidy of s^{HF} for each new recruit hired. It is straightforward to show that our two fundamental equations giving the reduced-form ZP and IW schedules become, respectively,

$$\begin{aligned} \frac{v_i^f}{\Lambda_i} &= 1-[\beta-\frac{s^{\text{HF}}}{\Lambda_i}] \\ &\quad \times[\zeta(N_i,\Omega(r^*-\rho,N_i))+r^*+\theta]; \qquad (35) \\ \frac{v_i^f}{\Lambda_i} &= [\beta-\frac{s^{\text{HF}}}{\Lambda_i}] \qquad (36) \\ &\quad \times[N_i\zeta_1(N_i,\Omega(r^*-\rho,N_i))+\Omega(r^*-\rho,N_i)\zeta(N_i,\Omega(r^*-\rho,N_i))]. \end{aligned}$$

Such a policy shifts up the ZP curve but shifts down the IW curve leading to an unambiguous expansion of equilibrium employment but possible decline of the product wage, v_i^f. (In contrast, under both the flat and graduated subsidy plans, the before-tax wage of the workers, $v_i^f+s_i$, unambiguously rises.) The take-home wage would accordingly fall further as the payroll tax is applied though this must be set against the subsidy that each new recruit receives when hired. We obtain the following derivative[7]:

$$\left.\frac{d[v_i^f/(1+\tau)+(r^*+\theta)s^{\text{HF}}]}{ds^{\text{HF}}}\right|_{s^{\text{HF}}=0} = (\zeta+\theta)[\mu-(\frac{\Lambda_i}{\Lambda_{\text{mean}}})]+(1+\mu)r^*+\theta-(1-\mu)[N_i\zeta_1+\Omega\zeta_2],$$

where

$$0<\mu\equiv\frac{(\zeta_1+\zeta_2\Omega_2)+(\zeta_{11}+\zeta_{12}\Omega_2)N_i+(\zeta_{21}+\zeta_{22}\Omega_2)\Omega}{2(\zeta_1+\zeta_2\Omega_2)+(\zeta_{11}+\zeta_{12}\Omega_2)N_i+(\zeta_{21}+\zeta_{22}\Omega_2)\Omega}<1.$$

[7] The balanced-budget condition with a hiring subsidy simplifies to $\tau=[(\zeta+\theta)s^{\text{HF}}/v_{\text{mean}}^h]$ around a zero-hiring-subsidy equilibrium, noting that in the steady state, the hiring rate equals $\zeta+\theta$ for every type of worker.

IV. Closed-Economy Incidence

We confine our analysis to a flat subsidy in the closed economy financed by a proportional payroll tax. For any r, our reduced form ZP and IW curves are written respectively as

$$\frac{v_i^f}{\Lambda_i} = 1 - \beta[\zeta(N_i, \Omega(r-\rho, N_i)) + \theta + r], \tag{37}$$

$$\begin{aligned} \frac{v_i^f}{\Lambda_i} + \frac{s^{\mathrm{F}}}{\Lambda_i} \quad = \quad & \beta[N_i \zeta_1(N_i, \Omega(r-\rho, N_i)) \\ & + \Omega(r-\rho, N_i)\zeta_2(N_i, \Omega(r-\rho, N_i))], \end{aligned} \tag{38}$$

where we have again substituted for y_i^w/v_i^h the function $\Omega(r-\rho, N_i)$ obtained from the Blanchardian relationship expressed as

$$r = \rho + \frac{\theta}{1 + (v_i^h/y_i^w)N_i}. \tag{39}$$

We note from (37) and (38) that by equating the required incentive wage to the demand wage, we can express the employment rate of any type-i worker as an implicit function of the interest rate and the subsidy relative to productivity level, namely,

$$N_i = \epsilon(r; (s^{\mathrm{F}}/\Lambda_i)); \ \epsilon_1 < 0; \ \epsilon_2 > 0. \tag{40}$$

The function ϵ is interpretable as the demand for the stock of employees in steady state. The value of the total stock of employees, which are the only form of asset in the closed economy, is $A \equiv \int_{\underline{\Lambda}}^{\infty} \beta \Lambda_i N_i g(\Lambda_i) d\Lambda_i$ since each employee is worth $\beta\Lambda_i$. By (40), A is a decreasing function of the rate of interest:

$$A = \int_{\underline{\Lambda}}^{\infty} \beta \Lambda_i \epsilon(r; (s^{\mathrm{F}}/\Lambda_i)) g(\Lambda_i) d\Lambda_i. \tag{41}$$

