Optimal Crawling Strategies for Web Search Engines

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ABSTRACT

Web Search Engines employ multiple so-called *crawlers* to maintain local copies of web pages. But these web pages are frequently updated by their owners, and therefore the crawlers must regularly revisit the web pages to maintain the freshness of their local copies. In this paper, we propose a two-part scheme to optimize this crawling process. One goal might be the minimization of the average level of staleness over all web pages, and the scheme we propose can solve this problem. Alternatively, the same basic scheme could be used to minimize a possibly more important search engine *embarrassment level* metric: The frequency with which a client makes a search engine query and then clicks on a returned url only to find that the result is incorrect. The first part our scheme determines the (nearly) optimal crawling frequencies, as well as the theoretically optimal times to crawl each web page. It does so within an extremely general stochastic framework, one which supports a wide range of complex update patterns found in practice. It uses techniques from probability theory and the theory of resource allocation problems which are highly computationally efficient – crucial for practicality because the size of the problem in the web environment is immense. The second part employs these crawling frequencies and ideal crawl times as input, and creates an optimal achievable schedule for the crawlers. Our solution, based on network flow theory, is exact as well as highly efficient. An analysis of the update patterns from a highly accessed and highly dynamic web site is used to gain some insights into the properties of page updates in practice. Then, based on this analysis, we perform a set of detailed simulation experiments to demonstrate the quality and speed of our approach.

Categories and Subject Descriptors

H.3 [**Information Systems**]: Information Storage and Retrieval; G.2,3 [**Mathematics of Computing**]: Discrete Mathematics, Probability and Statistics

General Terms

Algorithms, Performance, Design, Theory

1. INTRODUCTION

Web search engines play a vital role on the World Wide Web, since they provide for many clients the first pointers to web pages of interest. Such search engines employ *crawlers* to build local repositories containing web pages, which they then use to build data structures useful to the search process. For example, an *inverted index* is created that typically consists of, for each term, a sorted list of its positions in the various web pages.

On the other hand, web pages are frequently updated by their owners [8, 21, 27], sometimes modestly and sometimes more significantly. A large study in [2] notes that 23% of the web pages change daily, while 40% of commercial web pages change daily. Some web pages disappear completely, and [2] reports a half-life of 10 days for web pages. The data gathered by a search engine during its crawls can thus quickly become *stale*, or out of date. So crawlers must regularly revisit the web pages to maintain the *freshness* of the search engine's data.

In this paper, we propose a two-part scheme to optimize the crawling (or perhaps, more precisely, the *recrawling*) process. One reasonable goal in such a scheme is the minimization of the average level of staleness over all web pages, and the scheme we propose here can solve this problem. We believe, however, that a slightly different metric provides greater utility. This involves a so-called *embarrassment* metric: The frequency with which a client makes a search engine query, clicks on a url returned by the search engine, and then finds that the resulting page is inconsistent with respect to the query. In this context, *goodness* would correspond to the search engine having a fresh copy of the web page. However, *badness* must be partitioned into *lucky* and *unlucky* categories: The search engine can be bad but lucky in a variety of ways. In order of increasing luckiness the possibilities are: (1) The web page might be stale, but not returned to the client as a result of the query; (2) The web page might be stale, returned to the client as a result of the query, but not clicked on by the client; and (3) The web page might be stale, returned to the client as a result of the query, clicked on by the client, but might be correct with respect to the query anyway. So the metric under discussion would only count those queries on which the search engine is actually embarrassed: The web page is stale, returned to the client, who clicks on the url only to find that the page is either inconsistent with respect to the original query, or (worse yet) has a broken link. (According to [17], and noted in [6], up to 14% of the links in search engines are broken, not a good state of affairs.)

The first component of our proposed scheme determines the optimal number of crawls for each web page over a fixed period of time called the *scheduling interval*, as well as deter-

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mining the theoretically optimal (*ideal*) crawl times themselves. These two problems are highly interconnected. The same basic scheme can be used to optimize either the staleness or embarrassment metric, and is applicable for a wide variety of stochastic update processes. The ability to handle complex update processes in a general unified framework turns out to be an important advantage and a contribution of our approach in that the update patterns of some classes of web pages appear to follow fairly complex processes, as we will demonstrate. Another important model supported by our general framework is motivated by, for example, an information service that updates its web pages at certain times of the day, if an update to the page is necessary. This case, which we call *quasi-deterministic*, is characterized by web pages whose updates might be characterized as somewhat more deterministic, in the sense that there are fixed potential times at which updates might or might not occur. Of course, web pages with deterministic updates are a special case of the quasi-deterministic model. Furthermore, the crawling frequency problem can be solved under additional constraints which make its solution more practical in the real world: For example, one can impose minimum and maximum bounds on the number of crawls for a given web page. The latter bound is important because crawling can actually cause performance problems for web sites.

This first component problem is formulated and solved using a variety of techniques from probability theory [23, 28] and the theory of resource allocation problems [12, 14]. We note that the optimization problems described here must be solved for *huge* instances: The size of the World Wide Web is now estimated at over one billion pages, the update rate of these web pages is large, a single crawler can crawl more than a million pages per day, and search engines employ multiple crawlers. (Actually, [2] notes that their own crawler can handle 50-100 crawls per second, while others can handle several hundred crawls per second. We should note, however, that crawling is often restricted to less busy periods in the day.) A contribution of this paper is the introduction of state-of-the-art resource allocation algorithms to solve these problems.

The second component of our scheme employs as its input the output from the first component. (Again, this consists of the optimal numbers of crawls and the ideal crawl times). It then finds an optimal achievable schedule for the crawlers themselves. This is a parallel machine scheduling problem [3, 20] because of the multiple crawlers. Furthermore, some of the scheduling *tasks* have release dates, because, for example, it is not useful to schedule a crawl task on a quasi-deterministic web page *before* the potential update takes place. Our solution is based on *network flow theory*, and can be posed specifically as a *transportation problem* [1]. This problem must also be solved for enormous instances, and again there are fast algorithms available at our disposal. Moreover, one can impose additional real-world constraints, such as restricted crawling times for a given web page.

We know of relatively few related papers in the research literature. Perhaps the most relevant is [6]. (See also [2] for a more general survey article.) In [6] the authors essentially introduce and solve a version of the problem of finding the optimal number of crawls per page. They employ a staleness metric and assume a Poisson update process. Their algorithm solves the resulting continuous resource allocation problem by the use of Lagrange multipliers. In [7] the authors also study a similar problem (Poisson updates, but with general crawl time distributions), with weights proportional to the page update frequencies. They present a heuristic to handle large problem instances. The problem of optimizing the number of crawlers is tackled in [26], based on a queueing-theoretic analysis, in order to avoid the two extremes of starvation and saturation. In summary, there is some literature on the first component of our crawler optimization scheme, though we have noted above several potential advantages of our approach. To our knowledge, this is the first paper that meaningfully examines the corresponding scheduling problem which is the second component of our scheme.

Another important aspect of our study concerns the statistical properties of the update patterns for web pages. This clearly is a critical issue for the analysis of the crawling problem, but unfortunately there appears to be very little in the literature on the types of update processes found in practice. To the best of our knowledge, the sole exception is a recent study [19] which suggests that the update processes for pages at a news service web site are not Poisson. Given the assumption of Poisson update processes in most previous studies, and to further investigate the prevalence of Poisson update processes in practice, we analyze the page update data from a highly accessed web site serving highly dynamic pages. A representative sample of the results from our analysis are presented and discussed. Most importantly, these results demonstrate that the interupdate processes span a wide range of complex statistical properties across different web pages and that these processes can differ significantly from a Poisson process. By supporting in our general unified approach such complex update patterns (including the quasi-deterministic model) in addition to the Poisson case, we believe that our optimal scheme can provide even greater benefits in real-world environments.

The remainder of this paper is organized as follows. Section 2 describes our formulation and solution for the twin problems of finding the optimal number of crawls and the idealized crawl times. We loosely refer to this first component as the *optimal frequency* problem. Section 3 contains the formulation and solution for the second component of our scheme, namely the *scheduling* problem. Section 4 discusses some issues of parameterizing our approach, including several empirical results on update pattern distributions for real web pages based on traces from a production web site. In Section 5 we provides experimental results showing both the quality of our solutions and their running times. Section 6 contains conclusions and areas for future work.

2. CRAWLING FREQUENCY PROBLEM

2.1 General Framework

We formulate the crawling frequency problem within the context of a general model framework, based on stochastic marked point processes. This makes it possible for us to study the problem in a unified manner across a wide range of web environments and assumptions. A rigorous formal definition of our general framework and its important mathematical properties, as well as a rigorous formal analysis of various aspects of our general framework, are beyond the scope of the present paper. We therefore sketch here the model framework and an analysis of a specific instance of this model, referring the interested reader to the sequel for additional technical details. Furthermore, see [24] for additional details on stochastic marked point processes.

