

# Haircuts and Repo Chains\*

Tri Vi Dang  
Columbia

Gary Gorton  
Yale and NBER

Bengt Holmström  
MIT and NBER

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## Abstract

The recent financial crisis centered on the sale and repurchase (“repo”) market, a very large short-term collateralized debt market. Repo transactions often involve overcollateralization. The extent of overcollateralization is known as a “haircut.” Why do haircuts exist? And what determine the size of the haircut? We show that the existence of haircuts is due to sequential trade in which parties may default and intermediate lenders face liquidity needs. When there is a positive probability that the borrower will default, then the lender’s liquidity needs and own default risk in a subsequent transaction to sell the collateral become paramount. The haircut size depends on (i) the default probabilities of the borrower, (ii) the liquidity needs of the lender, (iii) the default probability of the lender in a subsequent repo transaction and (iv) the nature of the collateral. Chains of transactions involving risky counterparties with intermediate liquidity needs are the key to haircuts.

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## 1. Introduction

The recent financial crisis was precipitated by a run in the sale and repurchase (repo) market, and other short-term funding markets such as asset-backed commercial paper.<sup>1</sup> In repo markets the run corresponded to an increase in repo haircuts. That is, when a bond of a financial institution that was completely financed by borrowing in the repo market is subsequently only 90 percent financed, then there is a 10 percent repo haircut. This is the withdrawal from the financial institution of 10 percent, the amount that the financial institution (borrower) now has to fund in some other way. If there is no new funding forthcoming, the borrower must sell assets.

The (bilateral) repo market is a decentralized market for short-term, usually overnight, collateralized borrowing and lending that uses debt as collateral.<sup>2</sup> The repo market had an average daily trading volume of \$7.6 trillion in the first quarter of 2008.<sup>3</sup> The 10Qs filings of Goldman Sachs, Merrill Lynch, Morgan Stanley, JP Morgan, Lehman Brothers and Bears Stearns revealed that these six former investment banks together had financial assets worth \$1.379 trillion of which 47% were pledged as collateral at the end of 2006 (see King (2008) and Gorton and Metrick (2010a)). For example, the refinancing need of Merrill Lynch on the repo market in mid 2008 was \$194 billion, larger than the average total daily trading volume of stocks on NYSE and NASDAQ together. The March 2010 report of the Bankruptcy Examiners on Lehman Brothers' failure, states: "*Lehman funded itself through the short-term repo markets and had to borrow tens or hundreds of billions of dollars in those markets each day from counterparties to be able to open for business.*" (Report of Anton R. Valukas (2010) p.3)

There are two parts to a repo financial transaction. First, a cash lender lends to a borrower. The borrower will pay interest (the repo rate) on the money borrowed. In addition, to insure that the money lent out is safe, the borrower provides collateral to the lender. The collateral is valued at market prices and the lender takes physical possession of the collateral (via his clearing bank). The second part of the transaction consists of a contemporaneous agreement by the borrower to repurchase the securities at the original price on a specified future date. So, at the maturity of the repo transaction, the collateral is returned to the borrower when he repays the amount borrowed plus the repo rate to the lender.

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<sup>1</sup> See Hödahl and King (2008), Gorton (2010) and Gorton and Metrick (2010a,b) for descriptions of the run on repo. The run in the asset-backed commercial paper market is described by Covitz, Liang, and Suarez (2010). Also, see Aitken and Singh (2009).

<sup>2</sup> We focus on the bilateral market in which two parties privately contract. A smaller part of the repo market is the triparty repo market in which there is an intermediary, a triparty bank. See Copeland, Martin and Walker (2010).

<sup>3</sup> SIFMA (2008, p. 9) reported that: "The average daily volume of total outstanding repurchase (repo) and reverse repo agreement contracts totaled \$7.06 trillion in the first quarter of 2008, a 21.5 percent increase over the \$5.81 trillion during the same period in 2007." See the International Capital Markets Association (2009) repo survey for the size of the European repo market.

The standard repo contract states that if one party defaults, then the nondefaulting party can unilaterally terminate the agreement and keep the cash or the bond depending on their position.<sup>4</sup> Repo transactions are excluded from the U.S. Bankruptcy Code. Bankruptcy by a party to the repo transaction allows the nondefaulting party the option to unilaterally terminate the transaction and keep the money (if the depositor defaults) or sell the bond (if the borrower defaults). A repo transaction is a “safe” money-like instrument because there are no Chapter 11-type disputes over collateral or money.<sup>5</sup> The money is readily available to the lender, either because he is repaid or because he is expected to be able to recover the money easily by selling the (liquid) collateral

Repo transactions typically involve overcollateralization. The extent of overcollateralization is known as a “haircut.” A haircut is the case where less money is lent than the market value of the assets received as collateral. Suppose the lender lends \$90 million and receives \$100 million of bonds (at market value), then there is said to be a ten percent haircut. The haircut is distinct from “margin” which refers to maintaining the value of collateral should market prices adversely change after the contract is signed. Margining is standard practice in (longer term) repo, occurring during the whole period of the transaction. But this has nothing to do with the haircut which is a “price discount” relative to the current market price of the collateral and set when the contract is initially signed. The margin does not depend on the identities of the counterparties, but haircuts do, as we show below.

The existence of repo haircuts is a puzzle, as standard finance theory would suggest that risk simply be priced and the market price reflects risk and risk aversion of the market. Why is there a discount relative to the current market price? In addition, haircuts are not the same for all counterparties to repo trades, even for the same type of collateral. Table 1 shows repo haircuts on different types of collateral for two different sets of counterparties. In Panel A the collateral is non-mortgage asset-backed securities (backed by automobile receivable, credit card receivables, or student loans). Panel B is for corporate bonds. In each case, the haircuts are shown for different ratings. In each panel there are columns labeled “hedge funds” and columns labeled “banks.” The columns labeled hedge funds correspond to repo transactions between a high-quality dealer bank (cash lender) and a representative mid-sized hedge fund (roughly \$2-5 billion in size). The columns labeled banks correspond to repo transactions between two high quality dealer banks. The table shows that the haircuts vary depending on the identity of the counterparties, holding fixed the asset class and rating of the collateral. One of our goals is to explain these types of patterns in haircuts.

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<sup>4</sup> For more information about the repo market see Corrigan and de Terán (2007) and Garbade (2006).

<sup>5</sup> In bankruptcy, repurchase agreements are exempted from the automatic stay. The nondefaulting party to a repurchase agreement is allowed to unilaterally enforce the termination provisions of the agreement as a result of a bankruptcy filing by the other party. Without this protection, a party to a repo contract would be a debtor in the bankruptcy proceedings. The safe harbor provision for repo transactions was recently upheld in court in a case involving American Home Mortgage Investment Corp. suing Lehman Brothers. See, e.g., Schroeder (1999), Johnson (1997), and Schweitzer, Grosshandler, and Gao (2008).

It is important to understand the repo transaction process. Because repo transactions are largely overnight, there is little due diligence on the bonds offered as collateral. They typically must have a rating and must have a current market price from a third party. If so, then that is usually sufficient for it to be acceptable as collateral. Some lenders specify broad parameters for the types of collateral to be acceptable. In this market billions and billions of dollars of repo are rolled each business day based on little due diligence.

In this paper we address the following questions: What makes repo so “liquid”? That is, how can it happen that billions and billions of dollars of repo are rolled each business day based on little due diligence? Why do haircuts exist? And what determine the size of the haircut? We provide micro foundations for collateralized borrowing and lending (repo trading) and a theory of haircuts. We compare repo to an asset sale.

