Competition between auction houses: a shill bidding perspective

Laurent Lamy*

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Abstract

We analyze the competition between auction houses to organize an English auction in the independent private value model with participation costs when the seller is unable to commit not to participate in the same way as any potential buyer through a shill bidding activity. The seller may prefer to contract with auction houses with higher fees since they make the shill bidding activity more costly and thus enlarge the set of implementable participation cutoffs.

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JEL classification: D44, L81

1 Introduction

Auction houses' commission fees are usually very high compared to the marginal cost of organizing an auction: eBay charges from 1.50% to 5.25% of the winning price though the marginal cost of organizing such an electronic auction is null. Christie's and Sotheby's have jointly dominated the fine art auction market for more than a century with fees above 10%. Attracting new potential buyers and lowering their participation costs seem to be the main field of the competition between auction houses. eBay activities consist mainly in building a friendly auction site and developing electronic payment facilities as PayPal whereas auctioneers are also playing an expertise role in fine art auctions. Those stylized facts are hardly explained by standard auction theory models where a seller should prefer the auction house with the lowest fee. Bertrand competition between auction houses should lead to vanishingly low commission fees and zero profits. On the contrary, Vogel [30] reports that, in his negotiations with Christie's and Sotheby's, the CIO of a japanese company planning to sell its about $20 million-worth art collection

*PSE, 48 Bd Jourdan 75014 Paris. e-mail: laurent.lamy78@gmail.com

has ostensibly decided to make them play a game of chance to determine which one would organize the sale and pocket the fees.

Two kinds of arguments could match with high commission fees. First fees can be only a specific dimension of the competition between auction houses that are also competing to attract the largest set of potential buyers. In such a perspective, auction houses appear as the platform of a two-sided market as in Caillaud and Jullien [6] and the demand that a seller faces is supposed to depend not only on the auction she chooses, e.g. the announced reserve price if any, but also on how the platform conquers the other side of the market. We are not sure that such an argument would apply for top-valued fine art auctions where the set of potential buyers does not seem to depend on the chosen auction house: potential buyers chose to go to the auction house after having observed the seller's decision. Though the final value fee is often secretly negotiated at much lower rates for high-valued art compared to the official fees, the very significant fees above 10% remain a puzzle and we can wonder why sellers even do not choose to organize the auction themselves or on eBay where fees are much lower.[1] Furthermore, contrary to Caillaud and Jullien's [6] predictions for exclusives services, eBay Inc seems to be far away from zero profit with a 5-years-average net profit margin over 20%.[2] In the present paper, we consider a second kind of argument: we argue that commission fees may help the seller to comply with the auction rules and may thus enlarge the set of implementable mechanisms. In other words, ceteris paribus, the seller may surprisingly prefer to contract with an auction house with larger fees.

We consider the English auction in the symmetric independent private value model with two additional ingredients. First, following the literature on auctions with participation costs, we consider that, *after* being informed on their private valuations, potential buyers decide whether they participate in the auction, in which case they incur a positive participation cost. These costs can either correspond to the time consuming activity of submitting a bid on eBay or to the financial preparation of the bid for an expensive painting. Second, simultaneously to potential buyers' participation decisions, the seller decides whether she enrolls a shill bidder, in which case she incurs a positive shill bidding cost. Then the shill bidder is supposed to be able to bid in the auction as any other participant. In equilibrium, only bidders above a given cutoff that lies strictly above the reserve price will decide to participate. It means that there is a gap between the announced reserve price and the type of the lowest participant. As a corollary, if the cost to enroll a

[1] Ashenfelter [1] and Ashenfelter and Graddy [2] report that the total fee corresponds to the sum of the buyer's premium with the seller's commission. Only this latter part of the fee is negotiable.

[2] http://stocks.us.reuters.com/stocks/ratios.asp?rpc=66&symbol=EBAY.O (figures reported at november, 2007) The net profit margin has recently shrunk due to the diversification policy of eBay Inc outside its original marketplace activity, e.g. Skype.

shill bidder is sufficiently low, the seller should find profitable to bid at least until the participation cutoff: it never changes the final allocation and raises strictly the price in the event where only one buyer is participating.

In the first part of our analysis, we derive the whole set of buyer-symmetric equilibria: potential buyers use cutoff strategies, i.e. bid their valuation if it is greater than a cutoff point and do not participate otherwise, whereas the seller either never enrolls a shill bidder or uses a mixed strategy where she enrolls a shill bidder with some positive probability. The structure of the equilibrium set varies with the shill bidding cost. For low shill bidding costs, there is a unique mixed-strategy equilibrium. For a given reserve price, the shill bidding activity reduces potential buyers' gains from participation compared to the equilibrium where the seller could commit not to shill bid. Thus it increases the aforementioned gap between the announced reserve price and the lowest equilibrium bid which makes the shill bidding activity even more profitable. Due to this amplification effect, equilibria may raise very poor revenue: in the limit where the shill bidding cost goes to zero, the expected revenue of the seller goes to zero. Surprisingly, participation may be enhanced by higher reserve prices. The intuition is that the gap between the lowest equilibrium bid and the announced reserve price shrinks with the reserve price: limiting this initial incentive to shill bid reduces the amplification effect. For high shill bidding costs, only equilibria without shill bidding exist. For intermediate shill bidding costs, multiple equilibria may appear with one involving no shill bidding activity and which is Pareto-dominant. We characterize the optimal reserve price and the corresponding participation cutoff that the seller can implement. For small shill bidding costs, the seller is unable to implement the optimal cutoff as under commitment not to use shill bids. She can rather implement only cutoffs above a given threshold which is decreasing in the shill bidding cost and increasing in the participation cost and the number of potential buyers.

Those first theoretical results hardly fit with the success of electronic auctions which drive the cost of shill bidding almost to zero which would then cancel the benefits coming from the reduction of the participation costs. Ockenfels et al [25] report that, in Germany, a commercial company provides a service that automates the process of shill bidding. Moreover, empirical analysis of eBay auctions (e.g. Bajari and Hortacsu [4]) show that lowering the reserve price increases the entry into the auction.

The second part of our analysis considers the possibility of intermediation via competing auction houses. Auction houses appear endogenously as an organizational device that prevents from the hold-up of the participation costs by the seller, while we do not rely on any network externality hypothesis. We consider an imperfect Bertrand competition framework where auction houses compete on three parameters: (positive) insertion fees, final value fees (possibly non-linear at some stage) and the number of participants through a given marketing technology. The seller decides whether to sell at one of the auction houses or to organize herself the auction while we consider that she disposes of the same marketing technology in this latter case. The key difference is that auction houses allow the seller to 'burn money' and thus enrich the set of implementable participation cutoffs. Final value fees are making the shill bidding activity more costly: in the case where the seller is the winning bidder, she does not refund entirely the auction price. We show that if the shill bidding cost is sufficiently low, then the seller's preferred final value fee is strictly positive and thus positive fees arise in equilibrium. The occurrence of inefficiencies and 'positive profit' in the equilibrium under Bertrand competition is discussed.

1.1 Related Literature

Our model is related to three branches of literature. First shill bidding introduces an hold-up problem: for a given announced reserve price and after participation costs have been sunk, the seller wants to 'renegotiate' the reserve price and expropriate the rents that are needed to compensate some potential buyers for the investment to participate in the auction. On the contrary to Che and Sakovics's [10] dynamic treatment of investment decisions, it corresponds to a standard static framework as in Tirole's [29] seminal paper. The analog of the underinvestment result is how the set of implementable participation cutoffs shrinks when the shill bidding cost shrinks: in the limiting case where the shill bidding cost goes to zero, i.e. without any friction in the way the seller can expropriate the rents of the participants, then the hold-up problem unravels completely the market. In the same vein as the incomplete contract literature proposes organizational or contractual remedies for the holp-up problem (see Che and Sakovics [9]), auction houses fees are making the shill bidding activity more costly and thus mitigate the hold-up problem.

Second, our paper is related to the literature on the role of auction houses, their pricing policy and on how competition works between them. In auctions with an informed seller, Jullien and Mariotti [17] show that an auction house may reduce the lemon problem. The crucial point is the time where the commitment to the mechanism is made: *after* being informed if the seller organizes herself the auction or *before* being informed if she uses the auction house. The main bulk of the literature that endogenizes auction houses is the growing literature on two-sided markets where the success of a platform depends on its joint ability to attract both sellers and potential buyers. Attracting one extra seller on a platform has a leverage effect: it makes the platform more attractive to potential buyers and thus also to the other sellers. Deltas and Jeitschko [12] show that it severely limits a monopolist's ability to extract rents. Ellison et al [13] study the conditions under which an equilibrium with several auction houses exist. Closely related is the literature on competition among sellers as in Peters and Severinov [26], Burguet and Sakovics [5], Hernando-Veciana [15] and Damianov [11]. We emphasize that our model completely abstract from those issues by considering a unique seller.