An expression for the average supply of wealth per member of the type-i workforce is obtained from (39) as

$$W_i = (\frac{v_i^h N_i}{r+\theta})[\frac{r-\rho}{\theta+\rho-r}]. \tag{42}$$

As before, excluding the case where $r-\rho>\theta$, we have a well-defined steady state with the righthand side of (42) being unambiguously positive. The total supply of wealth per worker, under balanced budget, is given by

$$W=[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}]\int_{\underline{\Lambda}}^{\infty}v_i^f N_i g(\Lambda_i)d\Lambda_i. \tag{43}$$

Further using (37) and (40) in (43), we obtain an expression giving us total desired supply of wealth as a function of the rate of interest:

$$\begin{aligned} W &= [\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}] \qquad (44) \\ &\times\int_{\underline{\Lambda}}^{\infty}\{1-\beta[\zeta(\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i}),\Omega(r-\rho,\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i}))+r+\theta]\}\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i})\Lambda_i g(\Lambda_i)d\Lambda_i. \end{aligned}$$

Suppose that initially the subsidy and payroll tax are zero. In that case, we note from (37) and (38) that setting $s^{\mathrm{F}}=0$ implies that N_i and y_i^w/v_i^h are equal across all types of workers. Consequently, the quit rate is initially identical across all types of workers. As in our earlier discussion in the neoclassical case, we can argue that the per worker supply of wealth is upward-sloping initially but at very high r may bend backward as in Figure 1.[8] In the same plane, per worker demand for the domestic assets in value terms is downward sloping. We suppose that the equilibrium r is unique or that only the lowest equilibrium r is empirically relevant.

To see how the tax-subsidy policy affects the rate of interest, it will help to have a sharper characterization of this equilibrium. Since the quit rate is equal across all types of workers in the neighborhood of the zero-subsidy equilibrium, we can simplify the equilibrium condition to

$$\begin{aligned} & W\equiv[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}] \\ & \times\{1-\beta[\zeta(\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i}),\Omega(r-\rho,\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i}))+r+\theta]\}\int_{\underline{\Lambda}}^{\infty}\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i})\Lambda_i g(\Lambda_i)d\Lambda_i \\ & =\beta\int_{\underline{\Lambda}}^{\infty}\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i})\Lambda_i g(\Lambda_i)d\Lambda_i\equiv A. \end{aligned} \tag{45}$$

[8] Although the increase in r leads to a decline in the real demand wage, the fall in N_i acts to lower the quit propensity and hence indirectly acts to offset the fall in wage. We assume that the direct effect dominates.

The equilibrium r is therefore given by

$$[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}] \times\{1-\beta[\zeta(\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i}),\Omega(r-\rho,\epsilon(r;\frac{s^{\mathrm{F}}}{\Lambda_i}))+r+\theta]\}=\beta. \tag{46}$$

Thus we see that a tax-subsidy policy involving a small change in s^{F} financed by a proportional payroll tax has ultimately an influence on the interest rate only via its influence on the quit rate. The effects of introducing a small subsidy are as follows: At the original r, the subsidy, in expanding the demand for employees of all types, shifts the domestic asset demand schedule in Figure 1 to the right. (See the righthand side of (45).) As workers, finding the probability of obtaining employment improved, step up their saving accordingly, the supply of wealth schedule is also shifted to the right. (See the lefthand side of (45).) In fact, both rightward shifts are equal in magnitude, leaving the interest rate unchanged. But the rise in each N_i acts to tighten the labor market of each type-i worker. The resulting increase in the propensity to quit reduces the demand wage. This leads to a leftward shift of the supply of wealth schedule causing the interest rate to rise.[9] (When the zero profit curve is horizontal, however, this effect would be zero.) But clearly this effect can only moderate the net expansionary effect on employment of low-Λ_i workers. To show this we may calculate the total derivative of $\epsilon(r;(s^{\mathrm{F}}/\Lambda_i))$ evaluated at a low Λ_i with respect to s^{F}.