In the setting we study there will be two transactions sequentially, and three agents, called agents A, B, and C. The situation is one in which it is efficient for agent A to borrow from agent B and consume at the first date, and then for agent B to trade with agent C and consume at the second date. Agent C consumes at the final date. Agent A owns a security which he can offer as collateral. The security is a claim on a risky payoff. Agent C is a sophisticated agent in that he can produce private information about the security payoff, while the other agents cannot produce private information. Agents may default so there is counterparty risk. For example, if agents A and B enter into a repo transaction, then there is some chance that agent A will not be able to repurchase the security from agent B. If agent B faces liquidity needs at the second date, then he seeks to sell the security to agent C.

The chance that agent A defaults and agent B needs cash jointly determine the likelihood that agent B will face agent C. Secondly, if agent B is very likely to repurchase the asset from agent C, under a repo contract, at the final date, then there is little incentive for agent C to produce information. But, if agent B is likely to default in the second transaction, then agent C may be tempted to produce information. Worrying about this possibility, agent B will want to build in protection when he trades with agent A at the first date. So, counterparty risk of agents A at the first transaction and counterparty risk of agent B at the second transaction are very important.

The nature of the collateral is also important. Collateral is characterized by its *information sensitivity* (IS), a concept formalized by Dang, Gorton, and Holmström (2012a) (DGH1). IS is a property of any security; it corresponds to the utility value of learning information about the security’s payoff distribution. IS measures the value of information in the sense that an informed lender learns and avoids lending and accepting the security as collateral.

Thus, we derive four determinants of a repo haircut: the default probability of the borrower, the liquidity needs of the lender and the default probability of the lender (when he needs to borrow subsequently) and the IS of the underlying collateral. These four parameters together yield a rich set of

implications for haircuts which can explain the qualitative feature of three stylized facts in Table 1, that: (i) assets with higher IS (i.e. collateral with lower ratings in Table 1) have weakly higher haircuts; (ii) borrower with higher default probability faces weakly higher haircuts (the hedge funds in Table 1), (iii) lender with higher intermediate liquidity needs demands higher haircuts. (iv) lender with higher default probability in subsequent repo trade demands higher haircuts for the initial repo trade.

Finally, we ask what can cause a crisis in the repo market. Public information which arrives at the interim date can change the IS of the collateral. A run on repo arises in this model if macroeconomic news causes information insensitive collateral asset to become information sensitive (see Dang, Gorton, and Holmström (2012b) (DGH2)). As an endogenous response the lender (agent B) demands a higher haircut when he trades with agent A at  $t=0$ . This is equivalent to not rolling over repo in the same amount. To obtain the same amount of money the borrower has to come up with more collateral. A “repo run” occurs if bad news arrives and haircuts rise. If the IS of the collateral rises, this increases the haircut.

This paper builds on Dang, Gorton, and Holmström (2012a) (DGH1) who derive a utility-based measure, called information sensitivity (IS). This measure is a property of any security; it corresponds to the utility value of learning information about the security’s payoff distribution. We show that IS is an important determinant of haircuts. This paper is also related to Dang, Gorton, and Holmström (2012b) (DGH2) who analyze the question of the optimal design of a security for trading purposes that allows agents to transport excess cash through time. DGH2 show that debt is the optimal security because debt minimizes IS. The present paper analyzes a related question, namely how agents with a shortage of cash can obtain cash. We add the feature of collateralized borrowing and default risk to the model in DGH2.

Repo is an example of collateralized borrowing. There are many such examples. Others include borrowing to buy a home or buying stock on margin. Our central message is that collateralized borrowing allows the borrower to borrow more because it reduces the information sensitivity of the collateral or more precisely, repo reduces the tail risk of the cash flow distribution the lender is facing.

The literature on the rationale for collateral in borrowing and lending is scant and focuses on the role of collateral as a screening device in asymmetric information settings. See, e.g., Stiglitz and Weiss (1981), Bester (1985, 1987), and Manove, Padilla, and Pagano (2001). Geanakoplos and Fostel (2008) and Geanakoplos (2009) analyze the role of collateral and speculation in a competitive economy where traders have differences of opinions.

The paper proceeds as follows. The next section introduces the model. Section 3 analyzes the information sensitivity (IS) of the collateral and the haircuts. Section 4 shows that the counterparty risks of the borrower, intermediate liquidity needs of the lender and his default probability in the

subsequent repo trade are further determinants of haircuts. Section 5 compares repo with an asset sale. Section 6 analyzes a run on repo. Section 7 concludes.

## 2. The Model

We consider an exchange economy with one storable good, three dates  $\{t=0, 1, 2\}$  and three risk neutral agents  $\{A, B, C\}$  with utility functions:

$$U_A = C_{A0} + (1-l_A)(C_{A2} + C_{A3})$$

$$U_B = C_{B0} + C_{B1} + (1-l_B)C_{B2}$$

$$U_C = C_{C1} + C_{C2} + C_{C3}$$

where  $0 < l_A \leq 1$  and  $(1-l_A)$  is the marginal value of consumption of agent A at dates 1 and 2. Since  $l_A > 0$ , agent A prefers to consume at  $t=0$ , i.e. he has liquidity needs at  $t=0$ . Similarly,  $0 \leq l_B \leq 1$  or  $(1-l_B)$  is the marginal value of consumption of agent B at  $t=2$ . If  $l_B=0$ , agent B values consumption goods at  $t=1$  and  $t=2$  the same, i.e. he has no liquidity needs. If  $l_B > 0$ , he attaches less value of consumption at  $t=2$  than  $t=1$ . Or in other words, he needs liquidity at  $t=1$ . The endowments of the agents are given as follows:

Agent A obtains  $w$  units of goods at date 1 with probability  $(1-\phi_A)$ .

Agent B owns  $w$  units of goods at  $t=0$ .

Agent B obtains  $w$  units of goods at date 2 with probability  $(1-\phi_B)$

Agent C owns  $w$  units of goods at  $t=1$  and nothing at the other dates.

There are three key parameters in the model:  $\phi_A$ ,  $\phi_B$  and  $l_B$ . We interpret  $\phi_i$  as the default probability of agent  $i$ , the counterparty risk of agent  $i$ . In the context of a repo transaction,  $\phi_A$  is the probability that agent A defaults on his loan and is not able to repurchase the collateral at  $t=1$ . Similarly,  $\phi_B$  captures the default probability of agent B at  $t=2$ .  $l_B$  is a measure of the liquidity needs of agent B at  $t=1$ . Default probabilities and liquidity needs are common knowledge at  $t=0$ .

Given the assumed form of the utility functions and endowments, there are gains from trade. It is socially efficient for agent A to consume at  $t=0$ , for agent B to consume at  $t=1$  if  $l_B < 1$ , and for agent C to consume at  $t=2$ . A Pareto efficient allocation is the following: Agent A sells his security  $s(x)$  to agent B for agent B's  $t=0$  goods. Then at  $t=1$  agent B sells  $s(x)$  to agent C for agent C's  $t=2$  goods. Since agent C is indifferent about when to consume it is socially efficient for him to consume  $s(x)$  at  $t=2$ .

### Information about the security

It is common knowledge that the payoff of the security  $s(x)$  depends on the realization of an underlying asset  $X$  which is a random variable with distribution  $F(x)$ . At  $t=2$  there is public (and verifiable) information about the realization of  $x$ .

A security is a function that maps each realization of  $x$  to a payment  $s(x)$  in units of goods, with the restriction that  $s(x) \leq x$ . Here are three examples: (i) If  $s(x)$  is a debt contract, then  $s(x) = \min[x, D]$  where  $D$  is the face value of debt; (ii) if  $s(x)$  is equity, then  $s(x) = \beta x$  where  $\beta$  ( $0 < \beta \leq 1$ ) is the share of  $x$ ; (iii) if  $s(x)$  is an AAA rated mortgage backed security (MBS), then  $s(x)$  is senior debt and  $x$  the payoff of the pool of mortgages that backs the MBS.