Third, our analysis is related to the literature on shill bidding, also called phantom bids or lift-lining. The first contributions analyze the English auction and perceive this activity as an additional flexibility that raises the revenue, e.g. in Graham et al [14], Izmalkov [16], Lopomo [20]. On the contrary, our analysis lies in the same perspective as Chakraborty and Kosmopoulou [8] and Lamy [18] where the shill bidding activity deteriorates the seller's revenue and where she would prefer to commit not to use shill bids. In an interdependent value framework, Lamy [18] shows that the seller can not implement the optimal participation cutoff because she can not refrain from submitting shill bids in equilibrium in order to make believe to the potential buyers that the item worths more. In the present paper, similar insights are derived in a model with participation costs. We go further by considering that the shill bidding activity may be costly, by deriving related comparative statics results and also mainly by analyzing the consequences on the competition between auction houses.

This paper is organized as follows: Section 2 introduces the model and some preliminary lemmata for our equilibrium analysis. Section 3 derives the whole set of buyer-symmetric equilibria in the English auction when the seller can use shill bids. In section 4, the optimal reserve price is characterized and comparative statics results are derived in a slightly more general framework where the seller may have to pay a final value fee. Imperfect Bertrand competition between auction houses is analyzed in section 5. Section 6 concludes.

2 The Model

We consider a symmetric independent private value environment. There are $n \geq 1$ risk-neutral buyers and a risk-neutral seller who wants to sell an indivisible object for which her valuation is zero. Potential buyers' valuations are private and independently distributed with a common cdf $F(.)$ that has continuously differential density $f(.)$ and full support on $[0,1]$. The cdf F is assumed to satisfy Myerson's regularity assumption.

Assumption A 1 *Myerson's regularity:* $x \to x - \frac{1-F(x)}{f(x)}$ *is a strictly increasing function on* $[0,1]$.

The timing of the auction game is as follows. First, the seller announces a reserve price (or opening bid) r and potential buyers are privately informed about their valuations. Second, the potential buyers and the seller decide simultaneously whether to register in the auction and incur the respective costs $c_{part} > 0$ and $c_{shill} > 0$ to participate in the auction and to enroll a shill bidder. To guarantee that some participation could be possibly profitable, we further assume in the following that $c_{part} < 1$. Third, registered participants are playing an English button auction. Our analysis is restricted to so-called **buyer-symmetric equilibria** where potential buyers are using the same strategy.

Our analysis covers the different cases with regards to the disclosure rule concerning the set of participants before the auction starts and the (irrevocable) exits and thus includes the second price auction as a special case. Nevertheless we do not allow bidders to exit and re-enter the auction.[3] We emphasize that we consider that the decision to enroll a shill bidder is done *before* observing the set of participants. The case where this decision is done *after* participation decisions have been disclosed will be briefly discussed in remark 2.1. For expositional purposes, we also assume that the seller's instruction to the shill bidder is done before participation decisions have been disclosed.[4]

Without shill bidding, Celik and Yilankaya [7] show that the seller's expected revenue depends only on the participation cutoffs and on the allocation rule.[5] In the English auction that puts the object in the hand of the participant with the highest valuation, the seller's expected revenue is thus completely characterized by the participation cutoffs. For buyer-symmetric participation cutoffs as previously analyzed in Samuelson [27], the derivative of the expected revenue of the seller as a function of the participation cutoff is given by $\frac{1-F(x)}{f(x)} + \frac{c_{part}}{[F(x)]^{n-1}} - x$, which is strictly decreasing in x as guaranteed by assumption A1 and which changes sign in the range $[0, 1]$. The optimal participation cutoff x_{opt}^{com} is then uniquely characterized by:

$$x_{opt}^{com} - \frac{1 - F(x_{opt}^{com})}{f(x_{opt}^{com})} - \frac{c_{part}}{[F(x_{opt}^{com})]^{n-1}} = 0 \tag{1}$$

The optimal participation cutoff is strictly increasing in c_{part} and is thus higher than the one arising in Myerson's optimal auction when participation costs are null, which is denoted by $x_{Myerson}$. Moreover, the seller's expected revenue is a unimodal function of the participation cutoffs, which guarantees

[3]See Bikhchandani and Riley [3] for an exhaustive description of the different possible models for the English auction. Contrary to Lamy [18], no assumption on the anonymous nature of the shill bidding activity is required.

[4]The new equilibria that would arise under an alternative timing are strategically equivalent to the one we derive insofar as they would differ only for 'inconsequential' actions, i.e. actions that do not modify the final payoffs.

[5]They also show that after having fixed the participation cutoffs, optimal mechanisms are those that allocate the item efficiently, e.g. the English auction. Lu [21] extends this result to a model with private participation costs and ex ante symmetric buyers while restricting to symmetric participation cutoffs.

that for a given set of participation cutoffs $[x, 1]$ where $x > x_{opt}^{com}$, then x is the seller's most preferred cutoff. This is summed up in the following lemma.

Lemma 2.1 *The seller's expected revenue as a function of the participation cutoff is a unimodal function with mode $x_{opt}^{com} > x_{Myerson}$.*

The remaining part of this section is devoted to three preliminary lemmata. Buyer-symmetric equilibria are then characterized by two variables: the cutoff type that is indifferent between participating and not participating in the auction and the probability that the seller enrolls a shill bidder. Finally, we derive the two equilibrium equations linking those unknowns.

We first establish that the seller can not enroll a shill bidder with probability one in equilibrium by raising a contradiction if she does so. Suppose that the seller always enrolls a shill bidder. Denote by x_S the lower bound of the support of this shill bidding strategy. Equilibrium conditions require then that no potential buyers with a valuation in the neighborhood of x_S would find profitable to participate. If potential buyers participate with some positive probability then the seller would never find it optimal to raise a shill bid in the neighborhood of x_S since she would profitably deviate with a mildly higher shill bid that can only increase the seller's expected revenue. Thus potential buyers never participate in equilibrium and consequently the seller would profitably deviate by not enrolling a shill bidder since this activity is strictly costly. Thus we have raised a contradiction and established the following lemma.

Lemma 2.2 *In any equilibrium, the seller chooses not to enroll a shill bidder with a positive probability.*

Contrary to Tan and Yilankaya [28] who restrict their attention to equilibria where potential buyers use cutoff strategies, we do not need such a restriction after having restricted our attention to buyer-symmetric equilibria.

Lemma 2.3 *In a buyer-symmetric equilibrium, buyers' strategy profile corresponds to a cutoff point x^* such that a potential buyer chooses to participate and to bid his valuation if it is greater than x^* and does not participate otherwise.*

Proof 1 *We first note that positive participation costs imply that all types below the reserve price and also in the neighborhood of r should not find profitable to participate. Since valuation are drawn independently, we obtain that, with positive probability, all potential competitors of a given buyer will prefer not to participate. Combined with the previous lemma, we obtain that the payoff derived from participation is a strictly increasing function of one's* *valuation for types above r: by mimicking a lower type's strategy, a given type guarantees a strictly higher payoff than the equilibrium payoff of this type since the probability of buying the item is not null. We conclude that two events can occur: either the payoff from participation is always negative and participation is null in equilibrium or there exists a cutoff x^* such that buyers with valuation x^* are indifferent between participating or not, whereas buyers above (resp. below) do (resp. do not) participate. In the auction stage, the only symmetric equilibrium is the one where buyers are bidding their valuations as shown by Blume and Heidhues [4].*

Finally, we show that the shill bidding strategy (if any) is strategically equivalent to the following form: the seller instructs the shill bidder to bid up to the maximum between x^* and Myerson's optimal reservation price $x_{Myerson}$. After participation costs have been sunk, the seller faces an unknown number of participants with a cdf F_{x^*} that depends on the cutoff x^*: $F_{x^*}(u) = \frac{F(u)-F(x^*)}{1-F(x^*)}$ if $u \geq x^*$ and $F_{x^*}(u) = 0$ otherwise. Remark that $\frac{1-F_{x^*}(u)}{f_{x^*}(u)} = \frac{1-F(u)}{f(u)}$ for $u \geq x^*$. Then the 'virtual valuations' for the cdf F_{x^*} correspond exactly to the virtual valuations of the original cdf F for valuations above x^*. The optimal reserve price against some bidders with cdf F_{x^*} is thus equal to $\max\{x^*, x_{Myerson}\}$ and we obtain the following lemma.

Lemma 2.4 *In any equilibrium with some shill bidding activity, the instruction to the shill bidder is to bid until $x^*_{shill} = \max\{x^*, x_{Myerson}\}$.*

In equilibrium, the strategy of the shill bidder is pure on the contrary to the equilibria in Lamy [18] where it is always mixed.