Taking the total derivative in (45), we obtain

$$\frac{dr}{ds^{\mathrm{F}}}=\frac{(\zeta_1+\zeta_2\Omega_2)\epsilon_2}{\{[\frac{\theta(\theta+\rho)}{(r-\rho)^2}-\zeta_2\Omega_1]-(\zeta_1+\zeta_2\Omega_2)\epsilon_1\}\Lambda_{\mathrm{mean}}}. \tag{47}$$

In the Appendix, we show that a necessary condition for the aggregate supply of wealth schedule to be positively sloped under the proviso that the labor-cost elasticity of the zero-profit curve exceeds unity is that $[\theta(\theta+\rho)/(r-\rho)^2]>$

[9] In the Appendix, we calculate the extent of the horizontal shifts of the total supply of wealth and total asset demand schedules. When the labor cost elasticity of the zero-profit curve is greater than one, the net shift of the total supply of wealth schedule is rightward. When this elasticity is less than one, the net shift is leftward.

$\zeta_2\Omega_1$.[10] Hence the tax-subsidy scheme raises the rate of interest. To prove that for low-Λ_i workers, the rise of r only moderates but does not overturn the expansionary employment effect of s^{F}, we calculate the following derivative:

$$\begin{aligned} \frac{dN_i}{ds^{\mathrm{F}}} &= \epsilon_1(\frac{dr}{ds^{\mathrm{F}}}) + \frac{\epsilon_2}{\Lambda_i} \\ &= \frac{-(\zeta_1+\zeta_2\Omega_2)\epsilon_1\epsilon_2(\Lambda_i^{-1}-\Lambda_{\mathrm{mean}}^{-1}) + [\frac{\theta(\theta+\rho)}{(r-\rho)^2}-\zeta_2\Omega_1]\epsilon_2\Lambda_i^{-1}}{\{[\frac{\theta(\theta+\rho)}{(r-\rho)^2}-\zeta_2\Omega_1]-(\zeta_1+\zeta_2\Omega_2)\epsilon_1\}\Lambda_{\mathrm{mean}}}. \end{aligned} \quad (48)$$

In the homogeneous case, the tax-subsidy scheme unambiguously expands employment for everyone. In the heterogeneous case, all workers whose Λ_i is either below or equal to the mean find their employment expanded. It is straightforward to obtain an expression for the derivative of v_i^h with respect to s^{F}:

$$\left.\frac{dv_i^h}{ds^{\mathrm{F}}}\right|_{s^{\mathrm{F}}=0} = 1 - \frac{\Lambda_i}{\Lambda_{\mathrm{mean}}} - \Lambda_i\beta[(\zeta_1+\zeta_2\Omega_2)\frac{dN_i}{ds^{\mathrm{F}}} + \zeta_2\Omega_1\frac{dr}{ds^{\mathrm{F}}}]. \quad (49)$$

We see that for a worker whose Λ_i is sufficiently low, his v_i^h will rise as well.

V. The Case of Rising Marginal Training Cost

We will confine our discussion here only to the case of the small open economy. Suppose that the fraction of a type-i employee's working time devoted to training new hires is given by $\Phi(h_i)$, where $\Phi'(h_i) > 0$ and $\Phi''(h_i) > 0$. Solving the Hamiltonian problem, we would have $q_i = \Lambda_i\Phi'(h_i)$. With a flat (constant) subsidy, the two fundamental reduced-form ZP and IW relationships would now be given by

$$\frac{v_i^f}{\Lambda_i} = 1 - \Phi(h) + h\Phi'(h) - \Phi'(h_i)[\zeta(N_i, \Omega(r^* - \rho, N_i)) + \theta + r^*], \quad (50)$$

[10]The inequality could (but need not necessarily) be reversed when the labor cost elasticity of the zero-profit curve is less than one. If the inequality is reversed but the aggregate supply of wealth schedule remains positively sloped, there is an increased upward pressure on the interest rate. If the inequality is reversed and the aggregate supply of wealth becomes negatively sloped, there is the theoretical possibility that the rate of interest is lowered as a result of the tax-subsidy scheme. In such a case, employment is unambiguously expanded for workers of all types.

$$\frac{v_i^f}{\Lambda_i} + \frac{s^{\mathrm{F}}}{\Lambda_i} = \Phi'(h_i)[N_i \zeta_1(N_i, \Omega(r^* - \rho, N_i))$$
$$+\Omega(r^* - \rho, N_i)\zeta_2(N_i, \Omega(r^* - \rho, N_i))]. \tag{51}$$

