The Information Sensitivity of a Security*

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First Version: June 2011

This Version: March 2015

Abstract

In this paper we introduce a new characteristic of a security, its "information sensitivity" (IS). This measure has two components, the first component measures a security's expected monetary loss in low payoff states relative to its price ("tail risks") and the other component measures the expected monetary profit in high payoff states. We apply this measure in different illustrative applications. (i) IS captures the incentive of an agent to produce information about the payoff of the security. (ii) We use IS to solve an optimal security design problem and show that it is optimal for a buyer to purchase debt when he faces a seller who can acquire information and there is never information acquisition in equilibrium. Even if information cost is zero the optimal debt contract makes the seller indifferent between acquiring and not acquiring information. (iii) We use IS to formalize the notion that it is easier to buy than to sell a security. (iv) IS can explain the optimality of securitization. (v) IS is a sufficient statistic for expected utility maximization and a pricing factor if agents have a linear reference point utility function.

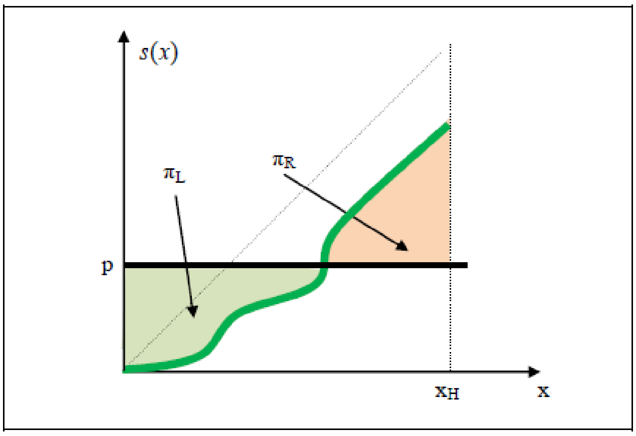
* Parts of this paper were previously contained in an earlier version of Dang, Gorton and Holmström (2010) entitled "Ignorance, Debt and Financial Crisis".

1. Introduction

Stochastic moments like mean and variance are important characteristics of securities. In this paper we introduce another characteristic of a security, its "information sensitivity" (IS). IS has two components, the first component measures a security's expected monetary loss in low payoff states relative to its price ("tail risks") and the other component measures the expected monetary profit in high payoff states. We employ this measure in different applications. In a trading context, we show that IS captures the incentive of an agent to produce private information about the payoff of a security. We analyze optimal security design with endogenous private information production. We use IS to formalize the notion that it is easier to buy than to sell a security. We examine the optimality of securitization using the concept of IS. In a portfolio choice and asset pricing setting, we show that IS is a sufficient statistic for expected utility maximization and a pricing factor if agents have a linear reference point utility function.

Consider an agent who buys a security for price p. The payoff of the security is $s(x)$ and backed by an asset x where x is a random variable with density $f(x)$.[1] If ex post $s(x)<p$, then the buyer incurs a loss of p-$s(x)$. By integrating over all x where $s(x)<p$ we can determine the expected loss of the buyer in low payoff states. We define $\pi_L(\cdot)=\int max[p-s(x),0]f(x)dx$ as the information sensitivity of a security in the loss region. If $s(x)>p$, then the buyer makes a profit of $s(x)$-p. We define the IS in the profit region as $\pi_R(\cdot)=\int max[s(x)-p,0]f(x)dx$ which measures the expected monetary profit of a security in high payoff states. See Figure 1 for an illustration showing π_L and π_R. The two measures are identical if $p=E[s(x)]$.

Figure 1



[1] For example, if $s(x)$ is an AAA-rated mortgage-backed security then $s(x)$ is senior debt backed by the payoff of a pool of underlying mortgages.

Dang, Gorton and Holmström (2012) show that if a risk neutral buyer can produce information about x which backs the security payoff $s(x)$, then information sensitivity measures the value of information. If $p \leq E[s(x)]$, then the value of information for the buyer is given by π_L. In other words, π_L captures the expected loss an informed buyer can avoid by not trading in low payoff states. If $p \geq E[s(x)]$, the value of information is π_R. Without information, an uninformed buyer does not trade. π_R captures the expected profit an informed buyer can make by trading in high payoff states. At $p=E[s(x)]$, $\pi_L=\pi_R$. In the present paper we extend this result and show that the value of information is $V=min\{\pi_L,\pi_H\}$ and the same whether the agent is a buyer or seller of the security and for any prices.

Dang, Gorton and Holmström (2012) also show that debt minimizes the value of information or the IS of a security. In this paper we give a complete characterization of the properties of IS. In addition, we derive the complete set of securities that minimizes π_L and π_R. The only restriction we impose on the set of securities (functions) is limited liability, i.e. $s(x) \leq x$ for all x. Among all securities with the same expected payoff $E[s(x)]$, we show that any security with $s(x)=x$ for all $x<p$ minimizes both π_L and π_R for all prices p and any distribution $F(x)$. We call such a security quasi-debt which is a security with "slope 1" up to the price and then can fluctuate arbitrary but never falls below the price for $x>p$. Debt is a special case of quasi debt. See Figure 2 below for an illustration. This result shows that it is not the "flat part" of debt that makes it least information sensitive but the key is seniority of payments in low payoff states.

We provide several applications of IS to illustrate its usefulness. First, we employ IS to solve an optimal security design problem with endogenous information acquisition. What is the optimal contract $(p,s(x))$ that an uninformed buyer proposes if he faces a seller who can choose to produce costly private information about the payoff x of the project that backs the security?

We show that quasi-debt is an optimal security for an uninformed buyer to purchase. Further, we show that there is never endogenous adverse selection in equilibrium even if the information cost is vanishingly small. To prevent adverse selection when the cost of information acquisition is low, the buyer either reduces the amount of quasi-debt to buy or bribes the seller not to acquire information by paying a price higher than the expected payoff of the quasi-debt. Even if the information cost is zero the buyer purchases a debt contract such that the seller is indifferent between acquiring and not acquiring information. In such a case debt is the uniquely optimal security.

This result is different from the result in Dang, Gorton and Holmström (2012) who show that if an uninformed seller faces a buyer who can acquire information then there is adverse selection in equilibrium if the information cost is low. In such a setting there is positive probability that there is no trade. Combining these two results we formalize the notion that it is easier to buy than to sell a security.

In a second application, we apply the IS concept to securitization and show that the creation of a securitized asset, i.e. debt backed by a pool of projects or loans, minimizes the IS.[2]

Another application of IS concerns its use in optimal portfolio choice under symmetric information. We show that IS is not ranked-correlated with variance and skewness and is thus a new characteristic of a security. We show that IS is a sufficient statistic for expected utility maximization if the agent has a linear reference point utility.[3] Finally, we show that IS is a pricing factor in a representative agent economy where the representative agent has a linear reference point utility function. There is a trade-off between IS and expected return.