By gathering the previous lemmata, any possible equilibrium is fully characterized by the participation cutoff x^* and the probability denoted by p that the seller does not enroll a shill bidder. If the equilibrium involves some participation, i.e. $x^* < 1.$, the potential buyers' equilibrium equation is given by:

$$p.(x^* - r).[F(x^*)]^{n-1} = c_{part} \tag{2}$$

If the equilibrium involves no participation from potential buyers the equality should be replaced by the inequality $p.(1-r) \leq c_{part}$. For any equilibrium where the seller strictly mix, i.e. if $0 < p < 1$, the seller's equilibrium equation is characterized by the indifference between enrolling a shill bidder that bids until x^*_{shill} and staying outside the auction with the reserve price r. The expected benefit from submitting the shill bid x^*_{shill} is equal to the corresponding increase of the expected auction price. It is denoted by $H(x^*, r)$ such that:

$$\begin{aligned} H(x^*, r) \quad = \quad & n.(1 - F(x^*_{shill})).(x^*_{shill} - r).[F(x^*_{shill})]^{n-1} \\ - \quad & \int_{x^*}^{x^*_{shill}} (u-r)n(n-1)[F(u)]^{n-2}(1-F(u))f(u)du. \end{aligned}$$

The cost of submitting a shill bid is resumed to c_{shill}. The indifference equation is thus given by:

$$H(x^*, r) = c_{shill} \tag{3}$$

For an equilibrium where the seller does not enroll a shill bidder, only the following inequality needs to be satisfied:

$$H(x^*, r) \leq c_{shill} \tag{4}$$

Remark 2.1 *If the choice to enroll a shill bidder were made after the disclosure of the participation decisions, then the unique equilibrium is full non-participation if the shill bidding costs are smaller than the participation costs for any announced reserve price. Buyers with a valuation below $r + c_{part}$ would never find profitable to participate which means that the gap between the reserve price and the lowest equilibrium bid is greater than c_{part}. The seller would find profitable to enroll a shill bidder if $c_{shill} < c_{part}$. The holdup problem is more severe since the seller can target more precisely the situations where she makes the costly investment to enroll a shill bidder.*

3 Equilibrium Set with Shill Bids

This section is devoted to the characterization of the equilibrium set while we fix the number of potential buyers, the announced reserve price and participation costs. Under a mild technical assumption, the equilibrium set is fully characterized for any shill bidding cost in Proposition 3.1. Multiple equilibria may arise. It results from the strategic complementarity between potential buyers' participation decisions and the seller's shill bidding activity. Equilibria with a low shill bidding activity and a low participation cutoff may coexist with one with a high shill bidding activity and a high participation cutoff.

Assumption A 2 *The map $x \to n[F(x)]^{n-1}(1 - F(x)(x - r)$ is a strictly unimodal (or strictly quasi-concave) function on $[r, 1]$ for any $0 \leq r < 1$.*

When x^* lies below $x_{Myerson}$ (and above r as it is required by equation (2)), $H(x^*, r)$ is strictly decreasing in x^*. When x^* lies above $x_{Myerson}$, $H(x^*, r)$ reduces to $n.(1-F(x^*)).(x^*-r).[F(x^*)]^{n-1}$ which is a strictly quasi-concave function as assumption A2 guarantees. On the whole, $x^* \to H(x^*, r)$ is thus strictly quasi-concave if assumption A2 is satisfied. Denote by x^{mod} its mode.

Three kinds of equilibria can emerge: equilibria where potential buyers never participate and the seller never enrolls a shill bidder, equilibria with some participation and where the seller never enrolls a shill bidder (also called equilibria without shill bids) and finally equilibria where the seller uses a mixed strategy.

After remarking that the seller should not find profitable to enroll a shill bidder if buyers never participate, we conclude from equation (2) that the first aforementioned equilibria emerge if and only if $r \geq 1 - c_{part}$ independently of the shill bidding cost. Now we consider that the participation costs are small enough such that equilibria with some participation occur, i.e. we consider $c_{part} < 1 - r$ and we denote by x_{low} the unique solution of the implicit equation $(x - r).[F(x)]^{(n-1)} = c_{part}$ as it is guaranteed by the intermediate value theorem and the fact that the left-hand side is a strictly increasing function as a function of x on $[r, 1]$.[6] Equilibria without shill bids are characterized by the participation cutoff x_{low} and the equilibrium inequality (4). Denote by c^*_{shill} the lowest shill bidding cost such that an equilibrium without shill bids exists, i.e. $H(x_{low}, r) = c^*_{shill}$.

For any strictly mixed strategy equilibrium, $p \in (0, 1)$ implies that the participation cutoff x^* should be in the interval $(x_{low}, 1)$. In any mixed strategy equilibrium, the seller's indifference condition gives $H(x^*, r) = c_{shill}$. In the quadrant (x^*, c_{shill}), candidates to be a mixed equilibria belong thus to the curve $x \to H(x, r)$ for $x > x_{low}$. Conversely, any point in such a location is an equilibrium. Under assumption A2, there are at most two candidates to be a mixed strategy equilibria for a given shill bidding cost: one for participation cutoffs on each side of the mode. Typical equilibrium sets in the quadrant (x^*, c_{shill}) are depicted by the thick line in Figure 1 when the mode x^{mod} is either on the right of x_{low} (left panel) or on the left of x_{low} (right panel). After noting that the map $x \to H(x, r)$ and thus x^{mod} do not depend on c_{part}, we define c^{mod}_{part} as the participation cost such x_{low} coincides with x^{mod}:

$$c^{mod}_{part} = (x^{mod} - r).[F(x^{mod})]^{n-1}$$

For $c_{part} < c^{mod}_{part}$ (resp. $c_{part} > c^{mod}_{part}(r)$), x_{low} is below (above) x^{mod}. In the case $c_{part} \geq c^{mod}_{part}$, the equilibrium is always unique. Note first that the candidate on the left of x^{mod} fail to satisfy the requirement $x \geq x_{low}$ and is thus never an equilibrium. Then note that the candidate mixed strategy equilibrium on the right of x^{mod} is actually an equilibrium if and only if $c_{shill} < c^*_{shill}$, i.e. exactly in the case where the equilibrium without shill bidding fails to exist. In the case $c_{part} < c^{mod}_{part}$, multiple equilibria may occur for intermediate shill bidding costs in the interval $[c^*_{shill}, c^{**}_{shill}]$ where $c^{**}_{shill} = H(x^{mod}, r)$.

Insert Figure [1]

The previous discussion is summarized in Figure 1 and in the following proposition.

[6] To alleviate notation we drop the dependence of x_{low} in the parameters r, n and c_{part}.

Proposition 3.1 *Consider the number of potential buyers and the reserve price as fixed and under assumption A2.*

- *For $c_{part} \geq 1 - r$, there is a unique equilibrium which involves no participation from the potential buyers and the seller.*
- *For $c_{part}^{mod} \leq c_{part} < 1 - r$, there is a unique equilibrium that involves no shill bidding if the shill bidding costs are high enough, $c_{shill} \geq c_{shill}^*$, and that is mixed otherwise.*
- *For $c_{part} < c_{part}^{mod}$, there is a unique equilibrium that involves no shill bidding if the shill bidding costs are high enough, $c_{shill} > c_{shill}^{**}$, and a unique equilibrium that is mixed if the shill bidding costs are small enough, $c_{shill} < c_{shill}^*$. For intermediate shill bidding costs in the interval $(c_{shill}^*, c_{shill}^{**})$, there are three equilibria: one without shill bidding and two in mixed strategy.*[7]

Remark 3.1 *In the limit case where the shill bidding cost is null, a similar equilibrium analysis shows that any equilibrium involves no trade, the seller enrolls a shill bidder with a probability greater than $1 - \frac{c_{part}}{(1-r)}$ and the payoff of the seller is null. Thus the equilibrium set presents no discontinuity when the shill bidding cost goes to zero.*

Remark 3.2 *In the general case beyond assumption A2, the structure of the mixed strategy equilibria is more complex. More than one mixed strategy equilibria may exist in the case with low shill bidding cost $c_{shill} < c_{shill}^*$. More than two mixed strategy equilibria may exist in the case with intermediate shill bidding costs in the interval $(c_{shill}^*, c_{shill}^{**})$. Nevertheless the set of the equilibria without shill bidding has the same structure: such equilibria exist if and only if the shill bidding cost is bigger than c_{shill}^*.*

The equilibrium set is continuous in the quadrant (x^*, c_{shill}) and following the equilibrium curve from an equilibrium without shill bidding on the left to the point $(1, 0)$ on the right, the probability to enroll a shill bidder rises continuously from 0 to $1 - \frac{c_{part}}{(1-r)} < 1$. Nevertheless, it is worthwhile to note that, when the shill bidding cost varies, the equilibrium can switch discontinuously from a mixed strategy equilibrium to an equilibrium without shill bidding.

Next proposition establishes the preferences of the agents on the so-called **equilibrium set curve**: the set of all equilibria when the shill bidding cost varies on $(0, \infty]$. Surprisingly, the seller and the potential buyers' preferences are varying in the same direction.

[7]For the degenerate cases, $c_{shill} = c_{shill}^*$ or $c_{shill} = c_{shill}^{**}$, there are two equilibria: one without shill bidding and one in mixed strategy.