The steady-state employment (SSE) condition is that

$$h_i = \zeta(N_i, \Omega(r^* - \rho, N_i)) + \theta. \tag{52}$$

From (51), we note that v_i^f/Λ_i is positively related to h_i and N_i and negatively related to s^{F}/Λ_i, written $v_i^f/\Lambda_i = V(h_i, N_i; s^{\mathrm{F}}/\Lambda_i)$. Using this relation in (50), we obtain a downward-sloping schedule in the (N_i, h_i) plane. Equation (52), on the other hand, gives us a positively-sloped schedule. (See Figure 5.) Starting from a zero-subsidy equilibrium, it is now clear that the implementation of the flat subsidy scheme results in a rightward, and hence upward, shift of the ZP curve in this plane. The result is that employment is expanded along with a rise in steady-state hiring. Moreover, for the same dollar amount of subsidy, the expansion is greatest for low-wage workers.

VI. Concluding Remarks

The pay open to the less advantaged is now so inadequate for meaningful self-suppport and their participation rates and job attachment, especially among men, are now so far from integrating poor communities in the nation's business life that, arguably, any remedy will require novel intervention. (If the goal is not far, just raising the level of familiar instruments may suffice to reattain it, but if the goal is far, designing de nuovo a more tailored instrument may be cheaper.) Any such innovation, however, may open the law of unintended consequences, since we do not know the scale and perhaps the nature of all the effects. This uncertainty leads to hesitation and disagreement over the intervention to select. An investment in education that would hypothetically restore low-end wages to their late-1970s level has been calculated to cost nearly two trillion dollars (Heckman, 1993). But the radical uncertainty over exactly what education reforms and expenditures to make may be a bigger drawback (along with the needed one-generation lead-time).

The employment subsidy instrument has the advantage that economists are familiar with the workings of corrrective taxes and subsidies—but mainly at the partial-equilibrium level of the individual industry. Massive and perhaps permanent low-wage employment subsidies would not likely prove an exception to the law of unanticipated effects. This paper has been addressed to the doubts over such subsidies that might arise at the level of general equilibrium. Is it theoretically possible in the context of our model of the natural rate that the rise of the wage rate relative to nonwage income initially achieved by the subsidies—recall that the increased payroll tax rate is ultimately neutral for that ratio—will induce worker-savers to build up nonwage income relative to the wage rate until incentive wages have been driven up and the demand-wage rate driven down by enough to nullify the expansion of employment? As the paper has shown, the adjustment of wealth in the small open economy does act to moderate the expansion of employment achieved by the subsidies in the "short run" but in the long run employment is increased throughout the distribution. In the closed-economy case, the interest rate is pulled up, which moderates employment expansion. Nevertheless, employment unambiguously expands for all workers whose productivity level is below the mean.

Other uncertainties must be left for future work. One of these, obviously, is the net budgetary cost of wage subsidies. In principle, employment subsidies could be targeted on groups who, if their employment were not subsidized, would otherwise cost the government as much or more in public support—single parents, generally mothers, with dependent children, for example. In America, however, it may be the increased difficulty of self-support and the increased disengagement from business life among *men* that is fundamental, since that may lie behind the rise of single-parenting as well as the rise of crime, violence and drug abuse. And men are not as eligible as women for most welfare outlays. So employment subsidies had better be untargeted. And the argument that their net budgetary cost will be small enough to satisfy taxpayers has to rest on estimates of the indirect savings and revenues achieved when entire poor communities are made self-supporting through work: the savings in welfare, crime prevention, administration of justice, unemployment compensation and other social insurance programs (under existing benefit schedules), and the revenues from the additional collection of income and sales taxes (under existing rates).

An attractive feature of hiring subsidies is that they can be targeted at those potential workers currently depending on unemployment compensation or welfare benefits for their support. So the budgetary savings achieved by stimulating their employment may equal or exceed the gross budgetary outlay for the subsidies. This feature has been used by Dennis Snower in designing a program whereby the unemployed worker creates his own hiring subsidy by trading in his unemployment benefits in return for a job. We found, however, that subsidies to hiring might actually reduce wage rates at the low end, perhaps, appreciably so, and this would be a serious drawback in the American context where, among the disadvantaged, low wages are as much in need of remedying as depressed employment. Furthermore, jobless American men receive little in entitlements that they could exchange for a job other than their unemployment compensation and those benefits are not long-term and not broad-based.