In the finance literature the notion of information sensitivity has different meanings in different contexts. In Gorton and Pennacchi (1990) information insensitivity means literally riskless. They show that banks produce such securities for use by uninformed agents to avoid adverse selection. In DeMarzo and Duffie (1999) information sensitivity of a security measures the "slope" of the inverse demand function in a signaling equilibrium of a game where an informed seller faces a set of competitive uninformed buyers. In the market microstructure literature starting with Kyle (1985, 1989) the notion of information sensitivity typically refers to the price impact of the "volume of informed trades".

There is large literature on optimal security design in finance. A class of papers discusses security design in a corporate finance context where a privately informed issuer (seller) faces

[2] Gorton and Pennacchi (1993) show that the creation of a composite security increases the utility of uninformed agents when they face privately informed buyers in secondary markets. In DeMarzo (2005), pooling reduces the adverse selection problem an uninformed agent faces when he sells to an informed intermediary while tranching increases the amount that the informed intermediary (seller) can sell to uninformed buyers subsequently. Both papers assume exogenous adverse selection. In our paper securitization maximizes trade and the utility of uninformed agents without triggering endogenous adverse selection.

[3] This result is similar in spirit to the result that mean and variance are sufficient statistics for expected utility maximization and optimal portfolio choice if the agent has a quadratic utility function.

a set of uninformed investors (buyer). See e.g. DeMarzo and Duffie (1999) who also give a brief survey of that literature. DeMarzo, Kremer and Skrzypacz (2005) analyze optimal security design as a mean of payment in a private value auction context and show that debt is the "worst" security for a seller to accept. Che and Kim (2010) add exogenous adverse selection to their setting and show that debt is the optimal security.[4]

Rather than analyzing security design under exogenous private information, we analyze how security design affects the demand for information production and thus endogenous adverse selection. In our setting, the optimal contract never induces adverse selection even if the information cost is vanishingly small.

Costly state verification models (e.g. Townsend (1979), Gale and Hellwig (1989)) show that debt is the optimal security because it minimizes ex post monitoring costs. There is no ex post verifiability problem in our model but we focus on potential information acquisition ex ante. In particular, the minimization problem of $\pi_R=\int max[s(x)-p,0]f(x)dx$, i.e. the incentive to learn about high states x ex ante, represents a very different economic issue. Regarding, the welfare implications, in costly state verification models the most efficient outcome is achieved if monitoring cost is zero while there is first best in our model if information cost is high.[5]

The mechanism design literature in economics is also large. Most papers assume exogenous private information, i.e. the types of agents are private information of the respective agents.[6] Our design problem is similar in flavor to Cremer and Khalil (1992) who analyze a principal-agent setting where the agent can acquire socially useless information before making a production decision. They show that in equilibrium the principal does not induce the agent to acquire information. In their paper the uninformed principal is uncertain about the total rents from optimal production. The focus of our paper is on a pure exchange setting and the uninformed agent is concerned about an endogenous lemons problem.[7] Further, they analyze a

[4] Innes (1990) analyzes security design, investment decision and moral hazard. Aghion and Bolton (1992) analyze security design and allocation of control rights. Our paper shows how to split an exogenous cash flow when there is endogenous adverse selection concern.

[5] Interestingly, asset backed securities (ABS) seem to be designed to maximize the verifiability of cash flow by having a servicer and trustee in any securitization process. Also, special purpose vehicles are bankruptcy remote.

[6] Bergemann and Valimaki (2002) discuss information acquisition and mechanism design.

[7] The result that there is no induced information acquisition in equilibrium is not driven by the assumption that information has no social value. Dang, Gorton and Holmström (2012) show that if an uninformed seller proposes a contract and faces a security buyer who can acquire information, then the optimal contract may induce the buyer to acquire socially useless information in equilibrium.

model with a discrete type space while in our model the type space is continuous and we put no restriction on the distribution *F(x)* of types and discuss implementation.

The next section defines information sensitivity (IS) and relates it to the value of information in a trading context. Section 3 derives a set of securities with minimal IS. In Section 4 we apply this concept to solve an optimal security design problem with endogenous information acquisition. Section 5 analyzes securitization as a means to reduce IS. In Section 6 we provide a generalized IS measure and its characterization for a broad class of utility functions and standard signal structures. Section 7 applies IS to optimal portfolio choices and asset pricing. Section 8 concludes.

2. Information Sensitivity and the Value of Information

We first define Information Sensitivity (IS) of a security and characterize some useful properties. Consider a security s that is backed by an asset x and pays off $s(x)$ at t=1 and has price p at t=0. x is a random variable with positive support on $[x_L, x_H]$ and distribution $F(x)$. Formally, $s(x)$ is a function that maps each realization of x to a repayment $s(x)$. The only restriction we impose on the function s is that $s(x)$ satisfies the resource feasibility (or limited liability) constraint, i.e. $s(x)\leq x$ for all x. Note, $s(x)$ can be non-monotonic.[8]

<u>Definition (Information Sensitivity (IS))</u>

Consider an arbitrary contract $(p, s(x))$. The information sensitivity (IS) of a security, s(x) at price p in the loss ("left") region (L) and profit ("right") regions (R) are defined as follows:

$$\pi_L(p, s(x)) = \int_{x_L}^{x_H} \max[p - s(x), 0] \cdot f(x) dx$$

$$\pi_R(p, s(x)) = \int_{x_L}^{x_H} \max[s(x) - p, 0] \cdot f(x) dx$$

See Figure 1 for an illustration. Consider a buyer of the security. If $s(x)<p$, he suffers a loss ex post. The expected loss in low payoff states is given by π_L and the expected profit in high payoff states is given by π_R. The following Lemma will prove useful for subsequent results.

[8] Some examples are: (i) Equity: $s(x)=\beta x$ where $\beta\in(0,1]$ is the share on the x; (ii) Debt: $s(x)=min[x, D]$ where D is the face value of the debt.

Lemma 1

(i) If $p=E[s(x)]$ then $\pi_L = \pi_R$.

(ii) $d\pi_L / dp > 0$ and $d\pi_R / dp < 0$.

Proof

Part (i). $E[s(x)]=p$ can equivalently be written as $E[s(x)-p]=0$

$$\Leftrightarrow \qquad \int_{\{x|s(x)<p\}} (s(x)-p)\cdot f(x)dx + \int_{\{x|s(x)\geq p\}} (s(x)-p)\cdot f(x)dx = 0$$

$$\Leftrightarrow \qquad \int_{\{x|s(x)\geq p\}} (s(x)-p)\cdot f(x)dx = \int_{\{x|s(x)<p\}} (p-s(x))\cdot f(x)dx.$$

Part (ii). This follows from simple inspection of the formulae.