We denote the expected payoff of the collateral,  $E[s(x)]$  with  $V$ , denoting the (expected) value of collateral. To save on notation, we assume that  $w=V$ , i.e. agent B has enough cash so that agent A is able to borrow fully against his collateral if agent B is willing to do that.

### Private information production

Only agent C can produce private information about the true realization  $x$  at  $t=1$  at the cost  $\gamma > 0$  in terms of utility.

Later, we will introduce a public signal as well.

### Repo contract

A repo contract consists of three parts: (i) the underlying collateral  $s(x)$  with value  $V$ ; (ii) the amount  $L$  lent (the loan size) which is also the repurchase price of  $s(x)$ ; and (iii) the repo interest rate  $r$ . The haircut is implicit in parts (i) and (ii). A repo contract is denoted by  $(L, s(x), r)$ .

The effective repayment is thus  $L(1+r)$ . Since we assume risk neutrality and storability the repo rate is zero. If the borrower repays  $L$  then he gets back  $s(x)$ . Otherwise the lender keeps  $s(x)$ .

Definition: A *haircut*  $H$  is defined as  $H = 1 - \frac{L}{V}$ .

We assume that only  $s(x)$  can be used as collateral. Future goods (i.e.  $w$  at  $t+1$ ) are not contractible at date  $t$ , i.e. agent A cannot borrow against his potential  $t=1$   $w$  endowment.

### Sequence of moves and events

$t=0.1$  Agent A makes a take-it-or-leave-it repo contract offer to agent B.

$t=0.2$  If agent B rejects, the game ends. Otherwise:

$t=1.0$  Agent C enters the economy.

If agent A obtains  $w$ , he decides whether to repurchase  $s(x)$ . If he repurchases  $s(x)$  the game ends. If he does not repurchase, then:

$t=1.1$  Agent B makes a take-it-or-leave-it repo contract offer to agent C.

$t=1.2$  Agent C decides whether to produce information and then whether to trade. If there is no trade, the game ends. Otherwise:

$t=2.0$  If agent B obtains  $w$ , agent B decides whether to repurchase  $s(x)$ .

The model is intended to capture the following situation. At  $t=0$ , agent A (say a dealer bank, like Bear Stearns) has a shortage of cash and wants to borrow from agent B (an institutional investor). In a repo transaction, the institutional investor “deposits” money with the dealer bank, and this bank then posts  $s(x)$  as collateral to the institutional investor. The collateral,  $s(x)$  in our notation, could be a U.S. Treasury bond, but it could also be corporate bond, an agency bond, or an asset-backed security. Both the dealer bank and the institutional investor might default. We will assume that the default probabilities are not correlated with the default risk of the collateral.

It is important for our purposes to note what happens if the dealer bank (the borrower) were to default and fail to repurchase the collateral. Then the institutional investor (the lender) owns the collateral  $s(x)$ . What if the institutional investor itself needs money the next period? The institutional investor does not have the money that was lent to the now defaulted dealer bank. But, it has the bond,  $s(x)$ , and can offer that as collateral in order to borrow from a third party. Given this potential problem for the institutional investor, a key question is: How much money is the institutional investor willing to lend to the dealer bank in the first place?

In this scenario the institutional investor corresponds to agent B and the dealer bank corresponds to agent A. If agent A defaults and does not repurchase the collateral, then agent B may want to borrow from agent C. But, we assumed that agent C is more sophisticated in the sense that only he can produce information about the realization of  $X$  and thus can learn about the payoff  $s(x)$  before he trades with agent B. It is this possibility which drives the results. To be clear, the possibility that agent B may have to trade with an agent (agent C) who could be better informed is the key problem. But, note that if the transaction between B and C is a repo, then the crucial question is whether agent B is likely to repurchase  $s(x)$  at the final date.

Note that if agent B is a very strong counterparty, in fact say that agent B will not default for sure. In that case, there is certainly no point for agent C to ever produce private information because agent B will always repurchase the security at the final date. The only reason that will be profitable for agent C to produce information is if agent B has a positive probability of defaulting and not repurchasing the

collateral. As we will see, this factor---the counterparty risk of agent B---will feedback to affect the terms of the initial transaction between agents A and B.

### 3. Information Sensitivity (IS) and Haircuts

In this section we analyze three benchmark cases: (i)  $l_B=0$  (agent B has no liquidity needs) and (ii)  $l_B=\phi_B=\phi_A=1$  (agent A has no endowment at  $t=1$ , agent B has liquidity needs at  $t=1$  and no endowment at  $t=2$ ). These cases highlight the economic intuition about why a haircut can arise in equilibrium.

**Proposition 1:** Suppose  $l_B=0$  (i.e. agent B has no liquidity needs). At  $t=0$ , agent B lends  $L_A=V$  to agent A and  $H=0$ .

**Proof:**  $l_B=0$  means that agent B has no urgency for consumption (money) at  $t=1$  so that he can just keep  $s(x)$  and consume  $V=E[s(x)]$  at  $t=2$  if agent A does not repurchase  $s(x)$  at  $t=1$ . Anticipating this, agent A proposes to borrow  $L_A=V$  by posting  $s(x)$  as repo collateral which agent B accepts.//

Proposition 1 is a special case of Proposition 4. This result is intuitive. If  $l_B=0$  (i.e., agent B is indifferent between consumption at  $t=1$  and  $t=2$ ), then the expected “user” value of  $s(x)$  for agent B is the same as the (objective) expected value of  $s(x)$ . In other words, agent B does not rely on the resale market so he is not concerned about any potential adverse selection. This highlights an interesting point. If the lender is not financially constrained in the sense that he can just keep the collateral until maturity, then there is no haircut. A central bank is an example of such a lender.

Proposition 1 highlights a potential reason for haircuts to arise in equilibrium. A haircut occurs in equilibrium because of the concern of agent B that if he needs to consume at  $t=1$ , he faces the potential adverse selection problem if he wants to borrow from agent C. But, when  $l_B=0$ , agent B need not transact with agent C, so the issue of adverse selection does not arise.

Subsequently we will consider the case where agent B has liquidity needs and is counterparty risk in that agent B might default at the final date. We formalize the potential problems that agent B then faces when dealing with agent C and how this feeds back to the repo trading between the identically informed agents A and B at  $t=0$ .

Now we provide a complete analysis of the equilibrium of the game under the assumption that  $l_A=l_B=\phi_A=\phi_B=1$ , i.e., agent A cannot repay when he borrows and agent B has no value of consumption at  $t=2$  and cannot repay at  $t=2$ . Agent B is first a lender or buyer (at  $t=0$ ) and then a borrower or seller (at  $t=1$ ).

We will show that a key determinant of haircuts is the value of information or information sensitivity (IS) of the collateral. DGH1 define IS as:

$$\pi = \int_{x_L}^{x_H} \max[L - s(x), 0] f(x) dx$$

with  $L = E[s(x)]$ . In words, IS measures the value in the tail of the distribution where the realizations of  $x$  are such that  $L$ , the amount lent, is greater than  $s(x)$ , the value of the collateral. So, for example, for an agent offered  $s(x)$  as collateral in repo, for a loan of  $L$ , the IS measures the value of information in the sense that an informed lender learns and avoids lending and accepting the security as collateral. The Appendix gives a brief overview of the IS concept.

**Proposition 2:** Suppose  $l_A = l_B = \phi_A = \phi_B = 1$ . Equilibrium at  $t=0$  has the following properties:

- (i) If  $\pi \leq \gamma$ , then agent B lends  $L_A = V$  to agent A and  $H=0$ .
- (ii) If  $\pi > \gamma$ , then agent B lends  $L_A < V$  to agent A and  $H > 0$ , where  $L_A = \max[p_I, E[C_{B2}(p_{II})]$  where

$$p_I \text{ solves } \int_{x_L}^{x_H} \max[p_I - s(x), 0] f(x) dx = \gamma$$

and

$$p_{II} \text{ maximizes } E[C_{B2}(p_{II})] = \int_{\{x: s(x) \geq p_{II}\}} p_{II} f(x) dx = p_{II} \cdot (1 - F(p_{II}))$$

$$\text{subject to: } \int_{x_L}^{x_H} \max[s(x) - p_{II}, 0] f(x) dx \geq \gamma.$$

**Proof:** We solve for a subgame perfect equilibrium of this game by backward induction. What happens in the subgame starting at  $t=1$  affects agent B's behavior at  $t=0$  when he trades with agent A.