We denote by N the total number of web pages to be crawled, which shall be indexed by i. We consider a *scheduling interval* of length T as a basic atomic unit of decision making, where T is sufficiently large to support our model assumptions below. The basic idea is that these scheduling intervals repeat every T units of time, and we will make decisions about one scheduling interval using both new data and the results from the previous scheduling interval. Let R denote the total number of crawls possible in a single scheduling interval.

Let $u_{i,n} \in \mathbb{R}^+$ denote the *point* in time at which the n^{th} update of page i occurs, where $0 < u_{i,1} < u_{i,2} < \ldots \leq T$, $i \in \{1, 2, \ldots, N\}$. Associated with the n^{th} update of page i is a *mark* $k_{i,n} \in \mathbb{K}$, where $k_{i,n}$ is used to represent all important and useful information for the n^{th} update of page i and $\mathbb{K}$ is the space of all such marks (called the mark space). Examples of possible marks include information on the probability of whether an update actually occurs at the corresponding point in time (e.g., see Section 2.3.3) and the probability of whether an actual update matters from the perspective of the crawling frequency problem (e.g., a minimal update of the page may not change the results of the search engine mechanisms). The occurrences of updates to page i are then modeled as a stationary stochastic marked point process $\mathcal{U}_i = \{(u_{i,n}, k_{i,n})\,;\, n \in \mathbb{N}\}$ defined on the state space $\mathbb{R}^+ \times \mathbb{K}$. In other words, $\mathcal{U}_i$ is a stochastic sequence of points $\{u_{i,1}, u_{i,2}, \ldots\}$ in time at which updates of page i occur, together with a corresponding sequence of general marks $\{k_{i,1}, k_{i,2}, \ldots\}$ containing information about these updates.

A counting process $N_i^u(t)$ is associated with the marked point process $\mathcal{U}_i$ and is given by $N_i^u(t) \equiv \max\{n : u_{i,n} \leq t\}$, $t \in \mathbb{R}_+$. This counting process represents the number of updates of page i that occur in the time interval $[0, t]$. The interval of time between the $n-1^{\text{st}}$ and n^{th} update of page i is given by $U_{i,n} \equiv u_{i,n} - u_{i,n-1}$, $n \in \mathbb{N}$, where we define $u_{i,0} \equiv 0$ and $k_{i,0} \equiv \emptyset$. The corresponding forward and backward recurrence times are given by $A_i^u(t) \equiv u_{i,N_i^u(t)+1} - t$ and $B_i^u(t) \equiv t - u_{i,N_i^u(t)}$, respectively, $t \in \mathbb{R}_+$. In this paper we shall make the assumption that the time intervals $U_{i,n} \in \mathbb{R}^+$ between updates of page i are independent and identically distributed (i.i.d.) following an arbitrary distribution function $G_i(\cdot)$ with mean $\lambda_i^{-1} > 0$, and thus the counting process $N_i^u(t)$ is a renewal process [23, 28], $i \in \{1, 2, \ldots, N\}$. Note that $u_{i,0}$ does not represent the time of an actual update, and therefore the counting process $\{N_i^u(t)\,;\, t \in \mathbb{R}_+\}$ (starting at time 0) is, more precisely, an equilibrium renewal process (which is an instance of a delayed renewal process) [23, 28].

Suppose we decide to crawl web page i a total of x_i times during the scheduling interval $[0, T]$ (where x_i is a non-negative integer less than or equal to R), and suppose we decide to do so at the arbitrary times $0 \leq t_{i,1} < t_{i,2} < \ldots < t_{i,x_i} \leq T$. Our approach in this paper is based on computing a particular probability function that captures, in a certain sense, whether the search engine will have a stale copy of web page i at an arbitrary time t in the interval $[0, T]$. From this we can in turn compute a corresponding *time-average* staleness estimate $a_i(t_{i,1}, \ldots, t_{i,x_i})$ by averaging this probability function over all t within $[0, T]$. Specifically, we consider the arbitrary time $t_{i,j}$ as falling within the interval $U_{i,N_i^u(t_{i,j})+1}$

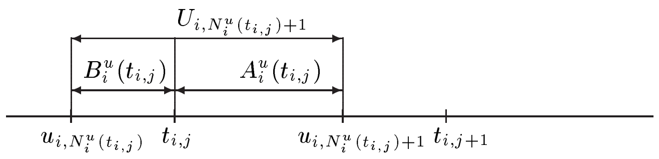


Figure 1: Example of Stationary Marked Point Process Framework

between the two updates of page i at times $u_{i,N_i^u(t_{i,j})}$ and $u_{i,N_i^u(t_{i,j})+1}$, and our interest is in a particular time-average measure of staleness with respect to the forward recurrence time $A_i^u(t_{i,j})$ until the next update, given the backward recurrence time $B_i^u(t_{i,j})$. Figure 1 depicts a simple example of this situation.

More formally, we exploit conditional probabilities to define the following time-average staleness estimate

$$a_i(t_{i,1}, \ldots, t_{i,x_i}) \;=\; \frac{1}{T} \sum_{j=0}^{x_i} \int_{t_{i,j}}^{t_{i,j+1}} \left(\int_0^\infty \frac{\mathsf{P}[\mathbf{A}, \mathbf{B}, \mathbf{C}]}{\mathsf{P}[\mathbf{B}]}\, g_{B_i^u}(v)\, dv \right) dt, \quad (1)$$

where $\mathbf{A} = \{U_{i,n_{i,j}+1} \leq t - t_{i,j} + v\}$, $\mathbf{B} = \{U_{i,n_{i,j}+1} > v\}$, $\mathbf{C} = \{k_{i,n_{i,j}+1} \in \mathcal{K}\}$, $t_{i,0} \equiv 0$, $t_{i,x_i+1} \equiv T$, $n_{i,j} \equiv N_i^u(t_{i,j})$, $g_{B_i^u}(\cdot)$ is the stationary density for the backward recurrence time, and $\mathcal{K} \subset \mathbb{K}$ is the mark set of interest for the staleness estimate under consideration. Note that the variable v is used to integrate over all possible values of $B_i^u(t_{i,j}) \in [0, \infty)$. Further observe the dependence of the staleness estimate on the update patterns for web page i.

When $\mathcal{K} = \mathbb{K}$ (i.e., all marks are considered in the definition of the time-average staleness estimate), then the inner integral above reduces as follows:

$$\int_0^\infty \frac{G_i(t - t_{i,j} + v) - G_i(v)}{1 - G_i(v)} g_{B_i^u}(v) dv \;=\; \int_0^\infty \left(1 - \frac{\overline{G}_i(t - t_{i,j} + v)}{\overline{G}_i(v)}\right) g_{B_i^u}(v) dv; \quad (2)$$

where $\overline{G}_i(t) \equiv 1 - G_i(t)$ is the tail distribution of the interupdate times $U_{i,n}$, $n \in \mathbb{N}$. From standard renewal theory [23, 28], we have $g_{B_i^u}(t) = \lambda_i \overline{G}_i(t)$, and thus after some simple algebraic manipulations we obtain

$$a_i(t_{i,1}, \ldots, t_{i,x_i}) \;=\; \frac{1}{T} \sum_{j=0}^{x_i} \int_{t_{i,j}}^{t_{i,j+1}} \left(1 \;-\; \lambda_i \int_0^\infty \overline{G}_i(t - t_{i,j} + v)\, dv\right) dt. \quad (3)$$

Naturally we would also like the times $t_{i,1}, \ldots t_{i,x_i}$ to be chosen so as to minimize the time-average staleness estimate $a_i(t_{i,1}, \ldots, t_{i,x_i})$, given that there are x_i crawls of page i. Deferring for the moment the question of how to find these optimal values $t^*_{i,1}, \ldots t^*_{i,x_i}$, let us define the function $\mathcal{A}_i$ by setting

$$\mathcal{A}_i(x_i) = a_i(t^*_{i,1}, \ldots, t^*_{i,x_i}). \quad (4)$$

Thus, the domain of this function $\mathcal{A}_i$ is the set $\{0, \ldots, R\}$.

We now must decide how to find the optimal values of the x_i variables. While one would like to choose x_i as large as possible, there is competition for crawls from the other web pages. Taking all web pages into account, we therefore wish to minimize the objective function

$$\sum_{i=1}^{N} w_i \mathcal{A}_i(x_i) \quad (5)$$

subject to the constraints

$$\sum_{i=1}^{N} x_i = R, \quad (6)$$

$$x_i \in \{0, \ldots, R\}. \quad (7)$$

Here the weights w_i will determine the relative importance of each web page i. If each weight w_i is chosen to be 1, then the problem becomes one of minimizing the time-average staleness estimate across all the web pages i. However, we will shortly discuss a way to pick these weights a bit more intelligently, thereby minimizing a metric that computes the level of *embarrassment* the search engine has to endure.