Uncertainty also hangs over the amount of abuse and fraud that wage subsidy programs would lead to. Hiring subsidies would apparently invite employers to swap employees, perhaps after the spell of unemployment required for eligibility, and to move employees more freely from corporation to corporation under the same parent company—all in order to collect increased hiring subsidies. An advantage of the employment subsidies studied here is that they would not encourage those abuses. However, employment subsidies (and possibly hiring subsidies too) would inspire firms, especially single-proprietor firms, to featherbed the payroll with phantom employees under the names of persons, such as family members, whose silence would be trusted. On balance, it might be advantageous for this as well as other reasons to restrict the subsidies to full-time jobs, to good-sized firms where whistle-blowers would be a deterrent, and to limit the subsidies to credits against the firms' tax liabilities. In another sort of abuse, the employer and employee would agree to a reduced wage, which would add to the subsidy earned, and a compensating increase in nonwage benefits, which, if undetected or not counted as compensation, would not add to the subsidy earned. For this reason, a graduated subsidy must decrease slowly with the wage rate so that this temptation is not too strong in relation to the monitoring powers of the tax authorities. Yet another abuse would draw upon the collusion of third parties. To earn increased employment subsidies an employer might reduce the wage rate of employees and compensate them with side jobs above their normal pay rates at a cooperating firm, which might do the same with the first firm or with other firms. Similiarly, under the exisiting Earned Income Tax Credit program awarding subsidies directly to the taxpayer reporting low earnings, the wage can be reduced and the employee compensated through special discounts obtained from third parties. It may be, however, that such abuses could be deterrred by punishing them with the same severity meted out to other kinds of tax fraud.

Appendix

1. To prove that $L_i/\bar{L}$ is decreasing in r, we obtain from (7) the relation $r = R(v_i^h\bar{L}/y_i^w, L_i/\bar{L})$ with $R_1 < 0$ and $R_2 < 0$. Substituting the function R for r in (8), we obtain a variable-interest-rate labor supply schedule in the $(L_i/\bar{L}, v_i^h\bar{L}/y_i^w)$ plane, whose slope is given by

$$\left.\frac{d(\frac{v_i^h\bar{L}}{y_i^w})}{d\frac{L_i}{\bar{L}}}\right|_{\mathrm{LS}} = \frac{1+[\frac{\theta+\rho}{r+\theta}]+[\frac{\theta(\theta+\rho)}{1+(v_i^h\bar{L}/y_i^w)(L_i/\bar{L})}]}{[\frac{\theta+\rho}{r+\theta}](\frac{v_i^h\bar{L}}{y_i^w})^{-2}\{1-[\frac{1}{(1+(r/\theta))(1+(v_i^h\bar{L}/y_i^w)^{-1}(L_i/\bar{L})^{-1})}]\}} > 0.$$

As we move northeast along the variable-interest-rate labor supply function, (7) tells us that the interest rate is declining.

2. To prove that at given r, the tax-subsidy policy shifts the aggregate supply of wealth schedule to the right, we express (44) as $W = \Psi(r, s^{\mathrm{F}})$. Taking a total derivative through (44) with respect to s^{F} we obtain

$$\Psi_2 = B\{[1-\beta(\zeta+r+\theta)] - \beta(\zeta_1+\zeta_2\Omega_2)\epsilon\}\epsilon_2,$$

where $B \equiv (r-\rho)/[(\theta+\rho-r)(r+\theta)]$. The assumption that the labor-cost elasticity of the zero-profit curve exceeds one implies that the reduced-form ZP curve cuts the hyperbola from below in the $(N_i, v_i^f/\Lambda_i)$ plane. The slope of the hyperbola is given by $-(v_i^f/\Lambda_i)N_i^{-1}$, which equals $-[1-\beta(\zeta+r+\theta)]N_i^{-1}$ around the equilibrium while the slope of the reduced-form ZP curve is given by $-\beta(\zeta_1+\zeta_2\Omega_2)$. Hence,

$$[1-\beta(\zeta+r+\theta)] > \beta(\zeta_1+\zeta_2\Omega_2)\epsilon.$$

Accordingly, $\Psi_2 > 0$.