Equilibrium of the subgame starting at  $t=1$

Agent B owns  $s(x)$  (agent B ends up owning  $s(x)$  because agent A is short-lived). Suppose agent B makes an offer to agent C to borrow  $L_B$  by posting  $s(x)$  as collateral. The IS of the collateral to agent C is given by:

$$\pi = \int_{x_L}^{x_H} \max[L_B - s(x), 0] f(x) dx.$$

If  $\pi \leq \gamma$  (i.e. information is too costly to produce), then the best response of agent C is to lend the amount  $L_B = V$  to agent B. As collateral agent C obtains  $s(x)$  which he consumes at  $t=2$ , since agent B does not appear to repurchase it since he is short-lived.

If  $\pi > \gamma$ , then agent C's best response is to produce information before deciding whether or not to trade with agent B (that is, lend him the amount  $L_B = V$  for collateral  $s(x)$ ). If agent C knows that  $s(x) < L_B$ , he

does not lend. If he knows that  $s(x) \geq L_B$ , he lends agent B the amount  $L_B$ . In this case, ex ante, a transaction only occurs with the probability that  $x$  is such that  $s(x) \geq L_B$ .

The maximum amount  $L_B = p_I$  that agent B can borrow from agent C with probability one is given by:

$$\pi(p_I) = \int_{x_L}^{x_H} \max[p_I - s(x), 0] f(x) dx = \gamma$$

since at that price agent C does not produce information. His utility is  $EU_B(p_I) = p_I$ . Call this Strategy I.

If  $\gamma$  is small, then the amount  $L_B$  that agent B can borrow is small using Strategy I. Thus, another potential best response of agent B is to induce agent C to produce information and possibly trade a larger amount (but with probability less than one). In this case, the optimal offer is to post (sell)  $s(x)$  for the amount  $p_{II}$  such that  $p_{II}$  maximizes expected amount to trade and consume:

$$E[C_{B2}(p_{II})] = \int_{\{x: s(x) \geq p_{II}\}} p_{II} \cdot f(x) dx$$

subject to:

$$\int_{x_L}^{x_H} \max[s(x) - p_{II}, 0] f(x) dx \geq \gamma.$$

Call this Strategy II.

Agent B compares the two strategies and chooses  $L_B = p_I$  if  $p_I \geq E[C_{B2}(p_{II})]$ . Otherwise he proposes to borrow  $L_B = p_{II}$ . In this case, agent C is informed and he does not lend if  $s(x) < p_{II}$ .

These arguments show that agent B may not be able to borrow (or sell  $s(x)$  for)  $L_B = V$  and consume that amount at  $t=1$ .

#### Equilibrium of the subgame starting at $t=0$ .

Agent B anticipates what he can consume at  $t=1$  when trading with agent C at  $t=1$ . Given that security  $s(x)$  is used as collateral at  $t=1$ , the maximum loan  $L_A$  agent B gives agent A at  $t=0$  is:

$$L_A = L_B = \max[p_I, E[C_{B2}(p_{II})].$$

If  $L_A < V$ , then agent B demands a haircut when lending to agent A, anticipating that he will have a problem reselling  $s(x)$  at  $t=1$ . //

The proposition highlights the problem of agent B, who is initially the lender to agent A and subsequently the borrower from agent C. In the initial transaction with agent A, the concerns of agent B with respect to the IS of the collateral play an important role. The above setting makes this manifest

because agent A needs to consume at  $t=0$  and has no  $t=1$  endowment so agent B will definitely have to sell the bond (or borrow against it) with agent C. Concerns about the IS of the collateral at  $t=1$  with agent C drive the haircut on the collateral in the initial transaction with agent A at  $t=0$ .

The volatility (i.e. variance) of the payoff of a security,  $s(x)$  itself cannot explain a haircut. To see this, suppose agent B and C are (equally) risk averse. Given any (concave) utility function we can calculate the market price  $V$  of  $s(x)$  as the certainty equivalent. Suppose no agent can produce information. In equilibrium agent B lends  $L_A=V$  to agent A and there is no haircut.

**Corollary 1:** Suppose  $l_A=l_B=\phi_A=\phi_B=1$  and debt (D) and equity (E) have the same expected payoff, i.e.  $E[s^D(x)]=E[s^E(x)]$ . Then in equilibrium at  $t=0$ ,  $L_A^E \leq L_A^D$  and  $H_A^E \geq H_A^D$ . Both inequalities are strict if  $\pi^D > \gamma$ .

**Proof:**  $X$  is the underlying asset that backs a security. Suppose  $s^D(x)=\min[x,D]$  where  $D < x_H$ .<sup>6</sup> We show that for any  $F(x)$ , (i)  $p_I^D > p_I^E$  and (ii)  $p_{II}^D > p_{II}^E$ .

(i) For  $s(x)$  debt,  $\pi^D = \int_{x_L}^p (p-x)f(x)dx$ . For,  $s(x)=\beta x$ ,  $\pi^E = \int_{x_L}^{\frac{p}{\beta}} (p-\beta x)f(x)dx$ . Since

$E[s^D(x)]=E[s^E(x)]=p$ ,  $\pi^D < \pi^E$ .<sup>7</sup> If debt triggers information production, then so does equity. If  $\pi^D > \gamma$ , this implies that  $p_I^D > p_I^E$ .

(ii) If  $s(x)$  is debt, then  $EU_B = \int_{p_{II}}^{x_H} p_{II} \cdot f(x)dx$  and the optimal  $p_{II}$  maximizes  $(1-F(p_{II}^D)) \cdot p_{II}^D$ .

If  $s(x)$  is equity, then  $EU_B = \int_{\frac{p_{II}}{\beta}}^{x_H} p_{II} \cdot f(x)dx$  and the optimal  $p_{II}$  maximizes  $(1-F(\frac{p_{II}^D}{\beta})) \cdot p_{II}^D$ . This

implies that  $p_{II}^D > p_{II}^E$ . //

This corollary shows that debt is traded with a lower haircut than equity. This also implies that the expected return on debt is less than the expected return on equity in a setting where agents are risk neutral.

#### 4. Information Sensitivity, Liquidity Needs, Counterparty Risks, and Haircuts

In the previous section we analyzed equilibrium lending at  $t=0$  for the polar cases where  $l_B=0$  (agent B has no liquidity needs) and  $l_A=l_B=\phi_A=\phi_B=1$  (agent A defaults at  $t=1$  and agent B has liquidity needs and defaults at  $t=2$ ). In this section we highlight an important incentive effect of repo in dealing with potential adverse selection when  $0 < \phi_A, \phi_B < 1$ , both agents A and B have a positive probability of

<sup>6</sup> If  $D=x_H$ , then  $s(x)=\min[s,D]=x$ , i.e. equity with  $\beta=1$ .

<sup>7</sup> DGH2 show debt is a least information-sensitive security among all possible securities.

defaulting as a borrower. In addition, we show how the IS of  $s(x)$  and the default probability of the borrower (agent A) at  $t=1$ , the liquidity needs of agent B at  $t=1$  and his default probability at  $t=2$ , generally affect the equilibrium haircut at  $t=0$ .