The optimization problem just posed has a very nice form. Specifically, it is an example of a so-called *discrete, separable resource allocation problem*. The problem is separable by the nature of the objective function, written as the summation of functions of the individual x_i variables. The problem is discrete because of the second constraint, and a resource allocation problem because of the first constraint. For details on resource allocation problems, we refer the interested reader to [12]. This is a well-studied area in optimization theory, and we shall borrow liberally from that literature. In doing so, we first point out that there exists a *dynamic programming* algorithm for solving such problems, which has computational complexity $O(NR^2)$.

Fortunately, we will show shortly that the function $\mathcal{A}_i$ is *convex*, which in this discrete context means that the *first differences* $\mathcal{A}_i(j+1) - \mathcal{A}_i(j)$ are non-decreasing as a function of j. (These first differences are just the discrete analogues of derivatives.) This extra structure makes it possible to employ even faster algorithms, but before we can do so there remain a few important issues. Each of these issues is discussed in detail in the next three subsections, which involve: (1) computing the weights w_i of the *embarrassment level* metric for each web page i; (2) computing the functional forms of a_i and $\mathcal{A}_i$ for each web page i, based on the corresponding marked point process $\mathcal{U}_i$; and (3) solving the resulting discrete, convex, separable resource allocation problem in a highly efficient manner.

It is important to point out that we can actually handle a more general resource allocation constraint than that given in Equation (7). Specifically, we can handle both minimum and maximum number of crawls m_i and M_i for each page i, so that the constraint instead becomes $x_i \in \{m_i, \ldots, M_i\}$. We can also handle other types of constraints on the crawls that tend to arise in practice [4], but omit details here in the interest of space.

2.2 Computing the Weights w_i

Consider Figure 2, which illustrates a decision tree tracing the possible results for a client making a search engine query. Let us fix a particular web page i in mind, and follow the decision tree down from the root to the leaves.

The first possibility is for the page to be fresh: In this case the web page will not cause embarrassment to the search engine. So, assume the page is stale. If the page is never returned by the search engine, there again can be no embarrassment: The search engine is simply lucky in this case.

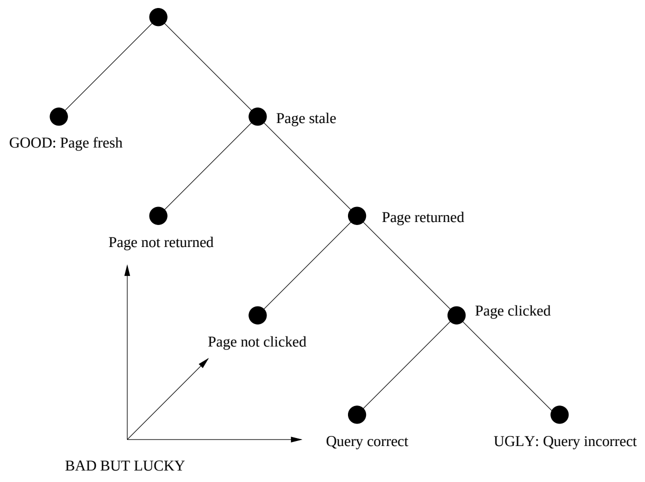


Figure 2: Embarrassment Level Decision Tree

What happens if the page is returned? A search engine will typically organize its query responses into multiple *result pages*, and each of these result pages will contain the urls of several returned web pages, in various *positions* on the page. Let P denote the number of positions on a returned page (which is typically on the order of 10). Note that the position of a returned web page on a result page reflects the ordered estimate of the search engine for the web page matching what the user wants. Let $b_{i,j,k}$ denote the probability that the search engine will return page i in position j of query result page k. The search engine can easily estimate these probabilities, either by monitoring all query results or by sampling them for the client queries.

The search engine can still be lucky even if the web page i is stale and returned: A client might not click on the page, and thus never have a chance to learn that the page was stale. Let $c_{j,k}$ denote the frequency that a client will click on a returned page in position j of query result page k. These frequencies also can be easily estimated, again either by monitoring or sampling.

One can speculate that this clicking probability function might typically decrease both as a function of the overall position $(k-1)P + j$ of the returned page and as a function of the page k on which it is returned. Assuming a Zipf-like function [29, 15] for the first function and a geometric function to model the probability of cycling through $k-1$ pages to get to returned page k, one would obtain a clicking probability function that looks like the one provided in Figure 3. (According to [4] there is some evidence that the clicking probabilities in Figure 3 actually *rise* rather than fall as a new page is reached. This is because some clients do not scroll down – some do not even know how to do so. More importantly, [4] notes that this data can actually be collected by the search engine.)

Finally, even if the web page is stale, returned by the search engine, and clicked on: The changes to the page might not cause the results of the query to be wrong. This truly *lucky loser* scenario can be more common than one might initially suspect, because most web pages typically do not change in terms of their basic content, and most client queries will probably try to address this basic content. In any case, let d_i denote the probability that a query to a stale

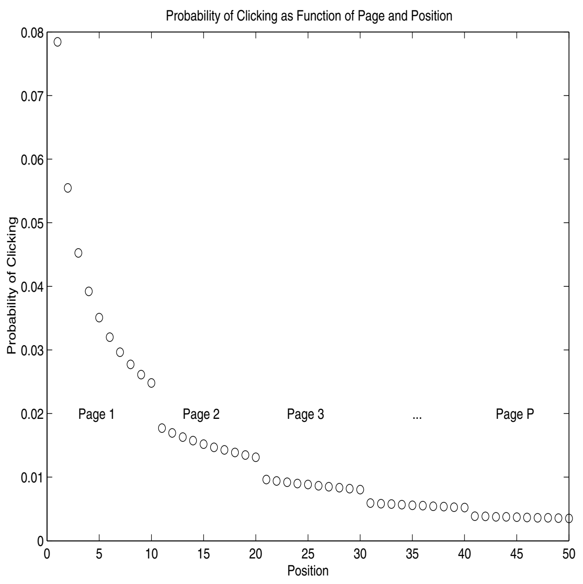


Figure 3: Probability of Clicking as Function of Position/Page

version of page i yields an incorrect response. Once again, this parameter can be easily estimated.

Assuming, as seems reasonable, that these three types of events are independent, one can compute the total level of embarrassment caused to the search engine by web page i as

$$w_i = d_i \sum_j \sum_k c_{j,k} b_{i,j,k}. \tag{8}$$

Note that although the functional form of the above equation is a bit involved, the value of w_i is simply a constant from the perspective of the resource allocation problem.

2.3 Computing the Functions a_i and $\mathcal{A}_i$

As previously noted, the functions to be computed in this section depend upon the characteristics of the marked point process $\mathcal{U}_i = \{(u_{i,n}, k_{i,n})\,;\, n \in \mathbb{N}\}$ which models the update behaviors of each page i. We consider three types of marked point processes that represent different cases which are expected to be of interest in practice. The first two cases are based on the use of Equation (3) to compute the functions a_i and $\mathcal{A}_i$ under different distributional assumptions for the interupdate times $U_{i,n}$ of the marked point process $\mathcal{U}_i$; specifically, we consider $G_i(\cdot)$ to be exponential and general distribution functions, respectively, where the former has been the primary case considered in previous studies and the latter is used to state a few important properties of this general form (which are used in turn to obtain the experimental results presented in Section 4). The third case, which we call *quasi-deterministic*, is based on a finite number of specific times $u_{i,n}$ at which page i *might* be updated, where the corresponding marks $k_{i,n}$ represent the probability that the update at time $u_{i,n}$ actually occurs.

2.3.1 *Exponential Distribution Function* G_i

Consider as a simple but prototypical example the case in which the time intervals $U_{i,n} \in \mathbb{R}^+$ between updates of page i are i.i.d. following an exponential distribution with parameter λ_i, i.e., $G_i(t) = 1 - e^{-\lambda_i t}$ and $\overline{G}_i(t) = e^{-\lambda_i t}$ [28]. In this case we also assume that all updates are of interest irrespective of their associated mark values (i.e., $\mathcal{K} = \mathbb{K}$).