A. The Value of Information in Trading

Suppose an agent with utility function $U=c_0+c_1$ can buy or sell a security $s(x)$ at price p at $t=0$. The security pays off $s(x)$ at $t=1$. At $t=0$, the agent can obtain private information before buying or selling the security. If the agent becomes informed, he learns the true realization of x.[9]

Consider an arbitrary contract $(p,s(x))$, i.e. an agent can trade a security $s(x)$ at price p. The value of information for a buyer (B) of $s(x)$ is defined as $EU_B(I)-EU_B(NI)$, where $EU_B(I)$ is the expected utility based on the optimal transaction decision in each state under perfect information about x (I), and $EU_B(NI)$ denotes the expected utility of an optimal transaction decision based on the initial information, i.e. no information about the true state (NI). Analogously, the value of information for a seller (S) of $s(x)$ is $EU_S(I)-EU_S(NI)$.

Proposition 1 (The Value of Information)

Consider an arbitrary contract $(p,s(x))$. The value of information (V) to a potential buyer (B) and a seller (S) is the same and is $V=\min\{\pi_L,\pi_R\}$.

Proof

Seller: Consider an agent who owns $s(x)$. (i) If $p \leq E[s(x)]$, an uninformed agent does not sell and $EU_S(NI, \text{no trade})$=E[s(x)]. If the agent is informed, he sells (does not sell) in state x

[9] Section 6 derives a generalized IS measure for arbitrary utility functions and a class of information structures.

where $p \geq s(x)$ ($p \leq s(x)$) so that $EU_S(I) = \int_{Q<} p \cdot f(x)dx + \int_{Q>} s(x) \cdot f(x)dx$ where $Q_< \equiv \{x \mid s(x) \leq p\}$ and $Q_> \equiv \{x \mid s(x) \geq p\}$. Therefore, $V_S = EU_S(I) - EU_S(NI, no\ trade)$ $= \pi_L$. If $p \geq E[s(x)]$, an uninformed agent sells and $EU_S(NI, trade) = p - E[s(x)]$. So $V_S = EU_S(I) - EU_S(NI, trade) = \pi_R$. Lemma 1 shows that at *p=E[s(x)]*, $\pi_L = \pi_R$ and for $p < E[s(x)]$, $\pi_L < \pi_R$ and for $p > E[s(x)]$, $\pi_L > \pi_R$. Therefore, $V_S = \min\{\pi_L, \pi_R\}$.

Buyer: The proof is analogous and first given in Dang, Gorton and Holmström (2012). **QED**

The intuition is simple. For *p<E[s(x)]*, an uninformed agent is not willing to sell. If he is informed, he sells in states *x* where *s(x)<p*. The value of information is that he avoids the "mistake" of not selling the security in low payoff states. Integrating over all *x* where *p-s(x)>0* yields π_L. For *p≥E[s(x)]*, an uninformed agent sells *s(x)*. Information changes his trading decision and he does not sell in states *x* where *s(x)>p*. The value of information is that he avoids selling the security for too little in high payoff states. Integrating over all *x* where *s(x)-p>0* yields π_R. For *p<E[s(x)]*, $\pi_L < \pi_H$ while for *p>E[s(x)]*, $\pi_L > \pi_H$. So the value of information is $V = \min\{\pi_L, \pi_R\}$.

Lemma 1 does not assume that *s(x)* is monotonic in *x*. *s(x)* can intersects with *p* arbitrarily often. Whenever *s(x)<p*, then *p-s(xp* "contributes" to π_L.

Corollary 1.1

Define $s_{\min} \equiv \min s(x)$ and $s_{\max} \equiv \max s(x)$. For all *s(x)* and any *F(x)* the following hold:

(i) $V = 0$ if $p \leq s_{\min}$ or $p \geq s_{\max}$.

(ii) V has its maximum at *p=E[s(x)]*.

Proof

Part (i) follows directly from inspection of the formulae in Lemma 1.

Part (ii). For $p \leq E[s(x)]$, $d\pi_L / dp > 0$; and for $p \geq E[s(x)]$, $d\pi_R / dp < 0$.**QED**

The intuition for Corollary 1.1 (i) as follows. At $p \leq s_{\min}$, an (uninformed) buyer always buys. A seller can never gain by selling the security. Therefore information has no value to them.

Analogously, at $p \geq s_{\max}$, an (uninformed) seller always sells and a buyer can never gain by buying the security.

3. The Class of Securities with Minimal Information Sensitivity

Proposition 1 shows that the value of information is $V = \min\{\pi_L, \pi_R\}$ what we call the information sensitivity (IS) of a security. In this section we ask which security minimizes the value of information or IS. We impose no restriction on the set of securities except resource constraint (limited liability), i.e. the security cannot repay more than the underlying *x*. We will compare all securities in this set with the same expected payoff *E[s(x)]=W*.

<u>Set of securities</u>

The set of securities is $S = \{(s(x) : s(x) \leq x\}$.

<u>Definition (Quasi Debt)</u>

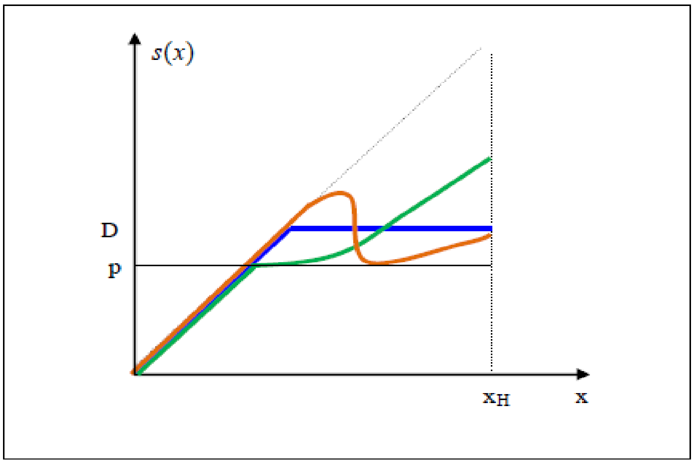
Define $S^{QD} = \{s(x) : s(x) = x \text{ if } x \leq p \text{ and } s(x) \geq p \text{ if } x > p\}$. A security $s(x) \in S^{QD}$ is called a quasi debt contract.

<u>Definition (Standard Debt)</u>

$s^D(x) = \min[x, D]$ is standard debt with face value D.

Standard debt (or just debt) is a special case of a quasi debt contract. Figure 2 illustrates three quasi debt securities with the same expected payoff *W* and where the "blue" security is standard debt.