Recall that the IS of the collateral is  $\pi$  and the probability that agent B defaults at  $t=2$  is  $\varphi_B$ . Given that the cost of producing information is  $\gamma$ , agent C finds information production at  $t=1$  profitable if  $\pi\varphi_B > \gamma$  and not otherwise.

**Proposition 3 (H=0)**: For any  $\{F(x), l_A, l_B, \varphi_A, \gamma\}$ , if  $\pi\varphi_B \leq \gamma$ , then:

- (i) Agent C does not produce information at  $t=1$ ;
- (ii) There is no haircut at  $t=0$ , i.e. agent B lends agent A the amount  $L_A=V$ .

**Proof**: Suppose agent B offers  $s(x)$  as collateral for a loan of  $L_B=V$  from agent C at  $t=1$ . There are two cases.

**Case I (No information production)**: We show that if agent C trades without information production, his expected payoff is  $EU_C = w$ . Note that at  $t=2$ , if agent B does not default, a best response of agent B is to repurchase  $s(x)$ . Thus, agent C gets back the amount  $L_B$  that he lent to agent B at  $t=2$ . If agent B defaults, agent C owns  $s(x)$  with  $E[s(x)]=L_B$ . Thus:

$$EU_C = w - L_B + \varphi_B \cdot E[s(x)] + (1 - \varphi_B)L_B = w.$$

**Case II (Information production)**: Agent C produces information before responding to agent B's offer. If agent C is informed and agent B offers  $s(x)$  for  $L_B=V$ , his expected profit from being informed is:

$$\pi = \int_{x_L}^p (p - x) \cdot f(x) dx$$

and his ex ante (prior to information production) expected utility is:  $EU_C = w + \pi - \gamma$ . Since the true type of agent B (whether B defaults at  $t=2$ ) is not known at  $t=1$ , the expected payoff to agent C from information production when seeing an offer  $s(x)$  for  $L_B=V$ , is given by:

$$EU_C = w + \varphi_B \pi - \gamma.$$

This formula highlights the point that repo changes the distribution of the payoff to agent C and thus the tail risk of payments in utility terms. For any realized  $x$ , there is a probability  $(1-\varphi_B)$  that agent C gets back  $L_B$ .

Thus, if  $\pi\varphi_B \leq \gamma$ , the best response of agent C is to not produce information. Anticipating this, agent B is willing to borrow  $L_A=L_B=V$  to agent A at  $t=0$ . //

Proposition 3 highlights two important insights. For any  $\gamma > 0$ , if the information sensitivity of the collateral is sufficiently low, there is no haircut at  $t=0$ . Or if agent B is sufficiently likely to be able to repay the loan at  $t=2$  (i.e.  $1-\varphi_B$  large so that counterparty risk is low), then there is no haircut at  $t=0$ , irrespective of the IS of collateral, the default probability of agent A, and the intermediate liquidity needs of agent B. If  $1-\varphi_B$  is large (i.e. agent B is likely to repurchase  $s(x)$  from agent C at  $t=2$ ), then agent C cannot exploit agent B by producing information and only trading if  $s(x) \geq L_B$ . At the same time, even if agent C learns  $x$  and knows that  $s(x) < L_B$ , he is willing to lend since agent B is likely to obtain endowment and is going to repurchase  $s(x)$  by repaying  $L$  at  $t=2$  since  $X$  is only revealed at the end of  $t=2$ .

The right to repurchase the collateral is a fundamental part of collateralized borrowing. A haircut provides an incentive to do so whenever the borrower is able to repay, i.e. he has money at the repayment date. Importantly, the option (and right) to repurchase the collateral, and the ability and incentive to do so, can eliminate the incentive of agent C to produce information, creating adverse selection in subsequent trading. To emphasize, it does not pay to produce private information about a security that a counterparty is likely to repurchase at a pre-agreed price.

**Proposition 4 ( $H > 0$ ):** If  $\pi\varphi_B > \gamma$ , then at  $t=0$  agent B is willing to lend to agent A the amount  $L_A = V - \varphi_A \cdot (V - L_B)$ , where  $L_B = \max[(1-l_B)V, p_I, E[C_{B2}(p_{II})]]$ , where  $p_I$  and  $p_{II}$  are defined in Proposition 2. Agent A does not conduct a repo trade with agent B if

$$l_A < 1 - \frac{L_A}{V + (1 - \varphi_A)(V - L_A)}.$$

**Proof:** It is easy to see that if there is a haircut, then the best response of agent A who obtains endowment at  $t=1$  is to repurchase  $s(x)$  at  $t=1$ . If he does not repurchase the security, then he consumes  $w$  units of goods. If he repurchases the security, then he consumes  $w - L_A + V$ . Since  $V > L_A$ , repurchasing is the best response.

If agent A does not default he repurchases  $s(x)$ ; agent B does not have to borrow from agent C and he can consume  $w$  at  $t=2$  (if he stores  $w - L_A$ ) or consumes  $L_A$  (if he has consumed  $w - L_A$ ).

On the other hand, if agent A defaults (has no endowment), then agent B is stuck with  $s(x)$ . He can keep the collateral and consume  $s(x)$  at  $t=2$  (with expected payoff  $(1-l_B)V$ ), or he can use  $s(x)$  as collateral to borrow from agent C. The maximum amount agent B gets is given in Proposition 2. The expected consumption of agent B when he lends at  $t=0$  is the linear combination of the two outcomes where agent A does not default or agent A defaults and agent B chooses either to keep the collateral or do repo with agent C at the terms in Proposition. Thus  $L_A = (1 - \varphi_A)V + \varphi_A \cdot L_B$ .

Agent A compares the following at  $t=0$ . If he does to trade with agent B his expected utility is  $(1-l_A)V$  and if he trades his expected utility is  $L_A + (1-\varphi_A)(1-l_A)(V-L_A)$

$$\text{So } L_A + (1-\varphi_A)(1-l_A)(V-L_A) < (1-l_A)V$$

$$\Leftrightarrow l_A < 1 - \frac{L_A}{V + (1-\varphi_A)(V-L_A)} //$$

Proposition 4 highlights the effect of the repo chain. When agent C expects private information to be valuable, then the amount that agent B is willing to lend to agent A is constrained by the strategy that agent B will adopt with respect to agent C at  $t=1$ . This is the term  $L_B = \max[(1-l_B)V, p_I, E[C_{B2}(p_{II})]$ . Note,  $V - L_B \geq 0$  can be thought of as an illiquidity discount. The higher the IS of  $s(x)$ , the smaller  $L_B$  and the larger is the illiquidity discount. Proposition 4 shows that even if there is potential adverse selection at  $t=1$ , this does not necessarily imply that agent B will demand a haircut that fully reflects the adverse selection problem at  $t=1$ . The haircut decreases in the likelihood that agent A is able to repurchase the collateral  $s(x)$  at  $t=1$ , so agent A's default probability also matters. In addition, note that if the borrower, agent A, is financially able, it is in his best interest to repurchase the collateral because he has sold at below market value initially when  $H > 0$ .

**Corollary 2:** Consider the set of debts with the same expected payoff but different distributions  $F(x)$ . The equilibrium amount of lending is weakly decreasing and the haircut is weakly increasing in the collateral's IAS.