Proposition 3.2 *In the equilibrium set curve, both the seller and each kind of potential buyers prefer the equilibria with the lowest participation cutoffs or equivalently with the lowest shill bidding activity.*

Proof 2 *Consider two equilibria 1 and 2 on the equilibrium set curve characterized by the participation cutoffs x_1 and x_2, where $x_1 > x_2 > r$. Denote by p_1 and p_2 the respective probabilities not to enroll a shill bidder in equilibrium.*

From lemma 2.2, the seller's expected payoff is equal to her expected revenue in the case where she does not enroll a shill bidder. In that case, it is equal to the expected revenue of an English auction with reserve price r and the given participation cutoff. It is straightforward that the expected payoff is ceteris paribus strictly decreasing in the participation cutoff: for a fixed reserve price, the expected payment in greater if there are more participants. Thus we have proved the proposition for the seller.

Consider a potential buyer with type $x < x_1$, then he clearly prefers equilibrium 2 to equilibrium 1 where he prefers not to participate and thus obtains a null payoff. Consider a potential buyer with type $x > x_1$. Two events may occur: either he is the bidder with the highest type or not. In this latter case, his payoff is null independently of the equilibrium. Two events may occur: either $x > x_{Myerson}$ or $x \leq x_{Myerson}$.

First we consider the case $x > x_{Myerson}$ such that the highest bidder with type x always win the auction. It is sufficient to show that his expected payment is smaller in equilibrium 2 conditionally on the fact that he has the highest type. First note that the equilibrium payment is the same in both equilibria if the second highest potential buyer has a valuation above x_1. Thus it is sufficient to compare the expected payment of the winning bidder conditionally on the event where the second highest bidder has a valuation below x_1. In equilibrium 1, the conditional expected payment is equal to:

$$p_1.r + (1 - p_1). \max\{x_1, x_{Myerson}\}.$$

He pays the announced reserve price with probability p_1 and the shill bid otherwise. In equilibrium 2, the conditional expected payment is equal to:

$$p_2.r.\frac{[F(x_2)]^{n-1}}{[F(x_1)]^{n-1}} + (1-p_2). \max\{x_2, x_{Myerson}\}.\frac{[F(\max\{x_2, x_{Myerson}\})]^{n-1}}{[F(x_1)]^{n-1}} + ...$$

$$+p_2. \int_{x_2}^{x_1} u\frac{d[F(u)]^{n-1}}{[F(x_1)]^{n-1}} + (1 - p_2). \int_{\max\{x_2, x_{Myerson}\}}^{x_1} u\frac{d[F(u)]^{n-1}}{[F(x_1)]^{n-1}}.$$

The first term corresponds to the case where the buyer pays the reserve price. The second term to the one where he pays the shill bid. The fourth and third terms are corresponding to the cases where the price is fixed by another *potential buyer and when the seller respectively does and does not enroll a shill bidder. Both expectations are corresponding to a weighted average of values between r and* $\max\{x_1, x_{Myerson}\}$. *We show that the cdf of the weights in the equilibrium 1 dominates the one in equilibrium 2 according to first-order stochastic dominance. For this we note that $p_1 < p_2 . \frac{[F(x_2)]^{n-1}}{[F(x_1)]^{n-1}}$ by combining the equilibrium equations (2) in the equilibria 1 and 2 with the fact that $x_1 > x_2$.*

Second, we consider the case $x \leq x_{Myerson}$. We compute the expected payoff of a potential buyer with valuation x in the auction. In equilibrium i and for $x \in [x_1, x_{Myerson})$, the conditional expected payoff is equal to:

$$[F(x)]^{n-1}.p_i.(x - r.\frac{[F(x_i)]^{n-1}}{[F(x)]^{n-1}}) - \int_{x_i}^{x} u.\frac{d[F(u)]^{n-1}}{[F(x_1)]^{n-1}}.$$

For $x = x_1$, buyer expected payoff is bigger in equilibrium 2 than in equilibrium 1: in equilibrium 1, he just refunds his participation costs, whereas his expected payoff is strictly higher in equilibrium 2 where a buyer with type x_2 refunds his participation costs. The derivative of the expected payoff with respect to x is equal to p_i. Since $p_2 > p_1$, the expected payoff of a buyer with a type $x \in [x_1, x_{Myerson})$ is thus bigger in equilibrium 2 than in equilibrium 1.

Equilibria with the same participation cutoffs on the equilibrium set curve are equilibria without shill bidding which are all equivalent from buyers' perspective. Thus we have proved the proposition for the buyers.

As a corollary, we obtain that equilibria without shill bidding are Pareto-dominant, which gives support to our 'weak' implementation perspective where we consider that the seller is able to select that the equilibrium without shill bidding is to be played in the case of multiple equilibria.

Corollary 3.3 *Equilibria without shill bidding are Pareto-dominant in the equilibrium set curve.*

Contrary to Myerson's [24] optimal auction where there is a trade-off between welfare and revenue, the seller and the buyer's preferences appear as more congruent in our framework with shill bidding as it will be further developed in next section with respect to the choice of the reserve price. It is reminiscent of Lamy [18] where the seller's most preferred equilibrium with shill bidding corresponds to the one that implements the lowest implementable participation cutoff.

4 The optimal reserve price

We now consider the strategic choice of the announced reserve price by the seller: we characterize the participation cutoffs that the seller can implement and the corresponding optimal reserve price that maximizes her expected revenue.

From now on we consider the more general framework where the seller may have to pay a final value fee $\alpha.R$, where R is the revenue of the auction. The final value fee does not modify potential buyers' equilibrium condition (2). On the contrary, the seller's indifference equation (3) is modified and becomes now:

$$(1-\alpha).H(x^*,r) = c_{shill} + \alpha.[r.[F(x^*)]^n + \int_{x^*}^{x^*_{shill}} ud[F(u)]^n], \qquad (5)$$

where $x^{\alpha}_{Myerson}$ corresponds to the optimal reserve price under commitment not to shill bid when the seller receives only $1-\alpha$ of the auction revenue, i.e. $x^{\alpha}_{Myerson}$ is the solution of the equation $x = (1-\alpha).\frac{1-F(x)}{f(x)}$, and x^*_{shill}, the seller's optimal shill bid, is equal to $\max\{x^*, x^{\alpha}_{Myerson}\}$. In an equilibrium without shill bidding, the seller's equilibrium condition is equation (5) where the equality has been replaced by the inequality $\leq$.

Ceteris paribus, high fees are making the shill bidding activity less profitable: first by reducing the gains in the auction as reflected by the multiplication by $(1-\alpha)$ of the left-hand term and second by making the shill bidding activity more costly since the shill bidder may buy the item and the seller does not refund entirely the auction price due to the final value fee as reflected by the new term added to the right-hand of (5).

The expected payoff of the seller is fully characterized by the participation cutoff and the probability to enroll a shill bidder. The following lemma show that there is no loss of generality to restrict the analysis to equilibria without shill bids when we are looking for an optimal reserve price.

Lemma 4.1 *If there is a reserve price r such that the participation cutoff x^* and the shill bidding probability $p < 1$ is an equilibrium profile, then there is a reserve price $r' > r$ such that the same participation cutoff x^* and no shill bidding activity is an equilibrium profile. Furthermore, this latter equilibrium raises a strictly higher revenue.*

Proof 3 *Consider the reserve price r' characterized by $(x^*-r').[F(x^*)]^{n-1} = c_{part}$ which guarantees that buyers' equilibrium condition is satisfied. From equation (2) and since $p < 1$, we have $r' > r$. Since the benefit from submitting a shill bid is decreasing in the reserve price -$\partial H/\partial r \leq 0$- and the marginal cost $\alpha.[F(x^*)]^n$ is positive, the expected payoff difference between raising or not a shill bid is decreasing in the announced reserve price for a given participation cutoff. We conclude that the seller prefers not to enroll a shill bidder.*

The revenue enhancing effect of the equilibrium without shill bids under the reserve price r' comes from the more general result that for a given participation cutoff, the seller's expected payoff is increasing in the probability *not to enroll a shill bidder. Consider two equilibria 1 and 2 characterized by the probabilities p_1 and p_2, where $p_1 > p_2$ and the same participation cutoff $x^* < 1$. Then the corresponding reserve prices are such that $r_1 > r_2$. The seller's expected payoff is equal to her expected revenue in the case where she does not enroll a shill bidder. In that case, it is equal to the expected revenue of an English auction with reserve price r_i and the given participation cutoff. It is straightforward that the expected payoff is strictly increasing in the reserve price provided that it is smaller than the participation cutoff. The expected revenue is thus bigger in environment 1.*

As a starting point of our analysis of the optimal announced reserve price, Proposition 4.2 gives some insights on the participation cutoffs that may arise in some equilibria. Such participation cutoffs are called 'implementable'.