Figure 2



Now we derive securities with the minimal IS. We provide a very simple characterization of the full set of solutions to the following two minimization problems in the functional space $S=\{s(x): s(x) \le x \text{ and } E[s(x)]=W\}$:

$$\min_{s(x)} \ \pi_L = \int_{x_L}^{x_H} \max[p - s(x),0] \cdot f(x)dx$$

and

$$\min_{s(x)} \ \pi_R = \int_{x_L}^{x_H} \max[s(x) - p,0] \cdot f(x)dx.$$

Lemma 2

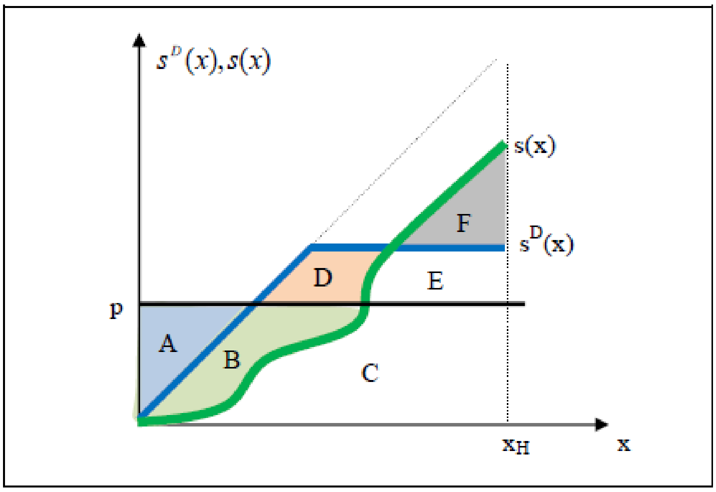
For any *f(x)* and price $p \le W$, the only securities that minimize $\pi_L(\cdot)$ are quasi debts contracts, $s^{QD}(x)$.

Proof

$IS^{QD} = \int_{x_L}^{x_H} \max[p - x,0] \cdot f(x)dx < \int_{x_L}^{x_H} \max[p - s(x),0] \cdot f(x)dx = IS^S$ since *s(x)<x* for some *x<p*. **QED**

This result is intuitive. In Figure 3, $IS^{QD} = A$ while $IS^S = A + B$. For $p \ge V$, the IS of quasi-debt is $\pi_R^{QD} = D + E$ while the IS of non-quasi debt is $\pi_R^S = E + F$. The next Lemma shows that the probability weighted area D is strictly smaller than the probability weighted area F in Figure 3. Thus any non-quasi-debt has a higher π_R than quasi debt. Surprisingly, at least to us, the solution to the minimization of π_R is also independent of *f(x)*.

Figure 3



Lemma 3

Define D as the solution to $\int_{x_L}^{x_H} \min[x,D]\cdot f(x)dx = W$.

(i) For any *f(x)* and price $p \in [W,D)$, the only securities that minimize $\pi_R(\cdot)$ are quasi debt contracts.

(ii) At *p=D,* (standard) debt is the unique minimizer of $\pi_R(\cdot)$.

(iii) If *p>D,* any security with $s_{\max} \le p$ has IS=0.

Proof

Case (i): For $p \in [W,D)$, IS=π_R where $\pi_R^{QD} = D+E$ and $\pi_R^S = E+F$. We compare all *s(x)* with *E[s(x)]=W.* For quasi-debt, the probability weighted integral (areas) is *B+C+D+E=W* while for *s(x),* it is *C+E+F=W.* This implies *F=D+B.* Since *B>0,* $\pi_R^{QD} < \pi_R^S$. Any *s(x)* with minimal π_R must have *s(x)=x* for $x \le p$ and $s(x) \ge p$ for $x > p$.

Case (ii): The maximum payoff of standard debt is *D* and smaller than the maximum payoff of any *s(x)* with the same expected payoff. Therefore, if *p=D*, then $\pi_R^D(D) = 0 < \pi_R^S(D)$.

Case (iii): Follows from Corollary 1.1 (i). **QED**

Intuitively speaking, a non-quasi debt security does not minimize π_R because one can move information value to the area below the price *p* and above *s(x).* In Figure 3 one can move information value from the area *F* to area *B* until *B* vanishes. Lemmas 2 and 3 imply the following result.

Proposition 2

Quasi-debt minimizes the value of information and is least information sensitive for all prices *p* and any *f(x).*

Proposition 2 shows that the set of solutions to both minimization problems $\pi_L(\cdot)$ and $\pi_R(\cdot)$ are quasi debt contracts and distribution free. In other words, seniority of payoffs is key. As the price increases the set of quasi debts “shrinks”. At *p=D,* debt is the unique solution, i.e. the only security with minimal IS.

4. Information Sensitivity and Optimal Security Design

Now we apply the IS measure to solve a new optimal security design problem: What is the optimal security for an uninformed buyer to transact with if he faces a seller who can acquire costly information about the payoff of the underlying asset? The literature on optimal security design and implementation theory in economics is large but to our knowledge, there is no answer to this question. Dang, Gorton and Holmström (2012) discuss optimal security design for a seller who faces a buyer who can acquire information. The equilibrium outcome in terms of information acquisition in these two settings will be very different.

A. The Setting

We consider two risk neutral agents {A,B} with utility functions

$$U_A = c_{A0} + \quad c_{A1}$$

$$U_B = c_{B0} + \alpha c_{B1}$$

where $\alpha > 1$ is a constant. Agent A owns a project X that pays off x units of goods at $t=1$. Agent B owns w units of perishable goods at t=0. w is a constant and X is a random variable with positive support on $[x_L, x_H]$ and distribution $F(x)$. We assume that the realization of x is verifiable at $t=1$. At $t=0$, agent A can acquire private information about the final payoff of X at the cost γ (in terms of utility). If agent A becomes informed, he learns the true realization x of X.

Given the preferences and endowments, there are gains from trade. It is socially efficient for agent B to consume at $t=1$ by exchanging his w units of $t=0$ goods for some of agent's A goods at $t=1$. In order to trade, agents need to write a contract which specifies what agent A should deliver at $t=1$.

Again, let $S = \{(s(x) : s(x) \leq x\}$ denote the set of all possible securities. In principle, agent A could promise whatever he wants, e.g. $s(x) > x_H$, but agent B would simply not believe it. The optimal design problem is to design a contract $(p, s(x))$ that maximizes agent B's expected utility subject to agent A obtaining at least his reservation utility $E[x]$ and subject to information acquisition constraint. This is equivalent to the following game.

1. Agent B makes a take-it-or-leave-it offer of $(p, s(x))$ to agent A.
2. Agent A chooses whether to produce private information.

3. Agent A accepts *(p,s(x))* or not.

If agent A accepts *(p,s(x)),* agent A consumes *p* units of good and agent B consumes the remaining *w-p* units of goods at *t=0*. At *t=1*, agent A consumes *x-s(x)* and agent B consumes *s(x).* If there is no trade, agents consume their endowments.

A. Intermediate Results

Consider *(p,s(x))* with *s(x)=x* and *p=E[x]*. If agent B buys the whole project *X*, then the IS of agent A is $IS^X=\int max[E[x]-x,0]f(x)dx$. Corollary 1.1 (iii) shows that IS^X is the maximal IS. Define

$$\bar{\gamma} \equiv \int_{x_L}^{x_H} \max[E[x]-x,0] \cdot f(x)dx .$$

Lemma 4

Suppose $\gamma \geq \bar{\gamma}$, any security *s(x)* with *E[s(x)]=min[E[x],w]* and *p=E[s(x)]* maximizes agent B's expected utility.