Suppose as before that we crawl a total of x_i times in the scheduling interval at the arbitrary times $t_{i,1}, \ldots t_{i,x_i}$. It then follows from Equation (3) that the time-average staleness estimate is given by

$$\begin{aligned} &a_i(t_{i,1}, \ldots, t_{i,x_i}) \\ &\quad = \frac{1}{T} \sum_{j=0}^{x_i} \int_{t_{i,j}}^{t_{i,j+1}} \left(1 - \lambda_i \int_0^\infty e^{-\lambda_i (t - t_{i,j} + v)} dv \right) dt \\ &\quad = \frac{1}{T} \sum_{j=0}^{x_i} \int_{t_{i,j}}^{t_{i,j+1}} \left(1 - e^{-\lambda_i (t - t_{i,j})}\right) dt. \end{aligned} \tag{9}$$

After some minor algebraic manipulations, we obtain that the time-average staleness estimate is given by

$$a_i(t_{i,1}, \ldots, t_{i,x_i}) = 1 + \frac{1}{\lambda_i T} \sum_{j=0}^{x_i} \left(e^{-\lambda_i (t_{i,j+1} - t_{i,j})} - 1\right). \tag{10}$$

Letting $T_{i,j} = t_{i,j+1} - t_{i,j}$ for all $0 \le j \le x_i$, then the problem of finding $\mathcal{A}_i$ reduces to minimizing

$$1 + \frac{1}{\lambda_i T} \sum_{j=0}^{x_i} \left(e^{-\lambda_i T_{i,j}} - 1\right) \tag{11}$$

subject to the constraints

$$0 \le T_{i,j} \le T, \tag{12}$$

$$\sum_{j=0}^{x_i} T_{i,j} = T. \tag{13}$$

Modulo a few constants, which are not important to the optimization problem, this now takes the form of a *continuous*, convex, separable resource allocation problem. As before, the problem is separable by the nature of the objective function. It is continuous because of the first constraint, and it is a resource allocation problem because of the second constraint; it is also convex for the reasons provided below.

The key point is that the optimum value is known to occur at the value $(T^*_{i,1}, \ldots, T^*_{i,x_i})$ where the derivatives $T^{-1} e^{-\lambda_i T^*_{i,j}}$ of the summands in Equation (11) are equal, subject to the constraints $0 \le T^*_{i,j} \le T$ and $\sum_{j=0}^{x_i} T^*_{i,j} = T$. The general result, originally due to [9], was the seminal paper in the theory of resource allocation problems, and there exist several fast algorithms for finding the values of $T^*_{i,j}$. See [12] for a good exposition of these algorithms. In our special case of the exponential distribution, however, the summands are all identical, and thus the optimal decision variables as well can be found by inspection: They occur when each $T^*_{i,j} = T/(x_i + 1)$. Hence, we can write

$$\mathcal{A}_i(x_i) = 1 + \frac{x_i + 1}{\lambda_i T} \left(e^{-\lambda_i T/(x_i+1)} - 1\right), \tag{14}$$

which is easily shown to be convex.

2.3.2 *General Distribution Function* G_i

Now let us consider the same scenario as the previous section but where the distribution of the interupdate times $U_{i,n} \in \mathbb{R}^+$ for page i is an arbitrary distribution $G_i(\cdot)$ with mean λ_i^{-1}. Then we observe from Equation (3) a few important properties of this general form. First, it is clear from this formula that the summands remain separable. Given that all the summands are also identical, the optimal decision variables occur when each $T^*_{i,j} = T/(x_i + 1)$ as in the exponential case.

2.3.3 *Quasi-Deterministic Case*

Suppose the marked point process $\mathcal{U}_i$ consists of a deterministic sequence of points $\{u_{i,1}, u_{i,2}, \ldots, u_{i,Q_i}\}$ defining possible update times for page i, together with a sequence of marks $\{k_{i,1}, k_{i,2}, \ldots, k_{i,Q_i}\}$ defining the probability of whether the corresponding update actually occurs. Here we eliminate the i.i.d. assumption of Section 2.1 and consider an arbitrary sequence of specific times such that $0 \le u_{i,1} < u_{i,2} < \ldots < u_{i,Q_i} \le T$. Recall that $u_{i,0} \equiv 0$ and define $u_{i,Q_i} \equiv T$ for convenience. The update at time $u_{i,j}$ occurs with probability $k_{i,j}$. If $k_{i,j} = 1$ for all $j \in \{1, \ldots, Q_i\}$, then the update pattern reduces to being purely deterministic. We shall assume that the value $k_{i,0}$ can be inferred from the crawling strategy employed in the *previous* scheduling interval(s). Our interest is again on determining the time-average staleness estimate $\mathcal{A}_i(x_i)$ for x_i optimally chosen crawls.

A key observation is that all crawls should be done at the potential update times, because there is no reason to delay beyond when the update has occurred. This also implies that we can assume $x_i \le Q_i + 1$, as there is no reason to crawl more frequently. (The maximum of $Q_i + 1$ crawls corresponds to the time 0 and the Q_i other potential update times.) Hence, consider the binary decision variables

$$y_{i,j} = \begin{cases} 1, & \text{if a crawl occurs at time } u_{i,j}; \\ 0, & \text{otherwise.} \end{cases} \quad (15)$$

If we crawl x_i times, then we have $\sum_{j=0}^{Q_i} y_{i,j} = x_i$.

Note that, as a consequence of the above assumptions and observations, the two integrals in Equation (1) reduce to a much simpler form. Specifically, let us consider a staleness probability function $\bar{p}(y_{i,0}, \ldots, y_{i,Q_i}, t)$ at an arbitrary time t, which we now compute. Recall that $N^u_i(t)$ provides the index of the latest potential update time that occurs at or before time t, so that $N^u_i(t) \le Q_i$. Similarly, define

$$J_i(t) \;=\; \max\{j : u_{i,j} \le t,\ y_{i,j} = 1,\ 0 \le j \le Q_i\}^+, \quad (16)$$

which is the index of the latest potential update time at or before time t that is actually going to be *crawled.* Clearly we can also unambiguously use $J_{i,j}$ to abbreviate the value of $J_i(t)$ at any time t for which $j = N^u_i(t)$. Now we have

$$\bar{p}(y_{i,0}, \ldots, y_{i,Q_i}, t) \;=\; 1 - \prod_{j=J_i(t)+1}^{N^u_i(t)} (1 - k_{i,j}), \quad (17)$$

where a product over the empty set, as per normal convention, is assumed to be 1.

Figure 4 illustrates a typical staleness probability function $\bar{p}$. (For visual clarity we display the *freshness* function $1 - \bar{p}$ rather than the staleness function in this figure.) Here the potential update times are noted by circles on the x-axis. Those which are actually crawled are depicted as filled circles, while those that are not crawled are left unfilled. The freshness function jumps to 1 during each interval immediately to the right of a crawl time, and then decreases, interval by interval, as more terms are multiplied into the product (see Equation (17)). The function is constant during each interval – that is precisely why $J_{i,j}$ can be defined.

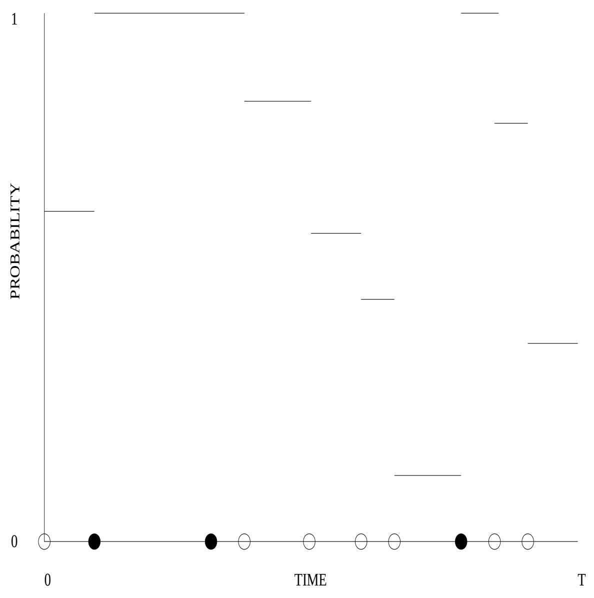


Figure 4: Freshness Probability Function for Quasi-Deterministic Web Pages

Now we can compute the corresponding time-average probability estimate as

$$\bar{a}(y_{i,0}, \ldots, y_{i,Q_i}) = \sum_{j=0}^{Q_i} u_{i,j} [1 - \prod_{k=J_{i,j}+1}^{j} (1 - k_{i,j})]. \quad (18)$$

The question of how to choose the optimal x_i crawl times is perhaps the most subtle issue in this paper. We can write this as a discrete resource allocation problem, namely the minimization of Equation (18) subject to the constraints $y_{i,j} \in \{0, 1\}$ and $\sum_{j=0}^{Q_i} y_{i,j} = x_i$. The problem with this is that the decision variables $y_{i,j}$ are highly intertwined in the objective function. While our optimization problem can be solved exactly by the use of so-called *non-serial dynamic programming* algorithms, as shown in [12], or can be solved as a general integer program, such means to obtain the problem solution will not have good performance. Hence, for reasons we shall describe momentarily, we choose to employ a *greedy* algorithm for the optimization problem: That is, we first estimate the value of $\mathcal{A}_i(1)$ by picking that index $0 \le j \le Q_i$ for which the objective function will decrease the most when $y_{i,j}$ is turned from 0 to 1. In the general inductive step we assume that we are given an estimate for $\mathcal{A}_i(x_i - 1)$. Then to compute $\mathcal{A}_i(x_i)$ we pick that index $0 \le j \le Q_i$ with $y_{i,j} = 0$ for which the objective function decreases the most upon setting $y_{i,j} = 1$. It can be shown that the greedy algorithm does not, in general, find the optimal solution. However, the average freshness can be easily shown to be an increasing, submodular function (in the number of crawls), and so the greedy algorithm is guaranteed to produce a solution with average freshness at least $(1 - 1/e)$ of the best possible [18]. For the special case we consider, we believe the worst-case performance guarantee of the greedy algorithm is strictly better. We therefore feel justified in suggesting the greedy algorithm as a heuristic to find $\mathcal{A}_i$ in general. Moreover, it is trivial to check that the function $\mathcal{A}_i$, when estimated in this way, is convex: If, given two successive greedy choices, the first difference decreases, then the second greedy choice would have been chosen before the first one.