Definition 1 *A participation cutoff is implementable if there exists a reserve price and an equilibrium supporting this participation cutoff.*

Denote by $\mathcal{S}$ the set:

$$\mathcal{S} = \{x \in [0,1] | (1-\alpha).[1-F(u)] \leq \frac{c_{shill}}{n.c_{part}} + \frac{\alpha}{n}.[\frac{u[F(u)]^{n-1}}{c_{part}} - 1].F(u), \forall u \geq x\}. \tag{6}$$

From our assumption $c_{part} < 1$, we obtain that $\mathcal{S}$ is nonempty. Then denote by x^*_{low} the infimum of the set $\mathcal{S}$. By continuity, if $x^*_{low} > 0$, we have thus:

$$(1-\alpha).[1 - F(x^*_{low})] = \frac{c_{shill}}{n.c_{part}} + \frac{\alpha}{n}.[\frac{x^*_{low}[F(x^*_{low})]^{n-1}}{c_{part}} - 1].F(x^*_{low}). \tag{7}$$

Proposition 4.2 *Consider the final value fee, the number of potential buyers and the participation and shill bidding cost as given.*

- *If $x^*_{low} \geq x^{com}_{opt}$, then the set of implementable participation cutoffs is the interval $[x^*_{low}, 1]$.*
- *If $x^*_{low} < x^{com}_{opt}$, then the set of implementable participation cutoffs contains the interval $[x^{com}_{opt}, 1]$.*[8]

Proof 4 *We first consider the case $x^*_{low} \geq x^{com}_{opt}$, which guarantees that $x^*_{low} \geq x^{\alpha}_{Myerson}$. We first show that all participation cutoffs in the interval $[x^*_{low}, 1]$ are implementable. Pick $x \in [x^*_{low}, 1]$ and consider the reserve price r such that $(x - r).[F(x)]^{n-1} = c_{part}$. Then we obtain $(1 -$*

[8]For $x^*_{low} < x^{com}_{opt}$, the set of implementable participation cutoff is not necessary an interval because the map $x \rightarrow H(x, x - \frac{c_{part}}{[F(x)]^{n-1}})$ is not guaranteed to be decreasing in the range $[0, x^{com}_{opt}]$.

$\alpha).H(x,r) = (1-\alpha).n.(1-F(x)).c_{part}$ which is smaller than $(1-\alpha).n.(1-F(x^*_{low})).c_{part}$. Since $x^*_{low} \in \mathcal{S}$, we obtain that $(1-\alpha).H(x,r)$ is smaller than $c_{shill} + \alpha.[x^*_{low}[F(x^*_{low})]^{n-1} - c_{part}].F(x^*_{low})$, which is smaller than $c_{shill} + \alpha.[x[F(x)]^{n-1} - c_{part}].F(x)$ since $x \to [x[F(x)]^{n-1} - c_{part}].F(x)$ is increasing in x in the interval $[x^{com}_{opt}, 1]$.[9] *For a participation cutoff $x \geq x^{\alpha}_{Myerson}$, the seller's optimal shill bids is x. On the whole, we have shown that shill bidding is not profitable. Now consider $x < x^*_{low}$. As a corollary to lemma 4.1, to show that x is not implementable there is not loss of generality to restrict ourselves to equilibria without shill bids. Equation (2) requires that the reserve price is such that $(x-r).[F(x)]^{n-1} = c_{part}$ and thus $(1-\alpha).H(x,r) > (1-\alpha).n.(1-F(x^*_{low})).c_{part} = c_{shill} + \alpha.[x^*_{low}[F(x^*_{low})]^{n-1} - c_{part}].F(x^*_{low})$. This latter term is bigger than $c_{shill} + \alpha.[x[F(x)]^{n-1} - c_{part}].F(x)$ (since $x < x^*_{low}$ and $[x^*_{low}[F(x^*_{low})]^{n-1} - c_{part}] \geq 0$) and thus to $c_{shill} + \alpha.r[F(x)]^n$ (using again equation (2)). On the whole, we have shown that the seller would find strictly profitable to enroll a shill bidder that bids until x.*

Second we consider the case $x^*_{low} < x^{com}_{opt}$. Pick $x \in [x^{com}_{opt}, 1]$ and consider the reserve price r such that $(x-r).[F(x)]^{(n-1)} = c_{part}$. Then we obtain $(1-\alpha).H(x,r) = (1-\alpha).n.(1-F(x))c_{part}$ which is smaller than $n.(1-F(x^{com}_{opt}))c_{part}$ and thus smaller than $c_{shill} + \alpha.[x^{com}_{opt}[F(x^{com}_{opt})]^{n-1} - c_{part}].F(x^{com}_{opt})$ since $x^{com}_{opt} \in \mathcal{S}$. Following the same arguments as above, we conclude that shill bidding is not profitable.

Insert Figure [2]

Proposition 4.2 stands in sharp contrast with the picture when shill bidding does not occur. Without shill bidding, the seller can select any cutoff in the interval $[x^o, 1]$, where $x^o[F(x^o)]^{n-1} = c_{part}$, by choosing appropriately the reserve price. Since $x^o < x^{com}_{opt}$, the seller is not constrained to implement the optimal cutoff. When the participation cost vanishes, it means that the seller can roughly chose any cutoff. Moreover the relation between equilibrium cutoffs and reserve prices is monotonically increasing. On the contrary, this relation is not monotonic with shill bidding as it is illustrated in Figure 2 (where $\alpha = 0$) which corresponds to the first case of Proposition 4.2 where the seller can not implement the optimal participation cutoff but only the ones in the interval $[x^*_{low}, 1]$ with $x^*_{low} > x^{com}_{opt}$. The equilibrium sets for three possible reserve prices $r_1 < r_2 < r_3$ are depicted. For r_1 and r_2, a single strictly mixed strategies equilibrium exists, whereas two equilibria exist for r_3, one without shill bidding and the smallest implementable participation cutoff x^*_{low} and one with a strictly mixed shill bidding strategy. For those reserve prices, the equilibrium participation cutoffs are decreasing in the reserve price. Nevertheless, for reserve prices above r_3 and if we restrict our

[9]The derivative with respect to x is equal to $[F(x)]^n + [F(x)]^{n-1}.f(x).[nx - \frac{c_{part}}{[F(x)]^{n-1}}]$ which is bigger than $[F(x)]^{n-1}.f(x).[x - \frac{c_{part}}{[F(x)]^{n-1}}]$, which is positive for $x \geq x^{com}_{opt}$

attention to the equilibrium without shill bidding which always exists, then participation cutoffs are increasing in the reserve price.

With shill bidding, the set of implementable participation cutoffs does not depend solely on c_{part} but also on c_{shill}, n and α. Moreover, it is not the absolute value of the shill bidding costs that plays a role but its relative value compared to the participation costs c_{part}. It means that our holdup problem can arise in seemingly very different environments, i.e. with very different participation and shill bidding costs. To illustrate our ideas, consider the case where $\alpha = 0$ such that the cost from enrolling a shill bidder comes only from c_{shill}. Then x^*_{low} is characterized by $(1 - F(x^*_{low})) = \frac{c_{shill}}{n.c_{part}}$. In particular, vanishingly participation costs are still constraining the set of implementable cutoffs if shill bidding costs are concomitantly vanishing at the same rate.

Insert Figure [3]

Though we did not fully characterize the set of implementable participation cutoffs, Proposition 4.2 is sufficient to characterize the optimal implementable participation cutoff x^*_{opt} and the corresponding optimal reserve price r_{opt}. Denote by c^{com}_{shill} the threshold such that x^*_{low} coincides with x^{com}_{opt}. This case is depicted in Figure 3 where r_2 is the corresponding optimal reserve price. We moved from Figure 2 to Figure 3 just by raising the shill bidding cost which decreases x^*_{low} and thus enlarges the set of implementable cutoffs. If the shill bidding cost is bigger than c^{com}_{shill}, we obtain from lemma 2.1 that the seller is constrained and that her most preferred equilibrium corresponds exactly to the potential buyer's most preferred equilibrium in the set of implementable mechanisms. This is another contrasting point with respect to the case without shill bids where the seller choses to implement the participation cutoff x^{com}_{opt} that is too high with respect to potential buyers' preferences who are preferring a null reserve price and the corresponding cutoff x^o. In a nutshell, the surprising coincidence of the seller and the potential buyers' objectives does not hold solely in the equilibrium set for a given reserve price as established in proposition 3.2 but also in the choice of the reserve price.

Next proposition establishes the comparative statics of the optimal auction mechanism chosen by the seller.

Proposition 4.3 *The seller's optimal reserve price and the corresponding optimal implementable participation cutoffs are:*

- *increasing in the participation costs*
- *decreasing in the number of potential buyers, the shill bidding costs and the final value fee.*

	x_{opt}^{com}	x_{opt}, r_{opt}	Seller's Expected Payoff	Seller & Auctioneer's Expected Payoff	Potential Buyers' Expected Payoff	Total Welfare
c_{part}	+	+	-	-	-	-
n	-	-	?	?	-	?
c_{shill}	0	-	+	+	+	+
α	0	-	?	+	+	+

Table 1: Comparative Statics with respect to c_{shill}, c_{part}, n and α.