Proof

If agent B proposes to buy the whole project *X* for the price *p=E[x]*, then a best response of agent A is to sell without acquiring information. If $w \geq E[s(x)]$, buying the whole *X* for a price *p=E[s(x)]* clearly maximizes agent B' utility subject to agent A obtaining his reservation utility *E[x]*. If $w < E[s(x)]$, then agent B buys what he can afford, namely a security *s(x)* with *p=E[s(x)]=w*. Any of these securities has $IS < \bar{\gamma}$. **QED**

So debt is an optimal security for agent B to buy. Note, *s(x)=x* can be interpreted as a (degenerated) debt contract with *s(x)=min[x,D]* where $D=x_H$.

Lemma 5

Suppose $\gamma < \bar{\gamma}$ and *p=E[s(x)]*. The maximal amount *E[s(x)]* that agent B can buy without triggering information acquisition and without giving agent A any rents is a quasi debt contract with $p=E[s^{QD}(x)]=min[w,v]$ where *v* solves

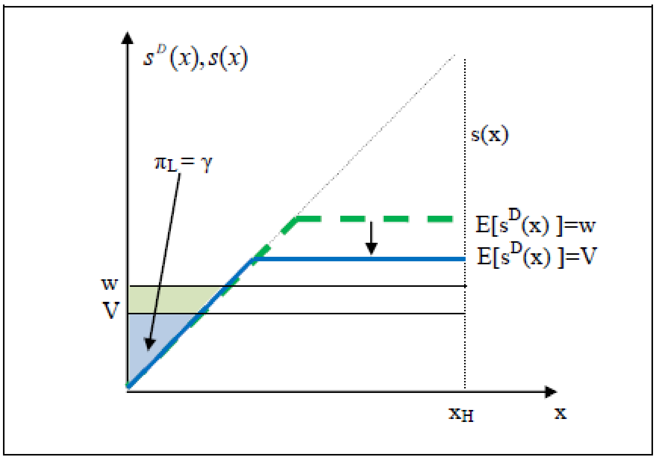
$$\int_{x_L}^{x_H} \max[v - s^{QD}(x),0] \cdot f(x)dx = \gamma .$$

Proof

Agent A gets no rents, if $p=E[s(x)]$. For $p=E[s(x)]$, $\pi_L=\pi_R$. We focus on π_L. Since $\gamma < \bar{\gamma}$, v as a solution to the above equation implies $v<E[x]$. Buying quasi debt with $p=E[s^D(x)]=v$ does not trigger information. Lemma 1 implies that any non quasi debt security with $E[s(x)]=v$ and price $p=v$ has $\pi_L^S(v) > \pi_L^{QD}(v) = \gamma$. Note, if agent B owns w<v, then the maximum he can pay is w which does not trigger information either. **QED**

Figure 4 illustrates the maximal amount agent B can buy without triggering information acquisition and where the contract does not give any rents to agent A. In other words, agent B reduces the amount of debt to purchase to avoid information acquisition. Note, if (quasi-)debt triggers information at p, then so do all other securities with the same expected payoff.

Figure 4



Lemma 6

Consider an arbitrary price p. A quasi debt contract with $E[s^{QD}(x)] \le p$ that just does not trigger information production by agent A maximizes agent B's expected utility at that price.

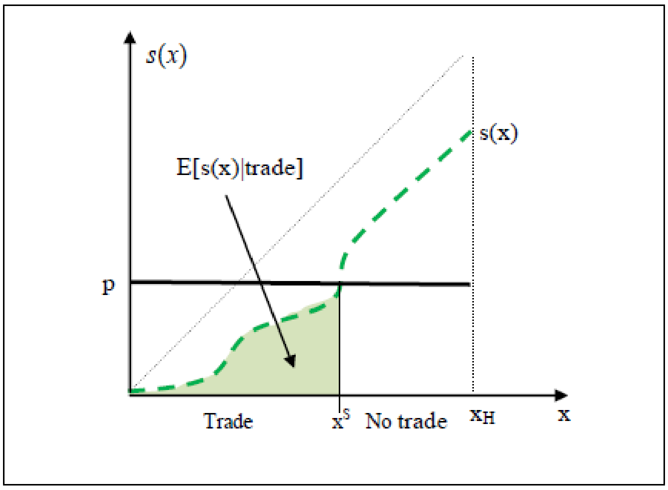
Proof

We divide the proof in four steps.

Step 1: Consider an arbitrary contract *(p, s(x))* that triggers information acquisition. Conditional on trade, the expected value of *s(x)* is given by $E[s(x)|trade] = \int_{\{x:s(x)\le p\}} s(x) f(x) dx$.

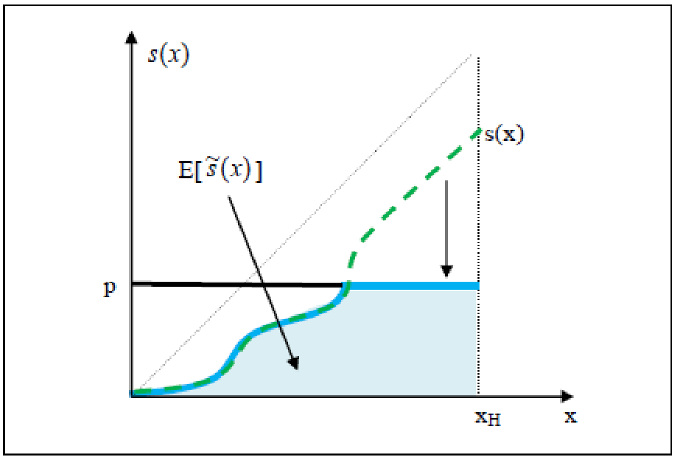
See Figure 5.

Figure 5



Step 2: We now construct another security that sells for the same price, and does not induce information acquisition, but strictly dominates the arbitrary contract considered above. Consider the contract (p, $\tilde{s}(x)$) with $\tilde{s}(x)=p$ for $x>x^S$ and $\tilde{s}(x)$=s(x) for $x<x^S$. See Figure 6, below. Under this contract, agent A sells without information acquisition since $p \le \tilde{s}(x)$ for all x. The expected value of $\tilde{s}(x)$ is $E[\tilde{s}(x)] = \int_{x_L}^{x_H} \tilde{s}(x) \cdot f(x)dx > E[s(x) \mid trade]$.