2.4 Solving the Discrete Separable Convex Resource Allocation Problem

As noted, the optimization problem described above is a special case of a discrete, convex, separable resource allocation problem. The problem of minimizing

$$\sum_{i=1}^{N} F_i(x_i) \tag{19}$$

subject to the constraints

$$\sum_{i=1}^{N} x_i = R \tag{20}$$

and

$$x_i \in \{m_i, \ldots, M_i\} \tag{21}$$

with convex F_is is very well studied in the optimization literature. We point the reader to [12] for details on these algorithms. We content ourselves here with a brief overview.

The earliest known algorithm for discrete, convex, separable resource allocation problems is essentially due to Fox [9]. More precisely, Fox looked at the continuous case, noting that the Lagrange multipliers (or Kuhn-Tucker conditions) implied that the optimal value occurred when the derivatives were as equal as possible, subject to the above constraints. This gave rise to a *greedy* algorithm for the discrete case which is usually attributed to Fox. One forms a matrix in which the (i,j)th term $D_{i,j}$ is defined to be the *first difference*: $D_{i,j} = F_i(j+1) - F_i(j)$. By convexity the columns of this matrix are guaranteed to be monotone, and specifically non-decreasing. The greedy algorithm initially sets each x_i to be m_i. It then finds the index i for which $x_i + 1 \leq M_i$ and the value of the next first difference D_{i,x_i} is minimal. For this index i^* one increments x_{i^*} by 1. Then this process repeats until Equation (20) is satisfied, or until the set of allowable indices i empties. (In that case there is no feasible solution.) Note that the first differences are just the discrete analogs of derivatives for the continuous case, and that the greedy algorithm finds a solution in which, modulo Constraint (21), all first differences are as equal as possible. The complexity of the greedy algorithm is $O(N + R\log N)$.

There is a faster algorithm for our problem, due to Galil and Megiddo [11], which has complexity $O(N(\log R)^2)$. The fastest algorithm is due to Frederickson and Johnson [10], and it has complexity $O(\max\{N, N\log(R/N)\})$. The algorithm is highly complex, consisting of three components. The first component eliminates elements of the D matrix from consideration, leaving $O(R)$ elements and taking $O(N)$ time. The second component iterates $O(\log(R/N))$ times, each iteration taking $O(N)$ time. At the end of this component only $O(N)$ elements of the D matrix remain. Finally, the third component is a linear time selection algorithm, finding the optimal value in $O(N)$ time. For full details on this algorithm see [12]. We employ the Frederickson and Johnson algorithm in this paper.

There do exist some alternative algorithms which could be considered for our particular optimization problem. For example, the quasi-deterministic web page portion of the optimization problem is inherently discrete, but the portions corresponding to other distributions can be considered as giving rise to a continuous problem (which is done in [6]). In the case of distributions for which the expression for $\mathcal{A}_i$ is differentiable, and for which the derivative has a closed form expression, there do exist very fast algorithms for (a) solving the continuous case, and (b) relaxing the continuous solution to a discrete solution. So, if all web pages had such distributions the above approach could be attractive. Indeed, if *most* web pages had such distributions, one could partition the set of web pages into two components. The first set could be solved by continuous relaxation, while the complementary set could be solved by a discrete algorithm such as that given by [10]. As the amount of resource given to one set goes up, the amount given to the other set would go down. So a bracket and bisection algorithm [22], which is logarithmic in complexity, could be quite fast. We shall not pursue this idea further here.

3. CRAWLER SCHEDULING PROBLEM

Given that we know how many crawls should be made for each web page, the question now becomes how to best schedule the crawls over a scheduling interval of length T. (Again, we shall think in terms of scheduling *intervals* of length T. We are trying to optimally schedule the *current* scheduling interval using some information from the last one.) We shall assume that there are C possibly heterogeneous crawlers, and that each crawler k can handle S_k crawl tasks in time T. Thus we can say that the total number of crawls in time T is $R = \sum_{k=1}^{C} S_k$. We shall make one simplifying assumption that each crawl on crawler k takes approximately the same amount of time. Thus we can divide the time interval T into S_k equal size time *slots*, and estimate the start time of the lth slot on crawler k by $T_{kl} = (l-1)/T$ for each $1 \leq l \leq S_k$ and $1 \leq k \leq C$.

We know from the previous section the desired number of crawls x_i^* for each web page i. Since we have already computed the optimal schedule for the last scheduling interval, we further know the start time $t_{i,0}$ of the final crawl for web page i within the last scheduling interval. Thus we can compute the optimal crawl times $t_{i,1}^*, \ldots, t_{i,x_i}^*$ for web page i during the current scheduling interval. For the stochastic case it is important for the scheduler to initiate each of these crawl tasks at approximately the proper time, but being a bit early or a bit late should have no serious impact for most of the update probability distribution functions we envision. Thus it is reasonable to assume a scheduler cost function for the jth crawl of page i, whose update patterns follow a stochastic process, that takes the form $S(t) = |t - t_{i,j}^*|$. On the other hand, for a web page i whose update patterns follow a quasi-deterministic process, being a bit *late* is acceptable, but being *early* is not useful. So an appropriate scheduler cost function for the jth crawl of a quasi-deterministic page i might have the form

$$S(t) = \begin{cases} \infty & \text{if } t < t_{i,j}^* \\ t - t_{i,j} & \text{otherwise} \end{cases} \tag{22}$$

In terms of scheduling notation, the above crawl task is said to have a *release time* of $t_{i,j}$. See [3] for more information on scheduling theory.

Virtually no work seems to have been done on the scheduling problem in the research literature on crawlers. Yet there

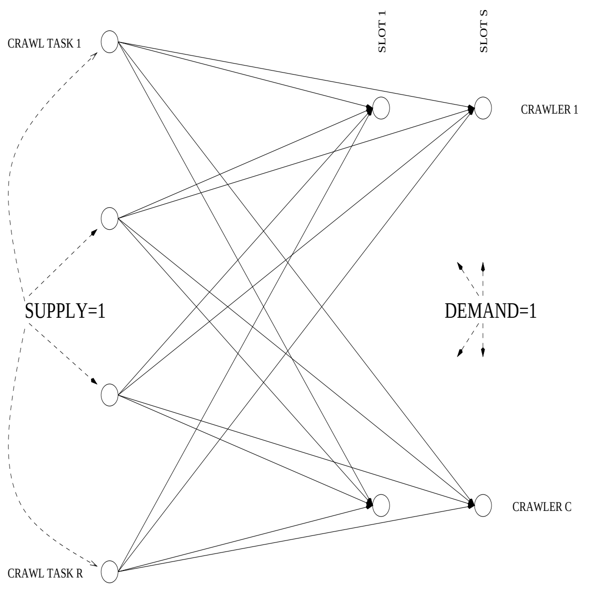


Figure 5: Transportation Problem Network

is a simple, exact solution for the problem. Specifically, the problem can be posed and solved as a *transportation problem* as we shall we now describe. See [1] for more information on transportation problems and network flows in general. We describe our scheduling problem in terms of a network.

We define a bipartite network with one directed arc from each supply node to each demand node. The R supply nodes, indexed by j, correspond to the crawls to be scheduled. Each of these nodes has a supply of 1 unit. There will be one demand node per time slot and crawler pair, each of which has a demand of 1 unit. We index these by $1 \leq l \leq S_k$ and $1 \leq k \leq C$. The cost of arc jkl emanating from a supply node j to a demand node kl is $S_j(T_{kl})$. Figure 5 shows the underlying network for an example of this particular transportation problem. Here for simplicity we assume that the crawlers are homogeneous, and thus that each can crawl the same number $S = S_k$ of pages in the scheduling interval T. In the figure the number of crawls is $R = 4$, which equals the number of crawler time slots. The number of crawlers is $C = 2$, and the number of crawls per crawler is $S = 2$. Hence $R = CS$.

The specific linear optimization problem solved by the transportation problem can be formulated as follows.

$$\text{Minimize} \sum_{i=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{M} R_i(T_{jk}) f_{ijk} \tag{23}$$

such that

$$\sum_{i=1}^{M} f_{ijk} = 1 \; \forall \; 1 \leq j \leq N \text{ and } 1 \leq k \leq M, \tag{24}$$

$$\sum_{j=1}^{N} \sum_{k=1}^{M} f_{ijk} = 1 \; \forall \; 1 \leq i \leq M, \tag{25}$$

$$f_{ijk} \geq 0 \; \forall \; 1 \leq i, k \leq M \text{ and } 1 \leq j \leq N. \tag{26}$$

The solution of a transportation problem can be accomplished quickly. See, for example, [1].