From (41), we can express total asset demand as $A = \Theta(r, s^{\mathrm{F}})$. We obtain the following derivative:

$$\Theta_2 = \beta\epsilon_2,$$

which is positive. Noting (46), it is clear that $\Theta_2 > \Psi_2$.

3. It is straightforward to show that

$$\Psi_1 = \beta B\{\epsilon[\frac{\theta(\theta+\rho)}{(r-\rho)^2} - \zeta_2\Omega_1] - \epsilon_1 B^{-1}[B\epsilon(\zeta_1+\zeta_2\Omega_2)-1]\}\Lambda_{\mathrm{mean}}.$$

Noting (46), the condition

$$[1 - \beta(\zeta + r + \theta)] > \beta(\zeta_1 + \zeta_2\Omega_2)\epsilon$$

can be re-expressed as

$$B\epsilon(\zeta_1 + \zeta_2\Omega_2) < 1.$$

Thus for Ψ_1 to be positive, it is required that

$$[\frac{\theta(\theta+\rho)}{(r-\rho)^2} - \zeta_2\Omega_1] > \frac{\epsilon_1}{\epsilon}[\frac{B\epsilon(\zeta_1 + \zeta_2\Omega_2) - 1}{B}] > 0.$$

Hence a necessary condition for the aggregate supply of wealth curve to be positively-sloped when the labor-cost elasticity of the zero-profit curve exceeds one is that $[\theta(\theta+\rho)/(r-\rho)^2] > \zeta_2\Omega_1$.

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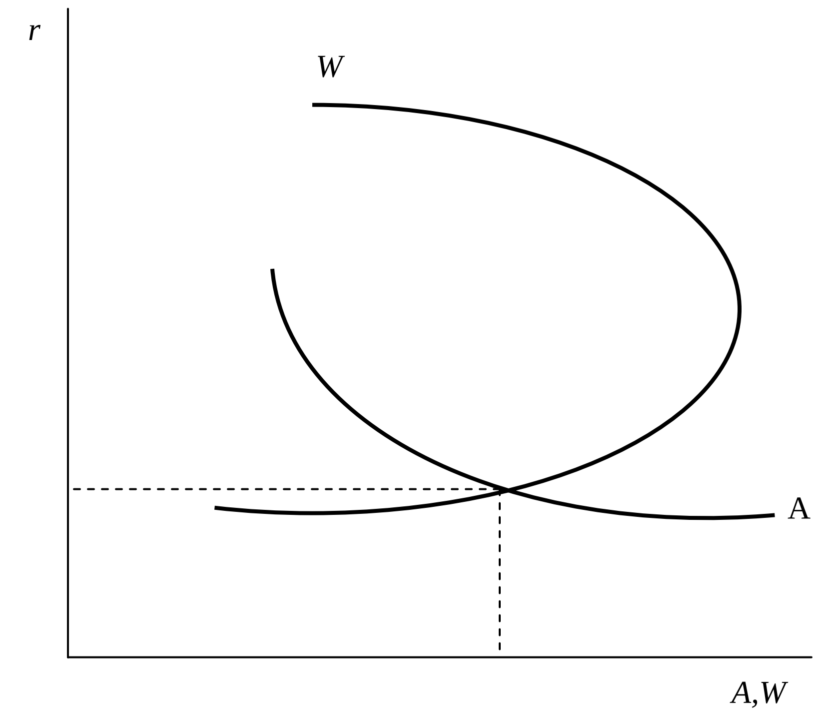