Proof 5 *We first show that those comparative statics hold for the threshold x_{low}^* which comes immediately from equation (7). From proposition 4.2, the optimal cutoff x_{opt}^{com} is implementable if $x_{low}^* \leq x_{opt}^{com}$ with the reserve price $r_{opt} = x_{low}^* - \frac{c_{part}}{[F(x_{low}^*)]^{n-1}}$. Combining the first case of proposition 4.2 with lemma 2.1, we obtain that the optimal implementable cutoff is x_{low}^* if $x_{low}^* \geq x_{opt}^{com}$. On the whole, we conclude by noting the optimal implementable cutoff and the corresponding reserve price are increasing with respect to x_{low}^* for which the comparative statics hold.*

Proposition 4.3 is silent whether those comparative statics are strict or not. Indeed the comparative statics with respect to c_{shill} is strictly decreasing until c_{shill}^{com} and then for $c_{shill} \geq c_{shill}^{com}$, shill bidding is no more a constraint in the implementation of the seller's preferred equilibrium. Note that c_{shill}^{com} depends on α and is strictly positive if α is small enough. The same remark holds for the parameter α, which appears as a substitute of c_{shill}. On the other hand the comparative statics with respect to c_{part} and n are always strict because independently of the fact that the seller is constrained, the optimal participation cutoff under commitment is strictly increasing in c_{part} and decreasing in n, as it is easily derived respectively from equation (1).

To end this section, we now turn to the comparative statics of the expected payoff of the different agents with respect to the key parameters of our model. All the results are summarized in Table 1.

From proposition 4.3, a decrease in the shill bidding costs or equivalently an increase in α has ceteris paribus a unilateral negative impact on the auction revenue and on potential buyers' expected payoffs. The comparative statics are immediately translated in term of seller's expected payoff, seller and auctioneer's expected payoff or total welfare except in the case of α and the seller's revenue due to the multiplication by the term $(1 - \alpha)$. Thus except in some limiting cases as considered in next section, the comparative statics of the seller's expected payoff with respect to α is undetermined. The comparative statics with respect to the final fee does not stand in line with the intuition that fees are preventing from some profitable trade between the seller and the potential buyers as it would be the case if the seller's reservation value were positive. We emphasize that by setting the seller's reservation value to zero, this effect is absent in our analysis.

An increase in the participation costs has two impacts. First the optimal participation cutoff increases which makes potential buyers worse-off, but we can not immediately conclude on the final impact of the auction revenue since the relation between the participation cutoff and the reserve price is also altered. For a given participation cutoff, an increase in the participation costs corresponds to an increase in the gap between the announced reserve price and the participation cutoff and thus this second effect is also negative for the auction revenue.

Finally to complete the picture, we report the comparative statics with respect to n which are mostly undetermined, in particular for the auction revenue as previously noted by Samuelson [27].

5 Bertrand competition between auction houses

In this section, we consider the role of auction houses with three pricing instruments. In a first step, we consider only two pricing instruments: $f \geq 0$ a positive insertion fee that the seller has to pay for auctioning the item and $\alpha \in [0, 1]$ the final value fee, which corresponds to the share of the auction price which accrues to the auction house. The constraint $f \geq 0$ reflects the fact that subsidizing the seller's entry can not be profitable if the auctioneer is unable to screen fictitious sellers that would enter the auction with dummy goods only with the purpose to capture the subsidy. In a second step, we enrich the analysis by adding a third instrument and thus endogenize the number of potential participants: we no longer consider that the number of potential buyers is fixed exogenously but we rather consider the availability of a convex marketing technology $c_{markt}(.) > 0$ such that $c_{markt}(n)$ represents the cost to obtain n potential buyers. Finally we consider competition with non-linear fees. We denote by $R(\alpha, n)$ the expected revenue of the optimal auction.

To abstract from the usual benefit of using a platform, we consider that the seller disposes also of this marketing technology if she decides to organize herself the auction. The only difference is that f and α are constrained to be equal to zero if she auctions herself the item. In other words, she can not 'burn money'. Competition between auction houses is modeled as a simultaneous Bertrand game on the pricing instruments. The seller chooses an auction house that maximizes her expected payoff, i.e. one that maximizes $(1 - \alpha)R(\alpha, n) - f$. Auction houses are restricted to make profit, i.e. $\alpha R(\alpha, n) + f - c_{markt}(n) \geq 0$.

5.1 Competition on the commission fees

The equilibrium under Bertrand competition is the solution of the following maximization program:

$$\max_{(f,\alpha)} (1-\alpha)R(\alpha,n) - f$$

subject to the positive profit constraint

$$\alpha R(\alpha,n) + f \geq 0.$$

From proposition 4.3, the seller is not constrained if shill bidding costs are high enough (it is also true if participation costs are null), then $R(\alpha,n)$ does not depend on α. Finally, the solution of this program without the constraint is: $f = 0$ and $\alpha = 0$. On the contrary, if the participation costs are positive and shill bidding costs are sufficiently low, then the seller is not only constrained in the implementation of the optimal cutoff if α is small enough: she also prefers strictly positive α to a null final value fee. It comes from the fact that x^*_{low} goes to 1 and thus the seller's expected payoff to zero when the shill bidding costs vanish under a null final value fee. Thus the solution of the program above without the constraint is: $f = 0$ and $\alpha > 0$. On the whole, the profit constraint is not binding at the optimum in both cases, which means that it is the equilibrium solution. Those results are summarized in proposition 5.1 and illustrated in Table 2 where numerical simulations for the uniform distribution give some support for fees in the range 1-6% for eBay auction. The simulations suggest that the shill bidding issue could be very important if there were no final value fee but easily tackled with small fees. For example, when the shill bidding cost is half of the participation costs and for five bidders, the revenue rises from 0.37 to 0.64 for $c_{part} = 10^{-3}$ (from 0.35 to 0.45 for $c_{part} = 5.10^{-2}$) when we go from the null fee to the equilibrium or seller-optimal fee which is equal to 2.2% (resp. 9.9%).

Proposition 5.1 Equilibrium under Bertrand competition on the commission fees

If the participation costs are null or if shill bidding costs are high enough, equilibrium fees and auctions houses profits are stuck to zero.

If the participation costs are strictly positive and shill bidding costs are sufficiently low, equilibrium fees and auctions houses profits are strictly positive.

The surprising 'positive profit result' comes from auction houses' impossibility to redistribute their profit resulting from how they solve the hold-up problem. With a marketing activity, auction houses may be able to 'burn their profit' to attract extra potential buyers, but not necessarily completely and, furthermore, the choice of the intensity of the marketing activity does not coincide with what the seller would do if she were responsible of the marketing activity.

n	$n=5$				$n=2$		
$\frac{c_{shill}}{c_{part}}$	0	1/2	1	∞	0	1/2	>1
$c_{part}=10^{-3}$							
α^*	0.027	0.022	0.016	0	0.007	0.004	0
x^*_{low}	0.638	0.629	0.619	0.508	0.512	0.509	0.501
Revenue with α^*	0.640	0.645	0.651	0.669	0.413	0.414	0.417
Revenue with $\alpha=0$	0	0.371	0.556	0.669	0	0.333	0.416
$c_{part}=10^{-2}$							
α^*	0.083	0.062	0.038	0	0.046	0.023	0
x^*_{low}	0.740	0.726	0.710	0.553	0.574	0.556	0.510
Revenue with α^*	0.552	0.572	0.597	0.648	0.383	0.395	0.407
Revenue with $\alpha=0$	0	0.366	0.547	0.648	0	0.328	0.407
$c_{part}=2*10^{-2}$							
α^*	0.111	0.079	0.040	0	0.072	0.035	0
x^*_{low}	0.775	0.760	0.740	0.585	0.611	0.586	0.519
Revenue with α^*	0.500	0.530	0.565242	0.627	0.359	0.378	0.397
Revenue with $\alpha=0$	0	0.361	0.537	0.627	0	0.323	0.397
$c_{part}=5*10^{-2}$							
α^*	0.159	0.099	0.023	0	0.119	0.048	0
x^*_{low}	0.825	0.807	0.783	0.645	0.674	0.639	0.546
Revenue with α^*	0.403	0.451	0.511	0.570	0.305	0.340	0.369
Revenue with $\alpha=0$	0	0.346	0.507	0.570	0	0.308	0.369

Table 2: Numerical example with the uniform distribution on $[0,1]$

5.2 Competition on the commission fees and the number of potential buyers

The equilibrium under Bertrand competition is the solution of the following maximization program:

$$\max_{(f,\alpha,n)} (1-\alpha)R(\alpha,n) - f$$

subject to the positive profit constraint

$$\alpha R(\alpha,n) + f - c_{markt}(n) \geq 0.$$

The corresponding program if the seller is responsible of the marketing activity (and independently of whether she choses the fee α or whether it results from Bertrand competition) is

$$\max_{(\alpha,n)} (1-\alpha)R(\alpha,n) - c_{markt}(n)$$

The solution is given by α_{sel} and n_{sel} with the corresponding first order equations:

$$(1-\alpha_{sel})\frac{\partial R}{\partial n}(\alpha_{sel},n_{sel}) = \frac{\partial c_{markt}}{\partial n}, \frac{\partial R}{\partial \alpha}(\alpha_{sel},n_{sel}) = \frac{R(\alpha_{sel},n_{sel})}{(1-\alpha)}.$$

Such a solution is labeled as **seller-efficient**.