Although not obvious at first glance, the nature of the transportation problem formulation ensures that there exists an optimal solution with integral flows, and the techniques in the literature find such a solution. Again, see [1] for details. This implies that each f_{ijk} is binary. If $f_{ijk} = 1$ then a crawl of web page i is assigned to the jth crawl of crawler k.

If it is required to fix or restrict certain crawl tasks from certain crawler slots, this can be easily done: One simply changes the costs of the restricted directed arcs to be infinite. (Fixing a crawl task to a subset of crawler slots is the same as restricting it from the complementary crawler slots.)

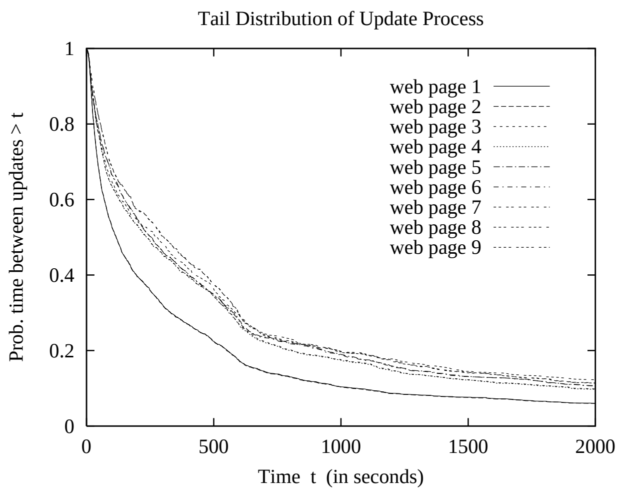
4. PARAMETERIZATION ISSUES

The use of our formulation and solution in practice requires calculating estimates of the parameters of our general model framework. In the interest of space, we sketch here some of the issues involved in addressing this problem, and refer the interested reader to the sequel for additional details.

Note that when each page i is crawled we can easily obtain the last update time for the page. While this does not provide information about any other updates occurring since the last crawl of page i, we can use this information together with the data and models for page i from previous scheduling intervals to statistically infer key properties of the update process for the page. This is then used in turn to construct a probability distribution (including a quasi-deterministic distribution) for the interupdate times of page i.

Another important aspect of our approach concerns the statistical properties of the update process. The analysis of previous studies has essentially assumed that the update process is Poisson [6, 7, 26], i.e., the interupdate times for each page follow an exponential distribution. Unfortunately, very little has been published in the research literature on the properties of update processes found in practice, with the sole exception (to our knowledge) of a recent study [19] suggesting that the interupdate times of pages at a news service web site are not exponential. To further investigate the prevalence of exponential interupdate times in practice, we analyze the page update data from another web site environment whose content is highly accessed and highly dynamic. Specifically, we consider the update patterns found at the web site for the 1998 Nagano Olympic Games, referring the interested reader to [16, 25] for more details on this environment. Figure 6 plots the tail distributions of the time between updates for each of a set of 18 individual dynamic pages which are representative of the update pattern behaviors found in our study of all dynamic pages that were modified a fair amount of time. In other words, the curves illustrate the probability that the time between updates to a given page is greater than t as a function of time t.

We first observe from these results that the interupdate time distributions can differ significantly from an exponential distribution. More precisely, our results suggest that the interupdate time distribution for some of the web pages at Nagano have a tail that decays at a subexponential rate and can be closely approximated by a subset of the Weibull distributions; i.e., the tail of the long-tailed Weibull interupdate distribution is given by $\overline{G}(t) = \mathrm{e}^{-\lambda t^{\alpha}}$, where $t \geq 0$, $\lambda > 0$ and $0 < \alpha < 1$. We further find that the interupdate time distribution for some of the other web pages at Nagano have a heavy tail and can be closely approximated by the



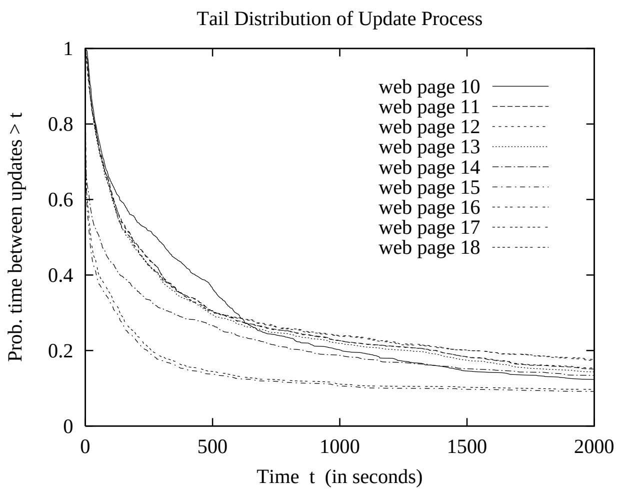


Figure 6: Tail Distributions of Update Processes

class of Pareto distributions; i.e., the tail of the Pareto interupdate time distribution is given by $\overline{G}(t) = (\beta/(\beta + t))^\alpha$, where $t \geq 0$, $\beta > 0$ and $0 < \alpha < 2$. Moreover, some of the periodic behavior observed in the update patterns for some web pages can be addressed with our quasi-deterministic distribution.

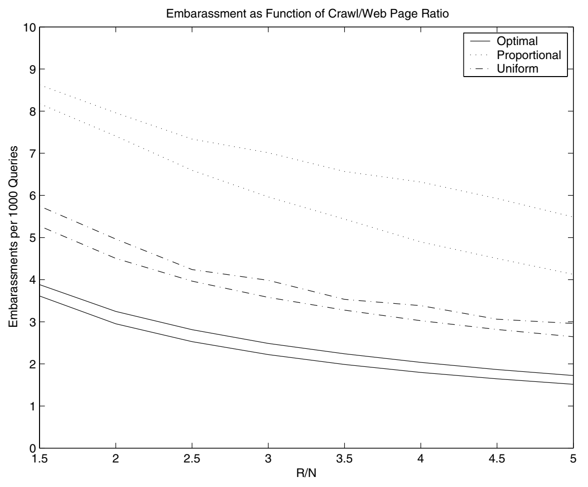
The above analysis is not intended to be an exhaustive study by any means. Our results, together with those in [19], suggest that there are important web site environments in which the interupdate times do not follow an exponential distribution. This includes the quasi-deterministic instance of our general model, which is motivated by information services as discussed above. The key point is that there clearly are web environments in which the update process for individual web pages can be much more complex than a Poisson process, and our general formulation and solution of the crawler scheduling problem makes it possible for us to handle such a wide range of web environments within this unified framework.

5. EXPERIMENTAL RESULTS

Using the empirical data and analysis of the previous section we now illustrate the performance of our scheme. We will focus on the problem of finding the optimal number of crawls, comparing our scheme with two simpler algorithms. Both of these algorithms were considered in [6], and they are certainly natural alternatives. The first scheme might be called *proportional*. We simply allocate the total amount of crawls R according to the average update rates of the various web pages. Modulo integrality concerns, this means that we choose $x_i \propto \lambda_i$. The second scheme is simpler yet, allocating the number of crawls as evenly as possible amongst the web pages. We label this the *uniform* scheme. Each of these schemes can be amended to handle our embarrassment metric weights. When speaking of these variants we will use the terms *weighted proportional* and *weighted uniform*. The former chooses $x_i \propto w_i\lambda_i$. The latter is something of a misnomer: We are choosing $x_i \propto w_i$, so this is essentially a slightly different proportional scheme. We can also think of our optimal scheme as *weighted*, solving for the smallest objective function $\sum_{i=1}^{N} w_i\mathcal{A}_i$. If we solve instead for the smallest objective function $\sum_{i=1}^{N} \mathcal{A}_i$, we get an *unweighted* optimal algorithm. This is essentially the same problem solved in [6], provided, of course, that all web pages are updated according to a Poisson process. But our scheme will have much greater speed. Even though this algorithm omits the weights in the formulation of the problem, we must compare the quality of the solution based on the *weighted* objective function value.