Figure 6

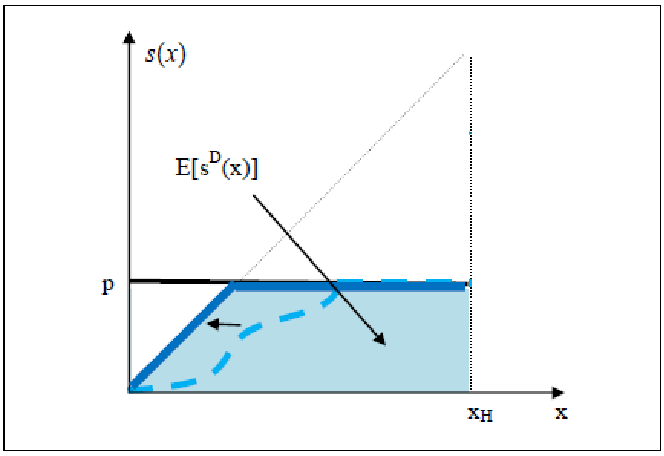


Step 3: We show that agent B can do even better by buying debt with $D=p$. (Note, $\tilde{s}(x)$ is not debt since $s(x)<x$ for some $x<p$.) Trade always occurs and the expected payoff of debt is larger than $E[\tilde{s}(x)]$. See Figure 7. Formally,

$$E[s^D(x) \mid trade] = \int_{x_L}^{x_H} \min[x, p] \cdot f(x)dx > E[\tilde{s}(x) \mid trade] = \int_{\{x: x \le x^S\}} \tilde{s}(x) \cdot f(x)dx$$

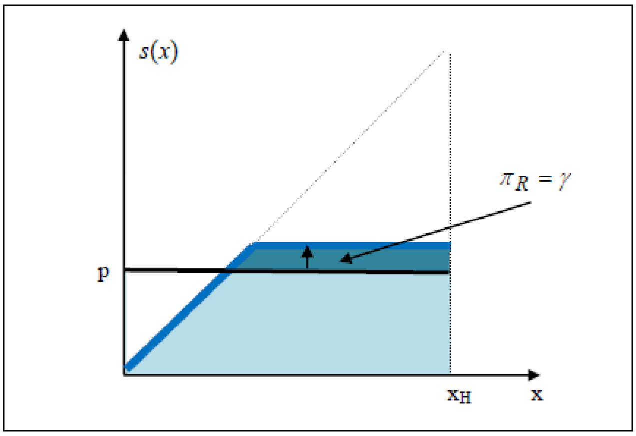
since $s^D(x) \ge \tilde{s}(x)$ and for some x the inequality is strict.

Figure 7



Step 4: We show that for any given price *p*, the optimal security is (quasi-) debt such that agent A is just indifferent between information acquisition and no information acquisition. See Figure 8.

Figure 8



At an arbitrary price *p*, a (quasi-) debt contract with either $E[s^{QD}(x)] = p$ and $\pi^R \leq \gamma$ or $E[s^{QD}(x)] < p$ and $\pi^R = \gamma$ maximizes the expected payoff of the buyer. **QED**

B. The Optimal Security

Now we are in a position to derive the optimal contract *(p,s(x))* that maximizes the expected utility of agent B, an uninformed buyer. We first define two strategies.

Strategy I: (Reduced trade)

Buy quasi-debt $s^I(x)$ with expected payoff $E[s^I(x)] = p_I$ where the price p_I solves

$$\int_{x_L}^{x_H} \max[p_I - s^I(x),0]\cdot f(x)dx = \gamma .$$

Note, in the case of a standard debt, the associated face value D_I is determined by

$$\int_{x_L}^{x_H} \min[x, D_I]\cdot f(x)dx = p_I$$

The expected utility of Strategy I is $EU_B(I) = w + \alpha E[s^I(x)] - p_I = w + \alpha p_I - p_I$.

Strategy II: (Bribe)

Buy quasi-debt $s^{II}(x)$ and price $p_{II} > E[s^{II}(x)]$ which maximize

$$EU_B = w + \alpha \int_{x_L}^{x_H} s^{II}(x) f(x)dx - p_{II}$$

s.t. $$\int_{x_L}^{x_H} \max\left[s^{II}(x) - p_{II}, 0\right] f(x)dx = \gamma .$$ [10]

Note, $p_{II} > E[s^{II}(x)] > E[s^I(x)] = p^I$.

Proposition 3

Agent B proposes the following contract *(p,s(x))* :

(i) If $\gamma \geq \bar{\gamma}$, any *s(x)* with *p=E[s(x)]=min[w,E[x]]* is optimal.

(ii) If $0 < \gamma < \bar{\gamma}$, depending on {α, γ, *F*, *w*} agent B buys (quasi-)debt according to either Strategy I or Strategy II.

Agent A does not acquire information in any equilibrium.

Proof

Part (i) follows from Lemma 4.

Part (ii): Information acquisition is a binary decision. So agent B has two types of potential best responses. Either he avoids information acquisition by agent A or he induces agent A to acquire information. Lemma 5 shows that inducing agent A to acquire information is a strictly dominated strategy. Lemma 4 shows that if agent B wants to avoid information acquisition and pay *p=E[s(x)]*, i.e. he does not give any trading surplus to agent A, then the maximal amount he can trade is to buy a quasi-debt contract and the price p_I is given in Lemma 5. Denote this amount as $E[s^I(x)]$. For γ small this amount is small. In order to trade more agent

[10] Note, for *p>E[s(x)]*, the IS is π_R.

B can "bribe" agent A not to acquire information by paying a price $p_{II}>E[s^{II}(x)]>E[s^{I}(x)]=p^{I}$. Strategy II characterizes the optimal contract $(p_{II}, s^{II}(x))$ with a bribe. Agent B chooses the strategy with the higher expected utility. **QED**

Proposition 3 shows that the optimal strategy for agent B is to avoid adverse selection. This can be achieved in two ways, either by (i) reducing the amount of trade or (ii) bribing. In the first case by proposing a security with $p=E[s(x)]$, agent B does not give agent A any rents. At $p=E[s(x)]$, $\pi_L=\pi_R$. Quasi-debt is optimal because it minimizes π_L for any $p=E[s(x)]$ so agent B can maximizes $E[s(x)]$ without triggering information production. If the optimal strategy is to avoid adverse selection by bribing, then quasi-debt is optimal because it has the lowest π_R for any $p>E[s(x)]$, i.e. for any p, the bribe is cheapest. If α is large and γ small, agent B chooses Strategy II.

Corollary 3.1

Suppose γ=0. Denote p_0 as the maximizer of $\alpha\int_{x_L}^{x_H}\max[x,p]f(x)dx-p$.

(i) The optimal contract is unique. Agent B buys debt with price p_0 and $D=p_0$.

(ii) For α sufficiently large, agent B buys debt with $D=p=w$.

The seller is indifferent between acquiring and not acquiring information.

Proof

For γ=0, the constraint in Strategy II yields the optimal face value $D=p$.