**Proof:** Proposition 2 shows that the amount lent to A at  $t=0$  is  $L_A = V - \varphi_A \cdot (V - L_B)$ , where  $L_B = \max[(1-l_B)V, p_I, E[C_{B2}(p_{II})]$ . Consider two debt contracts A and B, with the same fundamental value. Debt  $D_A$  has a higher IS than debt  $D_B$  if  $f_B(x)$  dominates  $f_A(x)$  in a first order stochastic dominance sense. Loosely speaking,  $f_A(x)$  has more mass on the left tail. For the two debt contracts to have the same fundamental value, the face value of debt A must be strictly higher than the face value of debt B. From simple inspection of the formulae in Proposition 2 and given above, it is easy to see that both  $p_I^{DB} \geq p_I^{DA}$  and  $p_{II}^{DB} \geq p_{II}^{DA}$ . Thus  $L_B^{DB} \geq L_B^{DA}$  and  $L_A^{DB} \geq L_A^{DA}$  which imply the  $H^{DB} \leq H^{DA}$ .  
//

## 5. Repo versus Asset Sales

As the equilibrium analysis revealed above, agent B is indifferent between lending and trading and not lending. Agent B can always consume his endowment  $w$  at  $t=1$ , so his reservation utility is  $EU_B = w$ . In this subsection we compare repo with asset sales, i.e., instead of borrowing, agent A sells  $s(x)$  to agent B. The focus now is on agent A; we ask how agent A can maximize the amount that he can borrow and consume.

**Proposition 5:** Agent A strictly prefers the existence of a repo market (for trade between agent B and C at  $t=1$ ) if  $l_B > 0$  and  $\pi\phi_B \leq \gamma$  or  $\phi_A < 1$ .

**Proof:** The case of asset sales is equivalent to the case where there is no repo market, that is when  $\phi_A = \phi_B = 1$  so both agents A and B do not obtain endowments to repay loans. Propositions 2 and 3 show that agent A can consume strictly more than when  $\phi_A = \phi_B = 1$ . //

In particular, the ability of agent B to engage in repo trading with agent C makes agent A strictly better off. If agent B has a sufficiently high probability of obtaining an endowment at  $t=2$  and thus is able to repay his borrowing from agent C, then there is also no price discount and no haircut at  $t=1$ , irrespective of the default probability of agent A and the IS of  $s(x)$ . Since agent B repurchases  $s(x)$  from agent C at  $t=2$  with a high probability, agent C has no incentive to produce information since he cannot exploit agent B because agent B will very likely reclaim  $s(x)$ .

As discussed above, a key difference between an asset sale and repo is that the borrower has no right to repurchase  $s(x)$  when there is an asset sale. But under a repo agreement the borrower has the right to repurchase  $s(x)$ . Further, even if the lender (agent B) goes bankrupt, the borrower (agent A) can reclaim the full  $s(x)$  without going into bankruptcy court. So, there are no bankruptcy costs.

Proposition 5 shows that agent A benefits from the fact that agent B can do repo with agent C at  $t=1$ . The next proposition gives a condition specifying when agent A also strictly wants to do repo with agent B at  $t=0$ .

**Proposition 6:** Suppose  $l_B > 0$  and  $\pi\phi_B > \gamma$  or  $\phi_A < 1$ . If there is a (vanishingly small) probability that at  $t=1$  agent B gets to make a take-it-or-leave-it offer to agent A in any new transaction, then agent A strictly prefers repo with agent B.

**Proof:** Proposition 3 shows, under the conditions of the proposition, that  $H > 0$  and  $L_A < E[s(x)]$ . With repo, agent B is contractually obligated to return the collateral in exchange for the amount lent to agent A at  $t=1$ . However, with an asset sale there is no obligation for agent B to sell the security back to agent A at the same price at which he bought it initially. In fact, there is a chance that agent B will have the bargaining power and will demand a higher price. //

Proposition 6 would be true in any setting where agent B was not able or willing to sell the asset back to agent A at the initial price (discount). Agent B might go bankrupt, for example, and ownership of the asset would then be under the control of a bankruptcy judge. Or, the outcome of the bankruptcy process is such that agent A receives a lower price than the price at which he originally sold the asset

to agent B. The main point is that with an asset sale there is no contractual obligation under which agent B has to sell the asset back for the original price.<sup>8</sup>

Propositions 5 and 6 show that repo protects the borrower when there is a haircut. In this model, in equilibrium, the lender is indifferent between any lending arrangements because he just breaks-even. But the borrower strictly prefers collateralized borrowing over an asset sale.

## 6. Repo Runs

In this section we show how there can be a run on repo. The run is not a coordination failure. The traditional story of a bank run with demand deposits relies on a common pool problem and sequential service constraint to generate the possibility of a run (Diamond and Dybvig (1983)). But with repo there is no common pool problem because each lender receives his individual collateral. A run on repo cannot be a coordination failure. Here, a run on repo can arise if macroeconomic news causes information insensitive collateral asset to become information sensitive. There is a kind of regime switch, analyzed in DGH2 with debt. As an endogenous response, the lender demands a higher haircut which is equivalent to not rolling over repo (or renewing the securitized loans) in the same amount. To obtain the same amount of money the borrower has to come up with more collateral.

The trigger for the increase in the haircut is the arrival of public news, which increases the IS of the security being used as collateral. We model the arrival of public news about the asset “quality” (i.e. payoff distribution of the underlying assets backing the collateral security) as a signal about payoff distributions. Suppose at  $t = -1$ , agents receive a public (nonverifiable) signal  $z$  about the distribution of  $x$ , resulting in the distribution  $F(x|z)$ . The set of posteriors  $F(x|z)$  are ordered by first order stochastic dominance (FOSD). Signal  $z$  is worse than  $z'$  if  $z$  is dominated by  $z'$  in a FOSD sense.

Briefly, imagine the following repeated version of the game at date 0 and 1. Given the initial information, at  $t=0.0$  agent B lends A the amount  $M_0$ . At date 0.1 agent B repays. At date 1.0 agent B lends  $M_0$  to agent A which he repays at date 1.1, etc. As long as there is no news, agent A is able to roll over the amount  $M_0$ .

Now suppose there is bad news in the FOSD sense prior to trade. This means that the fundamental value  $E[s(x)|z]$  decreases, which reduces the borrowing capacity (expected value) of the collateral. But on top of this negative effect, bad news can cause the IS to rise. We show that the best response of agent B is to lend less than the lower fundamental value of the collateral and thus exacerbate the cash

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<sup>8</sup> In fact, even if there was such a contractual obligation, under U.S. bankruptcy law, the asset would be part of the assets that would enter the bankruptcy process. Only repo transactions are outside this process.

problem of the agent A. Formally, IS is  $\pi = \int_{x_L}^{x_H} \max[L - s(x), 0] f_z(x) dx$  where  $L = E_Z[s(x)]$ . This

formula shows that L depends on  $f_z(x)$  and  $\pi$  depends on L and  $f_z(x)$ . For any monotone function  $s(x)$ , the fundamental value  $E_Z[s(x)]$  is monotone in z. However, the effect on  $\pi$  is ambiguous. See DGH2. Bad news reduces the fundamental value of  $s(x)$  as there is more probability mass in the left side. But, not all bad news necessarily causes a repo run. Bad news may just reduce the fundamental value without causing haircuts to rise.

Now we formalize the notion of a repo run. We assume that  $s(x)$  is debt. Suppose at the beginning of  $t=0$ , there is a public signal  $z$  about  $x$  which induces the posterior distribution of  $f(x|z) \equiv f_z(x)$ . After observing  $z$ , the agents play the game as specified in Section 2. To save on notation, we assume that the endowments of goods  $w$  also depends on  $z$  such that  $w_Z = E_Z[s(x)]$ . This means agent B is able to

lend  $L_A = E_Z[s(x)]$  if he is willing to do that. Define  $\pi(z) = \int_{x_L}^{x_H} \max[L_A - s(x), 0] f_z(x) dx$  where  $L_A = E_Z[s(x)]$ .

**Proposition 7 (Repo Run)**: Consider an economy with  $\{F_Z(x), l_A, l_B, \varphi_A, \varphi_B, w, \gamma\}$ .

- (i) The equilibrium haircut  $H$  agent A charges agent B at  $t=0$  is given as follows: If  $z$  is such that  $\pi(z) \leq \frac{\gamma}{\varphi_B}$  or  $\varphi_A=0$  then  $H=0$ . Otherwise  $H>0$ .
- (ii) The equilibrium amount of lending  $L_A$  is monotonic in  $z$ .