In our experiments we consider combinations of different types of web page update distributions. In a number of cases we use a mixture of 60% Poisson, 30% Pareto and 10% quasi-deterministic distributions. In the experiments we choose T to be one day, though we have made runs for a week as well. We set N to be one million web pages, and varied R between 1.5 and 5 million crawls. We assumed that the average rate of updates over all pages was 1.5, and these updates were chosen according to a Zipf-like distribution with parameters N and θ, the latter chosen between 0 and 1 [29, 15]. Such distributions run the spectrum from highly skewed (when $\theta = 0$) to totally uniform (when $\theta = 1$). We considered both the staleness and embarrassment metrics. When considering the embarrassment metric we derived the weights in Equation (8) by considering a search engine which returned 5 result pages per query, with 10 urls on a page. The probabilities $b_{i,j,k}$ with which the search engine returns page i in position j of query result page k were chosen by linearizing these 50 positions, picking a randomly chosen center, and imposing a truncated normal distribution about that center. The clicking frequencies $c_{j,k}$ for position j of query page k are chosen as described in Section 2.2, via a Zipf-like function with parameters 50 and $\theta = 0$, with a geometric function for cycling through the pages. We assumed that the client went from one result page to the next with probability 0.8. We choose the lucky loser probability d_i of web page i yielding an incorrect response to the client query by picking a uniform random number between 0 and 1. All curves in our experiments display the analytically computed objective function values of the various schemes.

Figure 7 shows two experiments using the embarrassment metric. In the left-hand side of the figure we consider the results under different mixtures of update distributions by



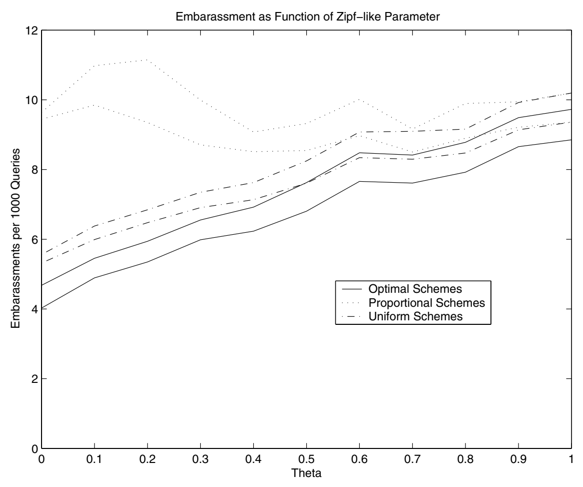


Figure 7: Two Embarrassment Metric Examples

varying the ratio of R to N from 1.5 to 5. We consider here a true Zipf distribution for the update frequencies – in other words we choose Zipf-like parameter $\theta = 0$. There are six curves, namely optimal, proportional and uniform in both weighted and unweighted versions. (The unweighted optimal curve is the result of employing unit weights during the computation phase, but displaying the weighted optimal objective function.) By definition the unweighted optimal scheme will not perform as well as the weighted optimal scheme, which is indeed the best possible solution. In all *other* cases, however, the unweighted variant does better than the weighted variant. So the true uniform policy does the best amongst all of the heuristics, at least for the experiments considered in our study. This somewhat surprising state of affairs was noticed in [6] as well. Both uniform policies do better than their proportional counterparts. Notice that the weighted optimal curve is, in fact, convex as a function of increasing R. This will always be true.

In the right-hand side of Figure 7 we show a somewhat differing mixture of distributions. In this case we vary the Zipf-like parameter θ while holding the value of R to be 2.5 million (so that $R/N = 2.5$). As θ increases, thus yielding less skew, the objective functions generally increase as well. This is appealing, because it shows in particular that the optimal scheme does very well in highly skewed scenarios, which we believe are more representative of real web environments. Moreover, notice that the curves essentially converge to each other as θ increases. This is not too surprising, since the optimal, proportional and uniform schemes would all result in the same solutions precisely in the absence of weights when $\theta = 1$. In general the uniform scheme does relatively better in this figure than it did in the previous one. The explanation is complex, but it essentially has to do with the correlation of the weights and the update frequencies. In fact, we show this example because it puts uniform in its best possible light.

In Figure 8 we show four experiments where we varied the ratio of R to N. These figures depict the average staleness metric, and so we only have three (unweighted) curves per figure. The top left-hand figure depict a mixture of update distribution types, and the other three figures depict, in turn, pure Poisson, Pareto and quasi-deterministic distributions. Notice that these curves are cleaner than those in Figure 7. The weightings, while important, introduce a degree of noise into the objective function values. For this reason we will focus on the non-weighted case from here on. The y-axis ranges differ on each of the four figures, but in all cases the optimal scheme yields a convex function of R/N (and thus of R). The uniform scheme performs better than the proportional scheme once again. It does relatively less well in the Poisson update scenario. In the quasi-deterministic figure the optimal scheme is actually able to reduce average staleness to 0 for sufficiently large R values.

In Figure 9 we explore the case of Pareto interupdate distributions in more detail. Once again, the average staleness metric is plotted as a function of the parameter α in the Pareto distribution; refer to Section 4. This distribution is said to have a heavy tail when $0 < \alpha < 2$, which is quite interesting because it is over this range of parameter values that the optimal solution is most sensitive. In particular, we observe that the optimal solution value is rather flat for values of α ranging from 4 toward 2. However, as α approaches 2, the optimal average staleness value starts to rise which continues to increase in an exponential manner as α ranges from 2 toward 0. These trends appear to hold for all three schemes, with our optimal scheme continuing to provide the best performance and uniform continuing to outperform the proportional scheme. Our results suggest the importance of supporting heavy-tailed distributions when they exist in practice (and our results of the previous section demonstrate that they do indeed exist in practice). This is appealing because it shows in particular that the optimal scheme does very well in these complex environments which may be more representative of real web environments than those considered in previous studies.

The transportation problem solution to the scheduling problem is optimal, of course. Furthermore, the quality of the solution, measured in terms of the deviation of the actual time slots for the various tasks from their ideal time slots, will nearly always be outstanding. Figure 10 shows an illustrative example. This example involves the scheduling of one day with $C = 10$ homogeneous crawlers and 1 million crawls per crawler. So there are 10 million crawls in all. Virtually all crawls occur within a window of plus or mi-

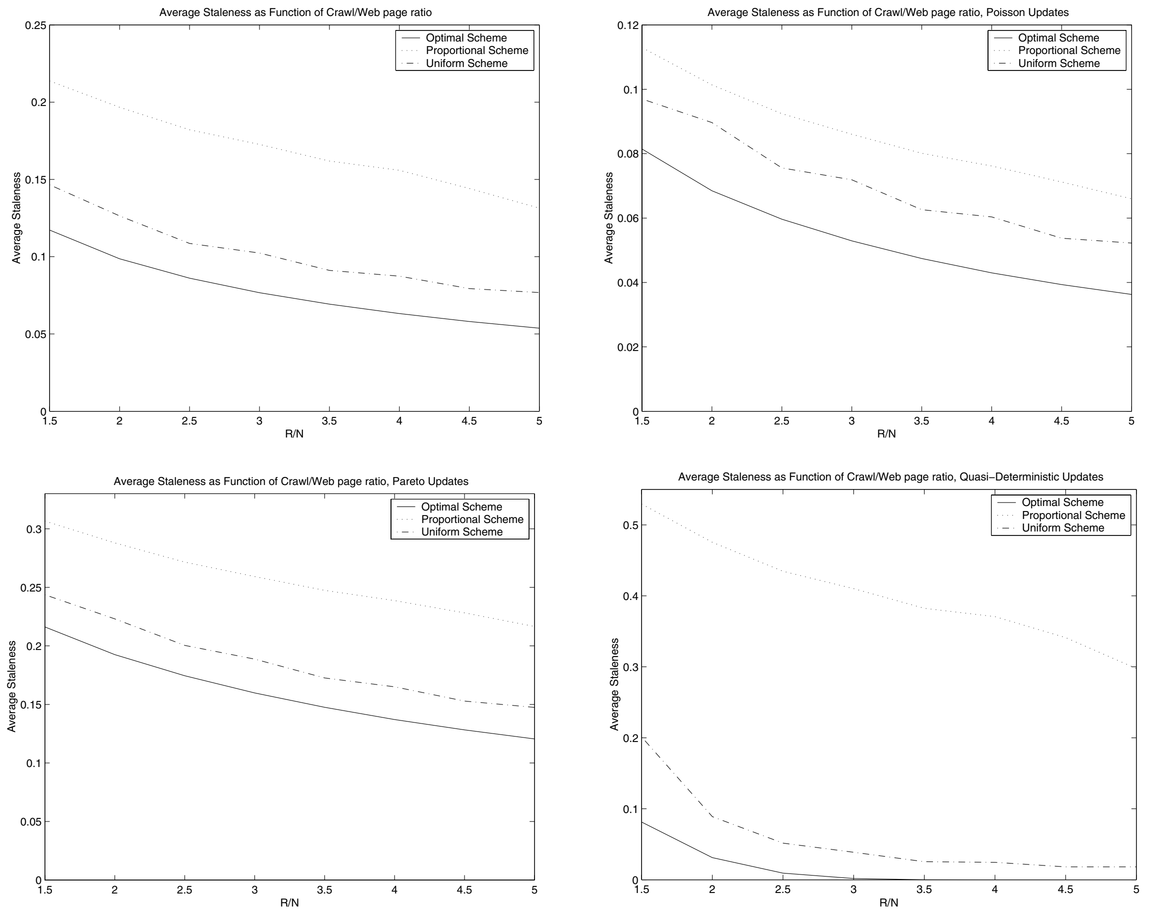


Figure 8: Four Average Staleness Metric Examples: Mixed, Poisson, Pareto and Quasi-Deterministic