Figure 1: Wealth Supply and Asset Demand

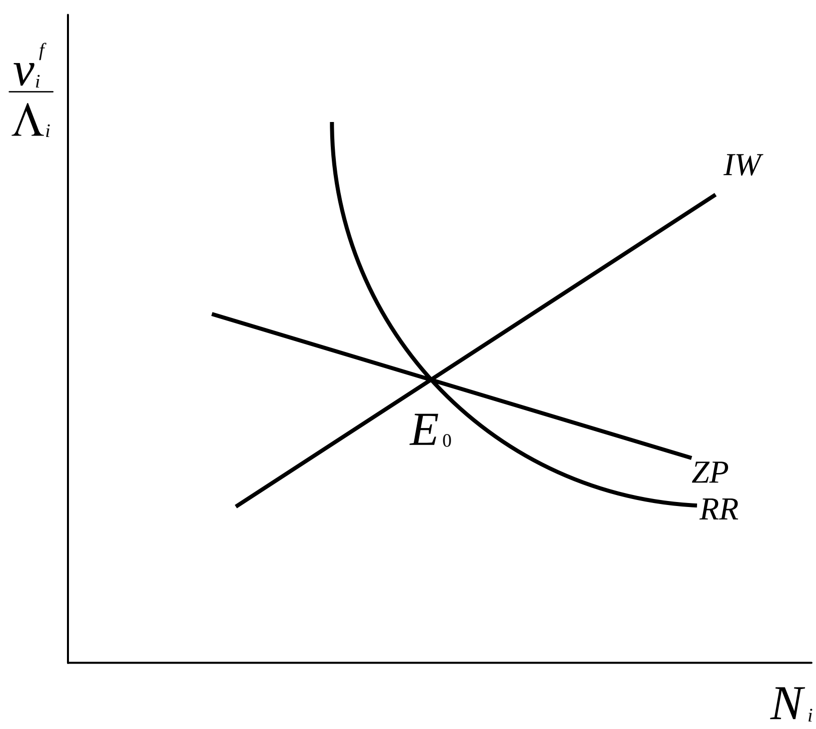

Figure 2: Labor, Product and Capital Market Equilibrium

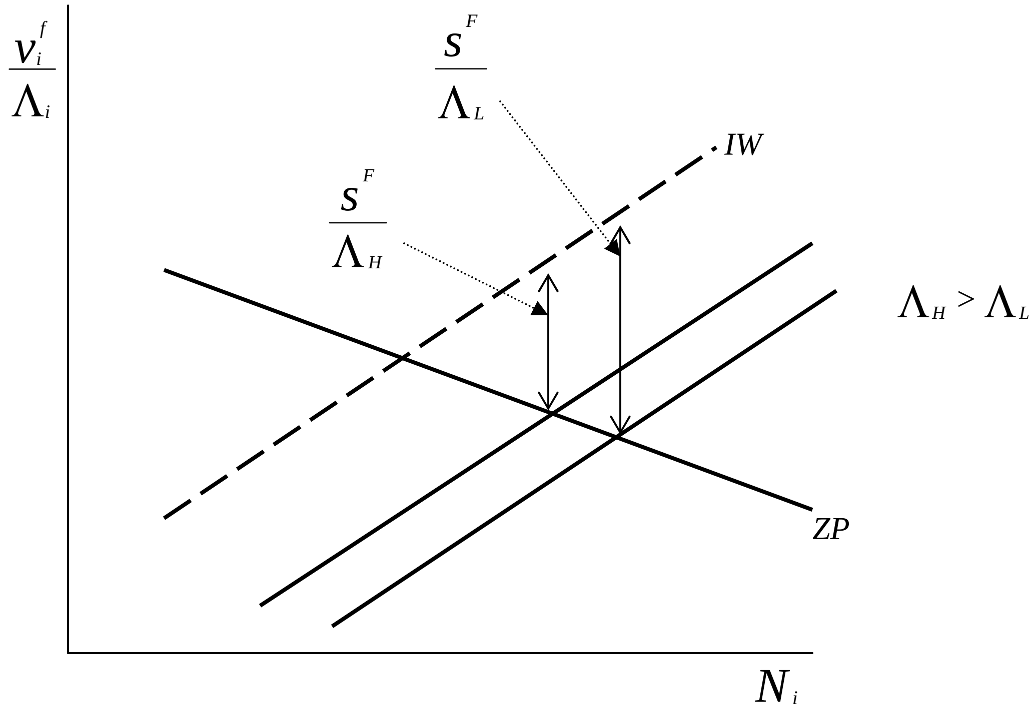