**Proof**: This follows from Propositions 2, 3, and 4 by replacing  $f(x)$  by  $f_z(x)$ . //

The following numerical example highlights the mechanics of a repo run. Suppose agent A owns a debt contract with face value  $D=1$ , i.e.  $\min[x, 1]$  and posts this as collateral. Depending on the signal  $z_i$  ( $i=1, \dots, 60$ ), the distribution of  $x$  is given as follows:  $F_1 \sim u[0, 0.1]$ ,  $F_2 \sim u[0, 0.2]$ , ...,  $F_{30} \sim u[0, 3]$ ,  $F_{31} \sim u[0.1, 3]$ ,  $F_{32} \sim u[0.2, 3]$ , ...,  $F_{60} \sim u[2.9, 3]$ . For each signal  $z$ , we can calculate the new fundamental value and the IS.

We derive equilibrium lending and haircuts for the cases where (i)  $\gamma=0.08$  and (ii)  $\gamma=0.01$ . The table below shows a few numbers for the fundamental value, IS, equilibrium lending and the haircut.

For  $\gamma=0.08$ , the best response of agent B is to choose Strategy I (see Proposition 2) if the collateral becomes information sensitive. For  $z \in [7, 32]$  agent B demands a haircut. For  $z \geq 40$ , debt is safe and

the market value is 1. The equilibrium amount if lending  $L$  in this range is determined by

$$\int_{x_L}^{x_H} \max[L - s(x), 0] f(x) dx = \gamma \text{ which yields } L = x_L + \sqrt{2(x_H - x_L)\gamma}.$$

For  $\gamma=0.01$ , there is no haircut if  $z \geq 38$ . For  $z \in \{1, 2, 3, 37\}$  the best response of agent B is to choose Strategy I. For  $z \in \{4, \dots, 36\}$ , agent B chooses Strategy II. The equilibrium  $L$  that agent B

proposes, and which induces agent C to produce information, maximizes  $E[L] = \left(1 - \frac{L - x_L}{x_H - x_L}\right)L$

which yields  $L = \min[1, \frac{1}{2}x_H]$ . Note, under uniform distribution, the constraint

$$\int_{x_L}^{x_H} \max[s(x) - p_H, 0] f(x) dx \geq \gamma \text{ is always satisfied.}$$

<b>Z</b>	<b>2</b>	<b>10</b>	<b>15</b>	<b>30</b>	<b>40</b>
Support	[0,0.2]	[0,1]	[0,1.5]	[0,3]	[1,3]
$E_z[s(x)]$	0.100	0.500	0.666	0.833	1.000
$\pi(z)$	0.025	0.125	0.148	0.116	0
<b><u>(i) <math>\gamma=0.08</math></u></b>					
L	0.100	0.400	0.489	0.693	1.000
H	0	20%	26.6%	16.8%	0
<b><u>(ii) <math>\gamma=0.01</math></u></b>					
L	0.050	0.250	0.375	0.666	1.000
H	50%	50%	43.7%	20%	0

The equilibrium haircut is non-monotonic in the above example because we are comparing haircuts for collateral with different fundamental values. But if we fix the fundamental value of the collateral (say, one million dollars), then when there is bad news, IS rises. In this sense the haircut weakly rises with the signal  $z$ . There is a monotone relationship between the haircut and news. See Proposition 7.

## 7. Amplification in a Financial Crisis

In this section we discuss how bad news leading to a repo run can be further amplified. We showed in previous sections how the IS of the collateral, liquidity needs of the lender and the counterparty default risks jointly determine haircuts. If the IS of the collateral rises, this increases the haircut. As discussed in the Introduction, an increase in haircuts is tantamount to a withdrawal from the bank, since the bank now has to come up with financing in the amount of the haircut. If no such financing is

forthcoming, and the bank must sell assets, then the resulting fire sales may result in losses, increasing the likelihood of default of the bank.<sup>9</sup>

In the previous sections we have assumed that the default probabilities  $\varphi_A$  is exogenous. Now we assume the default probability is not fixed but endogenous and given by  $\hat{\varphi}_A = g(L_A, \varphi_A)$  where  $\varphi_A$  is the baseline default probability that is the default probability that we assumed before, characterizing agent A. We impose two assumptions on the function  $g$ : (i)  $\hat{\varphi}_A = g(V, \varphi_A) = \varphi_A$  and (ii)  $d\hat{\varphi}_A / dL_A \leq 0$ .

The first assumption states that if agent A consumes the (maximum) amount  $L_A=V$ , then his default probability is the baseline default probability. The second assumption states that the default probability weakly increases in the amount  $L_A$  he can borrow. Literally speaking, the less he consumes the lower the probability that his production output is  $w$ .

**Proposition 8**

At  $t=0$ , agent B lends agent A the amount  $L_A^*$  which solves  $L_A = V - g(L_A, \varphi_A) \cdot (V - L_B)$  where  $L_B = \max[(1-l_B)V, p_I, E[C_{B2}(p_{II})]]$ , as defined in Proposition 2. If  $dg(\cdot)/dL_A|_{L_A=L_A^*} < 0$  then  $\hat{\varphi}_A^* > \varphi_A$ .

**Proof:** The equilibrium amount  $L_A^*$  of lending and the equilibrium default probability  $\hat{\varphi}_A^*$  are the solution to the following simultaneous equation system:

$$(i) \hat{\varphi}_A = g(L_A, \varphi_A)$$

$$(ii) L_A = V - \hat{\varphi}_A (V - L_B)$$

Substitution yields  $L_A = V - g(L_A, \varphi_A) \cdot (V - L_B)$ . The solution to this implicit function is  $L_A^*$ . If at  $L_A=L_A^*$ ,  $d\hat{\varphi}_A / dL_A < 0$  then  $\hat{\varphi}_A^* > \varphi_A$ . **QED**

The following example highlights the amplification mechanism. Suppose  $V=1$ ,  $\hat{\varphi}_A = \max(1 - L_A^3, \varphi_A)$ ,  $\varphi_A = 0.2$ , and  $\pi$  gives rise to  $L_B = 0.45$ . If there is no amplification, then  $\hat{\varphi}_A = \varphi_A$  and equilibrium lending is  $L_A = 1 \cdot 0.8 + 0.2 \cdot 0.45 = 0.89$  and the haircut is 11%. But if default probability depends on lending amount then there can be a reinforcing spiral between lending and default probability. At  $L_A=0.89$ ,  $\hat{\varphi}_A = 0.295$  which yields  $L_A=0.83$ , etc.

The solution is  $L_A^* = 0.5335$  and  $\hat{\varphi}_A^* = 0.8481$ . If agents have rational expectations than agent B demands a haircut of 46.65% immediately. If agents have adaptive expectations then the table below

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<sup>9</sup> Evidence of fire sales is presented and discussed by Manconi, Massa, and Yasuda (2011) and Boyson, Helwege, and Jindra (2010).

describes the adjustment process that exhibits a feature that is similar to Figure 1 which depicts a time series of repo haircuts on structured debt.

$L_A$	$\hat{\varphi}_A$	$H_A$
0.8900	0.2950	0.1100
0.8377	0.4121	0.1623
0.7734	0.5375	0.2266
0.7044	0.6505	0.2956
0.6422	0.7351	0.3578
0.5957	0.7886	0.4043
....		
0.5335	0.8481	0.4665

The amplification mechanism here depends on the link between a haircut increase and the resulting increasing in counterparty default risk, implicitly due to the fire sales. The channel here is distinct from that of Kiyotaki and Moore (1997), in which there is a feedback effect on the value of the underlying collateral. Here that need not be the case. In fact, in the crisis that does not seem to be the case. Manconi, Massa, and Yasuda (2011) show that mutual funds did not sell asset-backed securities, which were used as collateral in repo (see Gorton and Metrick (2010a)), but instead sold corporate bonds. Corporate bonds were not widely used as repo collateral. But, the yield spreads on corporate bonds increased for bondholders who held more asset-backed securities prior to the crisis. The haircuts on corporate bonds barely rose during the crisis (see Gorton and Metrick (2010b)). The default probabilities can rise for reasons other than the value of the collateral going down and still cause an amplification.