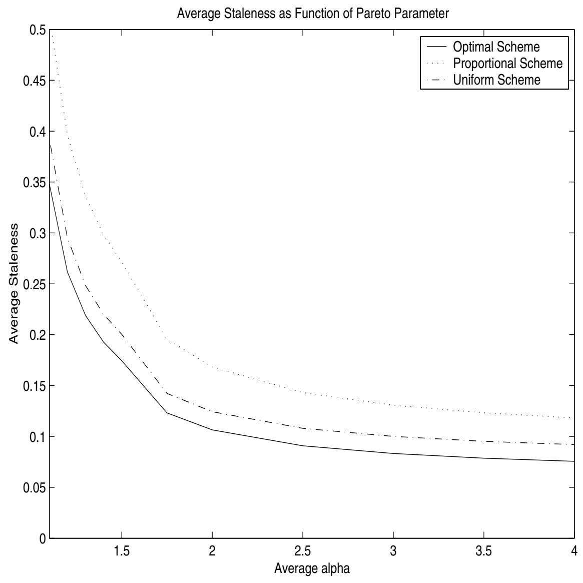


Figure 9: Pareto Example

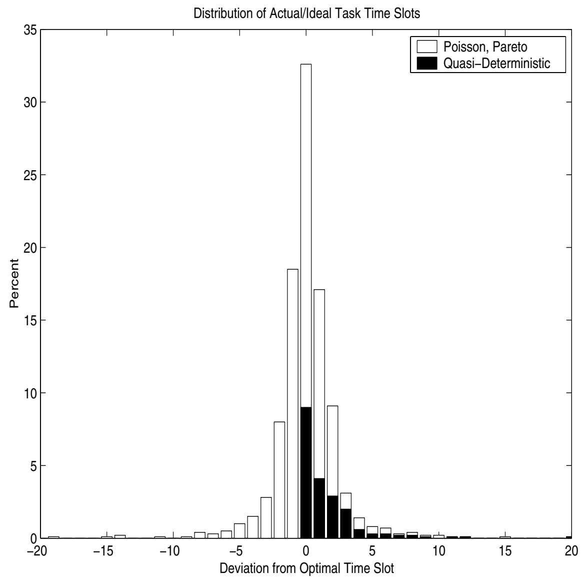


Figure 10: Transportation Problem Example

nus 20 time slots. We also highlight the quasi-deterministic tasks, to notice that they occur on or *after* their ideal time slots, as required. The quasi-deterministic crawls amounted to 20% of the overall crawls in this example. The bottom line is that this scheduling problem will nearly always yield optimal solutions of very high absolute quality.

The crawling frequency scheme was implemented in C and run on an IBM RS/6000 Model 50. In no case did the algorithm require more than a minute of elapsed time. The crawler scheduling algorithm was implemented using IBM's Optimization Subroutine Library (*OSL*) package [13], which can solve network flow problems. Problems of our size run in approximately two minutes.

6. CONCLUSION

Given the important role of search engines in the World Wide Web, we studied the crawling process employed by such search engines with the goal of improving the quality of the service they provide to clients. Our analysis of the optimal crawling process considered both the metric of staleness, as done by the few studies in this area, and the metric of embarrassment, which we introduced as a preferable goal. We proposed a general two-part scheme to optimize the crawling process, where the first component determines the optimal number of crawls for each page together with the optimal times at which these crawls should take place if there were no practical constraints. The second component of our scheme then finds an optimal achievable schedule for a set of crawlers to follow. An important contribution of the paper is this formulation which makes it possible for us to exploit very efficient algorithms These algorithms are significantly faster than those considered in previous studies. Our formulation and solution is also more general than previous work for several reasons, including the use of weights in the objective function and the handling of significantly more general update patterns. Given the lack of published data on web page update patterns, and given the assumption of exponential interupdate times in the analysis of the few previous studies, we analyzed the page update data from a highly accessed web site serving highly dynamic pages. The corresponding results clearly demonstrate the benefits of our general unified approach in that the distributions of the times between updates to some web pages clearly span a wide range of complex behaviors. By accommodating such complex update patterns, we believe that our optimal scheme can provide even greater benefits in real-world environments than previous work in the area.

Acknowledgement. We thank Allen Downey for pointing us to [19].

7. REFERENCES

[1] R. Ahuja, T. Magnanti and J. Orlin, *Network Flows*, Prentice Hall, 1993.

[2] A. Arasu, J. Cho, H. Garcia-Molina, A. Paepcke and S. Raghavan, "Searching the Web", *ACM Transactions on Internet Technology*, 1(1), 2001.

[3] J. Blazewicz, K. Ecker, G. Schmidt and J. Weglarz, *Scheduling in Computer and Manufacturing Systems*, Springer-Verlag, 1993.

[4] A. Broder, *Personal communication*.

[5] J. Challenger, P. Dantzig, A. Iyengar, M. S. Squillante, and L. Zhang. Efficiently serving dynamic data at highly accessed web sites. Preprint, May 2001.

[6] J. Cho and H. Garcia-Molina, "Synchronizing a Database to Improve Freshness", *ACM SIGMOD Conference*, 2000.

[7] E. Coffman, Z. Liu and R. Weber, "Optimal Robot Scheduling for Web Search Engines", *INRIA Research Report*, 1997.

[8] F. Douglas, A. Feldmann and B. Krishnamurthy, "Rate of Change and other Metrics: A Live Study of the World Wide Web", *USENIX Symposium on Internetworking Technologies and Systems*, 1999.

[9] B. Fox, "Discrete Optimization via Marginal Analysis", *Management Science*, 13:210-216, 1966.

[10] G. Frederickson and D. Johnson, "The Complexity of Selection and Ranking in X+Y and Matrices with Sorted Columns", *Journal of Computer and System Science*, 24:197-208, 1982.

[11] Z. Galil and N. Megiddo, "A Fast Selection Algorithm and the Problem of Optimum Distribution of Efforts", *Journal of the ACM*, 26:58-64, 1981.

[12] Ibaraki, T., and Katoh, N., "Resource Allocation Problems: Algorithmic Approaches", MIT Press, Cambridge, MA, 1988.

[13] International Business Machines Corporation, *Optimization Subroutine Library Guide and Reference*, IBM, 1995.

[14] N. Katoh and T. Ibaraki, "Resource Allocation Problems", in *Handbook of Combinatorial Optimization*, D-Z. Du and P. Pardalos, editors, Kluwer Academic Press, 2000.

[15] D. Knuth, *The Art of Computer Programming*, vol. 2, Addison Wesley, 1973.

[16] A. Iyengar, M. Squillante and L. Zhang, "Analysis and Characterization of Large-Scale Web Server Access Patterns and Performance", *World Wide Web*, 2:85-100, 1999.

[17] S. Lawrence and C. Giles, "Accessibility of Information on the Web", *Nature*, 400:107-109, 1999.

[18] G. Nemhauser and L. Wolsey, *Integer and Combinatorial Optimization*, J. Wiley, 1988.

[19] V. N. Padmanabhan and L. Qiu. "The Content and Access Dynamics of a Busy Web Site: Findings and Implications", *ACM SIGCOMM '00 Conference*, 2000.

[20] M. Pinedo, *Scheduling: Theory, Algorithms and Systems*, Prentice-Hall, 1995.

[21] J. Pitkow and P. Pirolli, "Life, Death and Lawfulness on the Electronic Frontier", *CHI Conference on Human Factors in Computing Systems*, 1997.

[22] W. Press, B. Flannery, S. Teukolsky and W. Vetterling, *Numerical Recipes*, Cambridge University Press, 1986.

[23] S. M. Ross. *Stochastic Processes.* John Wiley and Sons, Second Edition, 1997.

[24] K. Sigman. *Stationary Marked Point Processes: An Intuitive Approach.* Chapman and Hall, 1995.

[25] M. Squillante, D. Yao and L. Zhang, "Web Traffic Modeling and Web Server Performance Analysis", *IEEE Conference on Decision and Control*, 1999.

[26] J. Talim, Z. Liu, P. Nain and E. Coffman, "Optimizing the Number of Robots for Web Search Engines", *Telecommunication Systems Journal*, 17(1-2):234-243, 2001.

[27] C. Wills and M. Mikhailov, "Towards a Better Understanding of Web Resources and Server Responses for Improved Caching", *WWW Conference*, 1999.

[28] R. W. Wolff. *Stochastic Modeling and the Theory of Queues.* Prentice Hall, 1989.

[29] G. Zipf, *Human Behavior and the Principle of Least Effort*, Addison-Wesley, 1949.