(i) Agent B maximizes

$$EU_{=}\alpha\int_{x_L}^{x_H}\max[x,p]f(x)dx-p=\alpha\int_{x_L}^{p}xf(x)dx+\alpha(1-F(p))p-p=\alpha\int_{x_L}^{p}xf(x)dx+(\alpha-1)p-\alpha F(p)p.$$

There is always a positive amount of trade since the maximum value of EU_B is always positive because for p sufficiently small, $\alpha-1>F(p)$. The associated face value D solves $\int_{x_L}^{x_H}\max[x,D]f(x)dx=p_0$.

(ii) $EU_B=\alpha\int_{x_L}^{p}xf(x)dx+(\alpha-1)p-\alpha F(p)p$. Note, the first term is strictly increasing in p and the second and third term is $(\alpha-1)p-\alpha F(p)p=p(\alpha-1-\alpha F(p))$ $=p(\alpha(1-F(p)-1)$. This is positive if $\alpha\geq\frac{1}{1-F(p)}$. Thus he pays w. **QED**

This corollary shows that even if information costs is zero, the buyer purchases a debt contract such that the seller is indifferent between acquiring and not acquiring information. Furthermore, standard debt is the uniquely optimal security. If the private information of the seller is exogenous and perfect, Corollary 3.1 contains a result in Bias and Mariotti (2005) as a special case of γ=0 where the indifferent seller chooses to acquire costless information.

C. A Formalization of "It is easier to buy than to sell securities"

Proposition 3, which states that there is never information acquisition in equilibrium, is very different from the result in Dang, Gorton and Holmström (2012) who show that if the seller makes an offer and the buyer can acquire information then there is information acquisition in equilibrium when the information cost is small and the gains from trade are large. The economic intuition is interesting. If the seller is the proposer the only way to prevent information acquisition by the buyer is to ask for a price *p* such that $\pi_L(p) = \gamma$. Independent of the expected payoff *E[s(x)]* of the security or how much (more) the buyer can get in the high states, the buyer's best response is to acquire information if the loss he can avoid in low payoff states is smaller than the cost of information. The seller has no way to bribe the buyer other than asking for a low price. In some sense the limited liability of the seller is binding. If the gains from trade are large, then asking for a low price is dominated by triggering information acquisition and the probability to trade at a higher price. However, there is a positive probability that there is no trade.

In contrast, if the buyer makes the offer and the seller can acquire information, the buyer can prevent information acquisition at any price $p \leq E[s(x)]$, when he does not ask to get repaid too much. Note, the seller acquires information if $\pi_H(p) > \gamma$. By reducing the payoff of the security *s(x)* in high states he can make $\pi_H(p) = \gamma$. In the extreme case where *s(x)=min[x,p]*, one best response of the seller is not to acquire information even if $\gamma = 0$. In other words, the buyer can bribe the seller not to acquire information at any price *p* by reducing *E[s(x)]*. Besides being able to prevent information acquisition by the seller, Proposition 3 shows that avoiding information acquisition is a strictly dominant strategy for the buyer.

If we fix the expected payoff of an asset *E[x]* and compare the seller's ability to sell a security *s(x)* and the buyer' ability to buy *s(x)*, then we can show that it is easier to buy than to sell a security in the following formal sense. If the seller wants to sell for sure he can only ask for a price p such that $\pi_L(p)=\gamma$. If γ is small than p is small. The price the seller obtains converges to zero as γ goes to zero. In contrast, if the buyer wants to buy, there is never information acquisition in equilibrium and trade always occurs with probability one. Even if $\gamma=0$, the buyer can purchase an security *s(x)=min[x,p]* with probability one and has an expected payoff $E[s(x)]$ which is strictly bounded away from zero.

5. Information Sensitivity and Securitization

In this section we turn to a second application; we apply the IS measure to understand securitization. We show that the creation of a securitized asset, i.e. debt backed by a pool of projects, minimizes IS.

Consider a set $\{X_1,..,X_N\}$ of N projects with the joint distribution $F(x_1,,..,x_N)$ and marginal distributions $F_i(x_i)$. Suppose the seller owns N projects. He can sell (i) N arbitrary securities, (ii) N debt separate debt securities, (iii) or a securitized debt. Similarly to Section 4, we ask which of the three financial arrangements minimizes the seller's incentive to acquire information or equivalently which minimizes the expected loss of the buyer in low payoff states even when there is no endogenous lemons problem. In other words, what security does an uninformed agent want to buy?

Case 1: Suppose security $s_i(x_i)$ is backed by project x_i and all securities have the same expected payoff and the same price $p=E[s_i(x_i)]$. If agents pick and trade individual securities, then the total IS^{T0} is given by:

$$IS^{T0}=\sum_{i=1}^{N}\pi_i=\sum_{i=1}^{N}\int_{x_{iL}}^{x_{iH}}\max[p-s_i(x_i),0]\cdot f_i(x_i)dx_i\ .$$

Case 2: Seller can pick and sell N separate debt securities, i.e. $s_i(x_i)$ are debts. IS^{T1} is given by:

$$IS^{T1}=\sum_{i=1}^{N}\pi_i^D=\sum_{i=1}^{N}\int_{x_{iL}}^{x_{iH}}\max[p-\min[x_i,D_i],\ 0]\cdot f_i(x_i)dx_i$$

$$IS^{T1}=E\left[\sum_{i=1}^{N}\left[\max[p-\min[x_i,D_i],\ 0]\right]\right]$$

It is obvious that $IS^{T1}<IS^{T0}$ since debt minimizes $\pi_i^D\le\pi_i$ for all i. The inequality is strict if security i is not (quasi-)debt.

Case 3: Seller can sell a securitized debt with *E[s(x)]=Np*, that is backed by the pool of *N* projects, for the price *Np*. The expected loss of this securitized debt portfolio or the total IS^{T} is given by:

$$\mathrm{IS}^{\mathrm{T2}} = \int \max\left[Np - \min[\sum_{i=1}^{N} x_i, D],\ 0 \right] \cdot f(x_1,..,x_N) d(x_1,..,x_N)\,.$$

Proposition 4 (Optimality of Securitization)

Securitized debt reduces IS further.

Proof

We will show that $IS^{T2} < IS^{T1}$.

Step 1: Consider a portfolio of *N* debts each backed by one project. Agents cannot pick individual securities. Either the portfolio is sold or there is no trade. The IS of this portfolio is

$$\overline{IS} = \int \max\left[Np - \sum_{i=1}^{N} \min[x_i, D_i],\ 0 \right] \cdot f(x_1,..,x_N) d(x_1,..,x_N)$$

$$\overline{IS} = E\left[\max\left[Np - \sum_{i=1}^{N} \min[x_i, D_i],\ 0 \right] \right]$$

Step 2: The following arguments show that $\mathrm{IS}^{\mathrm{T1}} > \overline{IS}$.