## 8. Conclusion

Repo haircuts exist because of sequential transactions, i.e. trading chains. When an agent in the middle of the chain faces liquidity needs or is himself a risky counterparty, then he may not be around to repurchase the collateral in a subsequent repo trade and to that extent there is an incentive for his lender in that subsequent trade to produce private information. Repo haircuts exist to protect a lender who may have to sell the collateral if the borrower fails, facing the possibility of adverse selection. Chains of transaction involving risky counterparties with intermediate liquidity needs are the key to haircuts.

In addition, we show that the existence of a repo market makes a borrower (at the beginning of the chain) better off. Haircuts depend on the IS of the collateral, the default probabilities of the borrower, the liquidity needs of the lender and the default probability of the lender in a subsequent transaction. The haircut reduces the incentive of a potential lender to produce private information because the

borrower has the contractual right to buy back the collateral, and has an incentive to buy back the collateral if he is able. This makes repo an efficient form of transporting value through time

Our theory can provide an explanation for the cross sectional and time series behavior of haircuts data in the repo market. Haircuts differ depending on the identities of the counterparties and on the nature (i.e., the IS) of the collateral.

Finally, a public signal that alters the IS of the collateral can increase haircuts even if it does not change the default probabilities of the counterparties.

## Appendix

Dang, Gorton and Holmström (2012a) analyzes the value of private information acquisition in a trading context and defines information sensitivity (IS) of a security as follows. Consider an agent with utility function  $U=c_0+c_1$  and who can buy a security  $s(x)$  at price  $p$  at  $t=0$  which pays off  $s(x)$  at  $t=1$ . DGH1 show if  $p \leq E[s(x)]$ , then the IS of  $s(x)$  is given by

$$\pi_L(\cdot) = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x) dx.$$

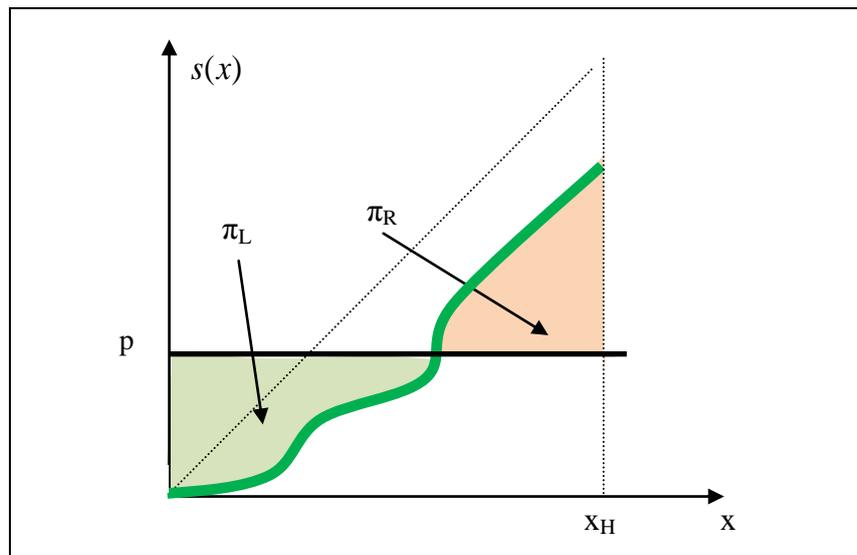
If  $p \geq E[s(x)]$ , then

$$\pi_R(\cdot) = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x) dx.$$

See Figure A for an illustration showing  $\pi_L$  and  $\pi_R$ . IS has a simple economic interpretation. Suppose the agent considers to buy  $s(x)$  and knows that  $x$  has the distribution  $F(x)$ . He is willing to buy the security for any price  $p \leq E[s(x)]$ . Now suppose he learns the true  $x$  before making the trading decision. Then the best response of an informed agent is not to buy if he learns  $x$  where  $s(x) < p$ . IS measures the value of information in the sense that an informed agent avoids buying the security for a price higher than actual payoff. The total expected “loss” a buyer can avoid by having information is  $p-s(x)$  integrated over all states  $x$  where  $s(x) < p$  which yields  $\pi_L$ .

If  $p > E[s(x)]$ , an uninformed agent is not willing to buy  $s(x)$ . But if he knows  $x$  then he buys if  $s(x) > p$ . In this case IS measures the value of information in the sense that an agent avoids the “mistake” of not buying the security for a price smaller than actual payoff. Integrating  $s(x)-p$  over all states where  $s(x) > p$  yields  $\pi_R$ .

**Figure A**



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**Table 1: Hedge Fund Haircuts vs. Bank Haircuts**

<b>Panel A: Asset-Backed Securities: Auto, Credit Cards, Student Loans</b>						
	<b>Hedge Funds</b>					<b>Banks</b>
<b>Date</b>	<b>AAA</b>	<b>AA</b>	<b>A</b>	<b>BBB</b>	<b>BB</b>	<b>A-AAA</b>
<b>Jan 2007</b>	3%	5%	10%	15%	25%	0%
<b>April 2007</b>	3%	10%	15%	20%		0%
<b>July 2007</b>	5%	15%				0%
<b>Oct. 2007</b>	5%	15%				0%
<b>Jan. 2008</b>	10%	20%				0-5%
<b>Feb. 2008</b>	15%					0-5%
<b>Mar. 2008</b>	25%					0-5%
<b>April 2008</b>	25%					0-5%
<b>May 2008</b>	25%					0-5%
<b>June 2008</b>	25%					5-10%
<b>July 2008</b>	25%					5-15%
<b>Aug. 2008</b>	25%					5-15%
<b>Sept. 2008</b>	30%					5-15%
<b>Oct. 2008</b>	30%					10-20%
<b>Nov. 2008</b>	30%					15-20%
<b>Dec. 2008</b>	30%					15-20%
<b>Jan. 2009</b>	30%					15-20%
<b>Feb. 2009</b>	30%					15-20%

<b>Panel B: Corporate Bonds</b>							
	<b>Hedge Funds</b>					<b>Banks</b>	
<b>Date</b>	<b>AAA</b>	<b>AA</b>	<b>A</b>	<b>BBB</b>	<b>BB</b>	<b>BBB+/A</b>	<b>AA-AAA</b>
<b>Jan 2007</b>	2%	5%	10%	12%	20%	0%	0%
<b>April 2007</b>	2%	5%	10%	12%	20%	0%	0%
<b>July 2007</b>	2%	5%	10%	12%	20%	0%	0%
<b>Oct. 2007</b>	10%	15%	25%	30%	40%	0%	0%
<b>Jan. 2008</b>	10%	15%	25%	30%	40%	0%	0%
<b>Feb. 2008</b>	10%	15%	25%	30%	40%	0%	0%
<b>Mar. 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>April 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>May 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>June 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>July 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>Aug. 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>Sept. 2008</b>	10%	20%	30%	35%	45%	0%	0%
<b>Oct. 2008</b>	15%	25%	35%	40%	60%	0-5%	0-5%
<b>Nov. 2008</b>	15%	25%	35%	40%	60%	0-5%	0-5%
<b>Dec. 2008</b>	15%	25%	35%	40%	60%	0-5%	0-5%
<b>Jan. 2009</b>	15%	25%	35%	40%	60%	0-5%	0-5%
<b>Feb. 2009</b>	15%	25%	35%	40%	60%	0-5%	0-5%