Figure 3: Effects of Flat Subsidy

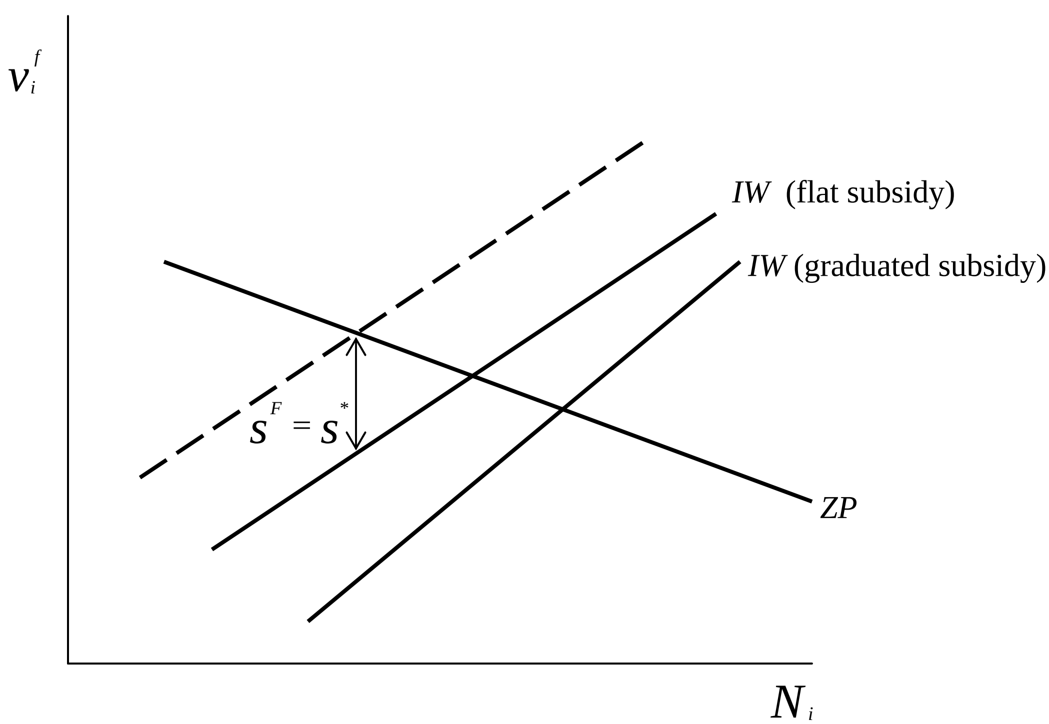

Figure 4: Comparison of Flat and Graduated Subsidies

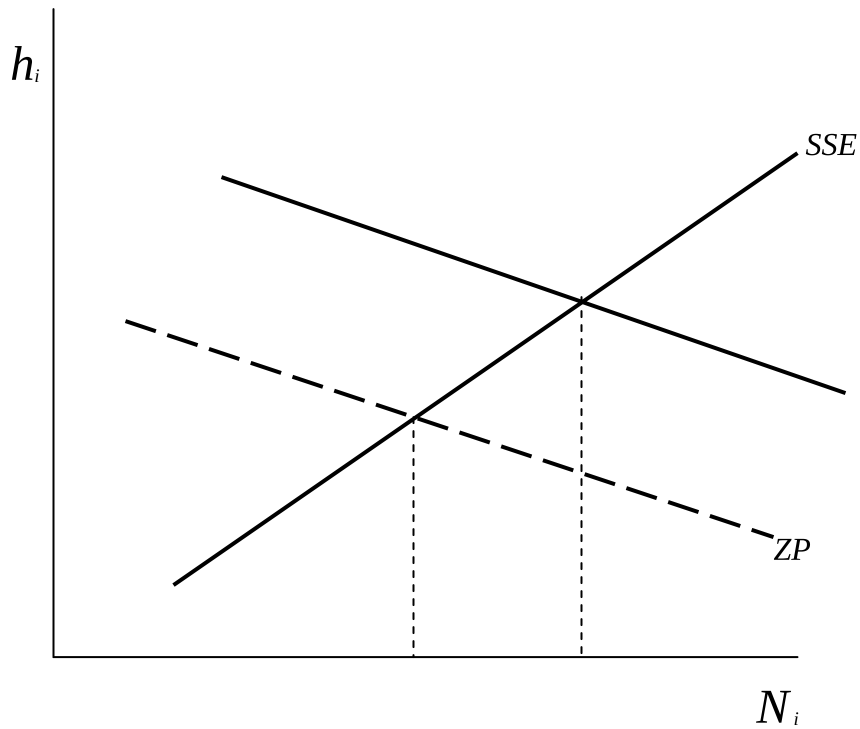

Figure 5: Effects of a Flat Subsidy