The properties of the solution α_{eq}, n_{eq} of the Bertrand competition depend on whether the profit condition is binding or not. The basic intuition would be that auction houses are burning entirely their profit in the marketing activity but it may be wrong if the intensity of the marketing activity reaches a counterproductive point where additional buyers are lowering the auction revenue. In such a case, auction houses can not spend entirely the profit coming from the holdup resolution. Both cases should then be considered. Anyway, in each circumstances, there is no loss of generality to set $f = 0$ since the insertion fee can always be substituted by a higher final value fee since the auction revenue is increasing in α from proposition 4.3.

If the constraint is binding, the first order equations are given by:

$$\frac{\partial R}{\partial n}(\alpha_{eq},n_{eq}) = \frac{\partial c_{markt}}{\partial n}, \frac{\partial R}{\partial \alpha}(\alpha_{eq},n_{eq}) = 0$$

If the profit constraint is not binding, the first order equations are given by:

$$\frac{\partial R}{\partial n}(\alpha_{eq},n_{eq}) = 0, \frac{\partial R}{\partial \alpha}(\alpha_{eq},n_{eq}) = \frac{R}{(1-\alpha)}(\alpha_{eq},n_{eq}).$$

Furthermore, if shill bidding does not matter, e.g. if participation costs are null, then there is no profit from the holdup problem and the profit constraint is always binding: fees are used only to reimburse the marketing costs.

Those results are summarized in next proposition.

Proposition 5.2 Equilibrium under Bertrand competition on the commission fees and the number of potential buyers

If the participation costs are null or if shill bidding costs are high enough, auction houses profits are stuck to zero. The number of potential buyers and participation cutoffs are seller-efficient.

If the participation costs are strictly positive and shill bidding costs are sufficiently low, auctions houses profits may be strictly positive and only in such a case will the commission fee be seller-efficient. In any case the number of potential buyers is higher than the seller-efficient solution.

Alternative efficiency criteria deserve some consideration: the one that considers the joint surplus of the seller and the auction house and also the total welfare that covers also potential buyers' surplus. The outcome of Bertrand competition is closely related to the former. When the profit constraint is binding, i.e. auction houses burn entirely their profit, then the outcome is efficient according to this criteria.[10] The welfare perspective adds potential buyers' surplus $\Pi(\alpha, n)$ which is increasing in α. The comparative statics of $\Pi(\alpha, n)$ with respect to n is undetermined. Nevertheless, if there is some monotonicity, it is necessarily decreasing in n. The intuition is that bidders impose a negative externality between themselves through the pricing policy and through the choice of the optimal participation cutoff which increasing in n. At the end, this externality will overwhelm the benefit from a highest valuation for the winning bidder. The following discussion assumes that $\Pi(\alpha, n)$ is decreasing in n. The outcome of Bertrand competition involves then too much marketing activity with respect to both the welfare or the seller's perspectives: first, the seller alone would not internalize the positive impact of the choice of an extra potential buyer on the auction house's revenue; second, Bertrand competition omits potential buyers' surplus and thus the negative impact of an extra potential buyer. The over-marketing is especially acute when the profit constraint is not binding such that Bertrand competition does not take into account the marketing costs. In term of final value fees, the related comparison depends on whether the profit constraint is binding or not. If it is binding, then the fee is such that the seller is not constrained to implement the optimal participation cutoff which maximizes the auction revenue and also the welfare. It is then higher than what the

[10]The profit constraint would always bind if auction houses could redistribute entirely their profits by means of the insertion fee's subsidy.

seller would chose. If it is not binding, it corresponds to the seller-efficient solution as it was also the case in proposition 5.1.

5.3 The declining fee anomaly?

In the previous analysis, we exclude the use of a system of non-linear fees as it is indeed often the case, e.g. on eBay where the final value percentage fee goes from 5.25% to 1.50% when the final price rises. This decreasing pattern is the usual pattern that holds also for art or real-estate auctions where sellers of high value items are negotiating rebates with the auctioneer. It has received no attention in the literature though it can not be based on the marginal cost of organizing such an auction. In next proposition we give some properties of the equilibrium fees when auction houses are allowed to any system of non-linear fees. In the following such a system is denoted by the function $\alpha : \mathcal{R}^+ \to \mathcal{R}^+$. We impose the constraint $\alpha(x) \leq x$ and thus we exclude the possibility that the seller can redistribute some of its profits by paying more that the final price to the seller in some price range. As the restriction of positive insertion fees, this assumption can be justified by a shill bidding perspective -otherwise the seller could make profit by selling dummy goods and fix the right price through a shill bidding activity- and it is necessary to obtain the 'positive profit' result under Bertrand competition.

We denote by $J(x, x^*, r, \alpha)$ the difference of the seller's expected payoff between enrolling a shill bidder that bids x and not enrolling a shill bidder. Generalizing equation (5), we obtain:

$$\begin{aligned} J(x, x^*, r, \alpha) = n.(1 - F(x).[F(x)]^{n-1}[(x^* - \alpha(x^*)) - (r - \alpha(r))] \\ - \int_{x^*}^{\max\{x^*, x\}} [(u - \alpha(u)) - (r - \alpha(r))]n.(n-1)(1 - F(u)[F(u)]^{n-2} f(u)du \\ -c_{shill} - \alpha(r)[F(x)]^n + \int_{x^*}^{\max\{x^*, x\}} \alpha(u)[dF(u)]^n. \end{aligned} \tag{8}$$

Proposition 5.3 Bertrand competition under a system of non-linear fees

The equilibrium is uniquely[11] *characterized by a participation threshold* x_{eq} *and a fee* $\alpha(r_{eq})$ *at the equilibrium reserve price. We have the following properties:*

- *The equilibrium involves no shill bidding activity and the equilibrium reserve price* r_{eq} *is thus characterized by* $r_{eq} = x_{eq} - \frac{c_{part}}{[F(x_{eq})]^{n-1}}$.

[11] By uniqueness, we refer to fees that arise with positive probability in equilibrium. The proposition is silent on how the system of fee α should be specified outside $[x_{eq}, 1]$ and for values different than r_{eq}, i.e. for those fees that are never used in equilibrium. Any values that guarantee than the shill bidding activity is not profitable will work, i.e. $J(x, x^*, r, \alpha) \leq 0$ for $x \in [r, x_{eq}]$. For example, these constraints are satisfied if $\alpha(x) \geq x - r + \alpha(r)$.

- *The fee at x_{eq} is given by:*

$$\alpha(x_{eq}) = \max\{0, \frac{n.(1-F(x_{eq})).c_{part} - c_{shill}}{n.(1-F(x_{eq})).[F(x_{eq})]^{n-1}} + \alpha(r_{eq})[1 - \frac{F(x_{eq})}{n.(1-F(x_{eq}))}]\} \tag{9}$$

- *α is continuous on $[x_{eq}, 1]$ and characterized by the differential equation on $[x_{eq}, 1]$:*

$$\begin{array}{ll} \frac{\partial \alpha}{\partial x} = -r_{eq}.\frac{f(x)}{1-F(x)} & \text{if } \alpha(x) > 0 \\ \frac{\partial \alpha}{\partial x} = 0 & \text{if } \alpha(x) = 0 \end{array} \tag{10}$$

If the shill bidding costs are small enough, auctions houses profits are strictly positive.

Proof 6 *Equations (9) and (10) guarantee that $J(x, x_{eq}, r_{eq}, \alpha) = 0$ (respectively ≤ 0) for $x \in [x_{eq}, 1]$ and such that $\alpha(x) > 0$ (resp. $\alpha(x) = 0$). Thus after announcing the reserve price r_{eq}, it is an equilibrium not to enroll a shill bidder. We then have to prove that there are no other equilibrium candidates. In the same vein as lemma 4.1, we show that there is no loss of generality to restrict the analysis to a system of fee that induces no shill bidding activity in the subsequent equilibrium of the auction. Consider a system of fee α and an equilibrium given by x_{eq} and $p_{eq} < 1$. The corresponding equilibrium reserve price, which is characterized by equation (2), is denoted by r_{eq}. From lemma 2.2, the seller should at least maximize her payoff by not enrolling a shill bidder. We thus have $J(x, x^*, r, \alpha) \geq 0$, which holds as an equality if the candidate $x = x_{shill}$ belongs to the support of the shill bidding activity. Consider the reserve price $r' = x_{eq} - \frac{c_{part}}{[F(x_{eq})]^{n-1}} > r_{eq}$ and the system of fee α' such that $r' - \alpha'(r') = r_{eq} - \alpha(r_{eq}) - \epsilon$ with $\epsilon > 0$, $\alpha'(x) = \alpha(x)$ for $x \geq x_{eq}$ and $\alpha'(x) = x$ otherwise. We show that, if the seller announces the reserve price r', then no shill bidding activity and the participation cutoff x_{eq} is an equilibrium that raises a higher revenue for the seller. If ϵ is small enough, then the inequalities $J(x, x^*, r, \alpha) \geq 0$ still hold. Combined with the way r' has been defined, the equilibrium property is proved. Moreover, since $\alpha(r') > \alpha(r)$ whereas the participation cutoff is the same and all fees above it remain equal, the seller raises a strictly higher revenue.*

Consider a system of non-linear fees $\beta(x)$ in the equilibrium with $\beta(r_{eq}) = \alpha(r_{eq})$. To simplify the proof we consider that β is continuous and has a left and a right derivative. It is left to the reader to extend the proofs beyond this case. We show that $\beta(u) = \alpha(u)$ on the range $[x_{eq}, 1]$ in several steps. We first show that $\beta(u) \geq \alpha(u)$ on $[x_{eq}, 1]$. Suppose on the contrary that $\beta(u) < \alpha(u)$ for some $u \in [x_{eq}, 1]$ and denote by x_{inf} the infimum of such u. Two events may happen. Either $\beta(u) = \alpha(u)$ on $[x_{eq}, x_{inf}]$ or $\beta(u) \geq \alpha(u)$ on $[x_{eq}, x_{inf}]$ and the inequality is strict on a positive measure of $[x_{eq}, x_{inf}]$.

Consider the first case. Then $x \to J(x, x_{eq}, r, \alpha)$ is null on $[x_{eq}, x_{inf}]$ and then strictly positive on the right neighborhood of x_{eq}. The seller can not find it optimal not to enroll a shill bidder which contradicts lemma 2.2. Consider the second case, we show that the payoff from shill bidding x_{inf} is strictly higher than from the one without shill bids. We subtract the equation (9) which holds between r_{eq} and x_{inf} for α in the corresponding inequality that holds for β and obtain then:

$$\int_{x_{eq}}^{x_{inf}} (\alpha(u) - \beta(u))[n(n-1)[F(u)]^{n-2}(1 - F(u))f(u)du - n[F(u)]^{n-1}f(u)du] \geq 0.$$

After a rewriting, we obtain:

$$\int_{x_{eq}}^{x_{inf}} (\alpha(u) - \beta(u))nd[F(u)^{n-1} - F(u)^n] \geq 0,$$

which can not hold and thus we have raised a contradiction. On the whole we have obtained that $\beta(u) \geq \alpha(u)$ on $[x_{eq}, 1]$. Suppose that $\beta \neq \alpha$ on $[x_{eq}, 1]$, then by proposing α instead of β, an auction house catch the whole auction market.

The remaining positive profit result follows the same argument as in proposition 5.1.

As an immediate corollary of proposition 5.3, we obtain the decreasing pattern of the final value percentage fee.

Corollary 5.4 *The final value percentage fee $\frac{\alpha(x)}{x}$ is decreasing on $[x_{eq}, 1]$.*

Remark that we do not obtain the decreasing pattern on the complete range of the final prices. In particular, if the equilibrium participation cutoff x_{eq} is such that $\frac{F(x_{eq})}{n.(1-F(x_{eq}))} < 1$, then we have $\alpha(r_{eq}) = 0$.[12] Moreover, our equilibrium prediction leads to the surprising feature that the total fee and not only the corresponding percentage should be decreasing in the final price since $\frac{\partial \alpha}{\partial x} < 0$. To the best of our knowledge, such a system of fee has never been implemented. Nevertheless, our analysis is lead for a given good and not for heterogeneous items as in real-life auctions where a unique system of fee is used for items with different distribution of valuations. Thus we believe that our result gives some rationale for the current decreasing pattern.

6 Conclusion

As emphasized previously, we give some rationale for some intriguing evidence about auction houses: high fees with a decreasing pattern and

[12] It comes from equation (9) which guarantees that $\alpha(x_{eq})$ is an increasing function of $\alpha(r_{eq})$ in such a case. If $\alpha(r_{eq}) > 0$, an auction house can profitably catch the whole market by a slight price-cut on $\alpha(r_{eq})$ while keeping equations (9) and (10) and thus without losing the incentives not to shill bid.

positive profits. In our model, fees are limiting the incentives to shill bid that are coming from the possibility to hold-up the participation costs. This general insight remains true for other models where the shill bidding activity deteriorates the seller's revenue as in Lamy [18].

Our model also explains the gap between auction theory's results on the desirability of entry fees versus reserve price and their quasi-absence in real-life auctions. In most auction design frameworks as Myerson [24], appropriate entry fees do as well as reserve prices: the same participation cutoffs can be implemented with both instruments and the Revenue Equivalence Theorem can be invoked. Moreover, once we introduce some variations, the former indifference is broken in favour of entry fees as in Milgrom and Weber [23], Levin and Smith [19] or Damianov [11].[13] Since an entry fee corresponds exactly to participation costs, the shill bidding commitment issue will grow with the entry fee.

Our model is limited insofar as we restrict our analysis to a restricted class of mechanism for the competition between auction houses. The analysis is confined to the English or second price auctions, whereas the first price auction would be immune to shill bidding in the same way as emphasized in Lamy [18]. However, if the seller is unable to commit not to solicit another round of offers in the first price auction, the equilibrium may closely correspond to the English auction with an endogenous and costly pace as shown by McAdams and Schwarz [22]. Enlarging the set of trading mechanism is left for future research. In this perspective, shill bidding may shed some light on the recent success of buy-it-now options in online auctions which makes the market closer to posted prices where the incentives to shill bid are null.

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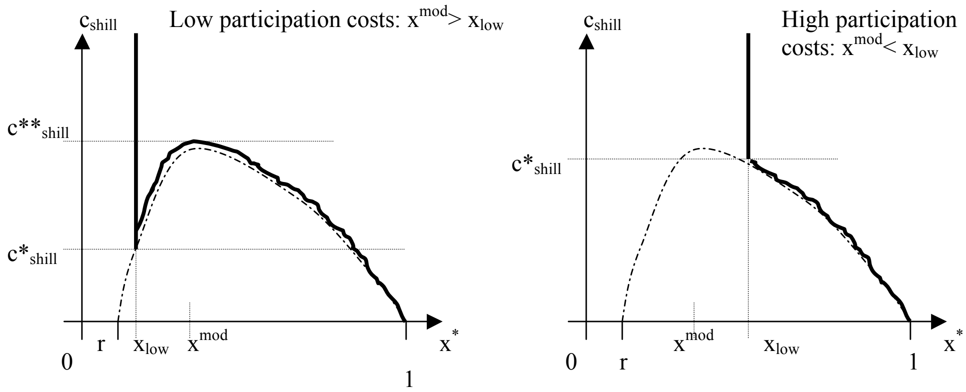
Figure 1: Equilibrium set under assumption A1



: Equilibrium set in the (x^{*}, c_{shill}) quadrant

: $H(x^{*}, r)$

Figure 2: Low shill bidding costs: $c_{shill}=c^*_{shill}$ for $r=r_3$

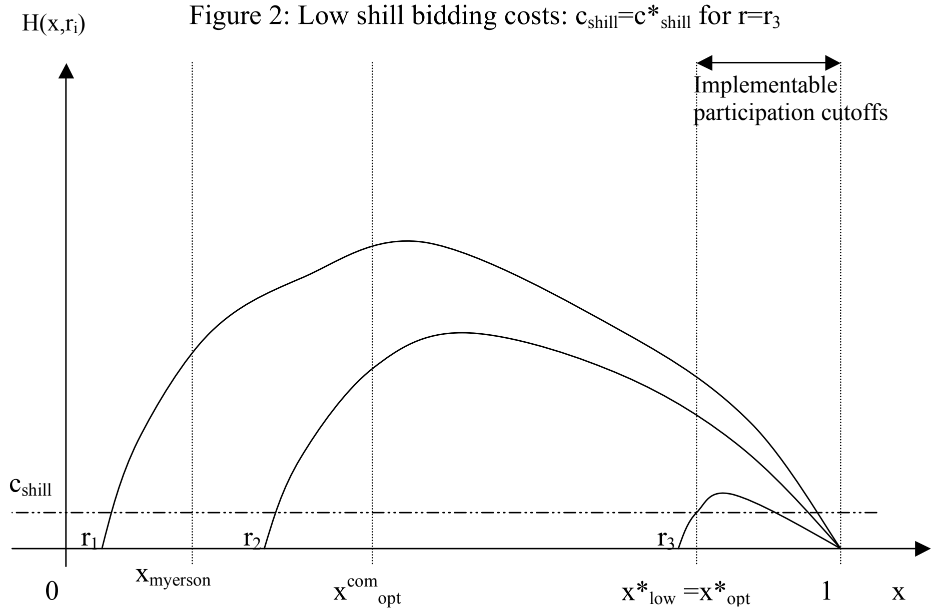


Figure 3: Medium shill bidding costs: $c_{shill}=c^*_{shill}$ for $r=r_2$