Define $$\kappa^{\mathrm{T1}}(x_1,..., x_N) = \sum_{i=1}^{N} \left[\max[p - \min[x_i, D_i],\ 0] \right]$$

$$\overline{\kappa}(x_1,.., x_N) = \max\left[Np - \sum_{i=1}^{N} \min[x_i, D_i],\ 0 \right]$$

$\kappa^{T1} = 0$ if and only if all $x_i \geq p$. If all $x_i \geq p$, then $\overline{\kappa} = 0$. In contrast, $\overline{\kappa} = 0$ if $\frac{1}{N}\sum_{i=0}^{N} x_i \geq p$. It is obvious that there are more states $(x_i,..,x_N)$, such that $\overline{\kappa} = 0$ than the case where $\kappa^{T1} = 0$. Also for each $(x_i,..,x_N)$, $\kappa^{T1} > \overline{\kappa}$. Therefore, $\overline{IS} < IS^{T1}$ since there is a continuous set of states where $\overline{\kappa} = 0$ but $\kappa^{T1} > 0$.

Step 3: The following arguments show that $I\bar{S} > IS^{T2}$. For a realization of $(x_1,..,x_N)$, $\max\left[Np - \min[\sum_{i=1}^{N} x_i, D],\ 0\right] > 0$ if $\Sigma x_i < Np$ while $\max\left[Np - \sum_{i=1}^{N}\min[x_i, D_i],\ 0\right] > 0$ if $\Sigma\min[x_i, D_i] < Np$. It is easy to see that $\Sigma\min[x_i, D_i] \le \Sigma x_i$. So $IS^{T2} < \overline{IS} < IS^{T1}$. **QED**

This proposition shows that securitization, i.e. the pooling of cash flows and then selling a senior debt backed by that pool of cash flows, minimizes the IS or endogenous adverse selection concerns. Note, if diversification is a prime concern, then agents trade equity shares backed by the pool of projects instead of senior debt.

6. Information Sensitivity, Risk Aversion and Imperfect Information

In this section we discuss some extensions. Subsection A provides a generalized IS formula for arbitrary utility functions and shows that quasi debt minimizes IS. Section B analyzes different information structures and gives conditions for (quasi-) debt to be least IS when agents are risk avers and the information acquired is not perfect. Section C discusses some further assumptions.

A. General Utility Functions

Suppose an agent with utility function $U(c_0,c_1)$ has an endowment of w_0 units of goods at date 0 and w_1 units of goods at date 1.[11] Define p' as the agent's maximum-willingness-to-pay for the asset, i.e. $EU(w_0 - p', w_1 + s(x)) = U(w_0, w_1)$. If $p \le p'$, an uninformed agent buys $s(x)$. Otherwise he does not buy $s(x)$. If the agent obtains perfect information, he buys $s(x)$ in state x if $U(s(x)) \equiv U(w_0 - p, w_1 + s(x)) \ge U(w_0, w_1) \equiv \underline{U}$. For a given p, $Q_{>}=\{x:\ U(s(x)) \ge \underline{U}\}$ denotes the set of states where an informed agent buys $s(x)$. At states $Q_{<}=\{x:\ U(s(x)) < \underline{U}\}$ an informed agent does not buy the security. From $EU(I)$-$EU(NI)$, the value of information is given as follows:

$$\begin{aligned}\pi_L &= \int_{Q<} \left(U(w_0, w_1) - U(w_0 - p, w_1 + s(x))\right) \cdot f(x)dx \\ &= \int_{z_L}^{x_H} \max[U(w_0, w_1) - U(w_0 - p, w_1 + s(x)), 0] \cdot f(x)dx\end{aligned} \qquad \text{if } p \le p'$$

[11] If w_1 is stochastic, we need the joint distribution of x and w_1. The reservation utility of the agent is $EU(w_0,w_1)$. To save on notation we assume w_1 is a constant.

and

$$\pi_R = \int_{Q>} \left(U(w_0 - p, w_1 + s(x)) - U(w_0, w_1)\right) \cdot f(x)dx \qquad \text{if } p \geq p'.$$
$$= \int_{z_L}^{x_H} \max[U(w_0 - p, w_1 + s(x)) - U(w_0, w_1), 0] \cdot f(x)dx$$

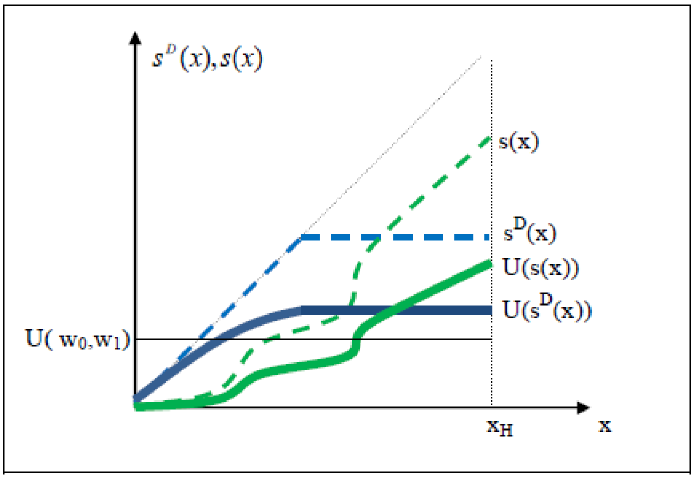
Proposition 5

Suppose the signal is perfect and the agent has $U(c_0,c_1)$. Any security that minimizes IS has $s(x)=x$ for all x where $U(\cdot, s(x)) < U(w_0,w_1)$.

Proof

For any strictly increasing utility function and $p<p'$, quasi-debt minimizes π_L since $x>s(x)$ for all $x<p'$ implies $U(\cdot, x) > U(\cdot, s(x))$. For $p>p'$, the slope of $U(\cdot, x)$ is weakly larger than the slope of $U(\cdot, s(x))$. Any $s(x)$ with slope of $U(\cdot, s(x))$ strictly smaller than the slope of $U(\cdot, x)$ has higher π_R because one can reduce information value by moving information value below the "price line" $U(w_0,w_1)$. See Figure 9. **QED**

Figure 9



B. Imperfect Information

Suppose the agent has an endowment of w_0 units of goods *at t=0* and w_1 units of goods at $t=1$, and utility function $U(c_0,c_1)$. Further, suppose the agent obtains a signal z that induces $F(x|z)$ and where z has density $g(z)$. Given a signal structure $\{F(x|z)\}$, the value of information is given as follows:

$$\pi_L = \int_Z \max\left[U(w_0, w_1) - E[U(w_0 - p, w_1 + s(x)) \mid z], 0\right] \cdot f(x \mid z)dz \qquad \text{if } p \leq p'$$

and

$$\pi_R = \int_Z \max\left[E[U(w_0 - p, w_1 + s(x)) \mid z] - U(w_0, w_1), 0\right] \cdot f(x \mid z) dz \qquad \text{if } p \geq p',$$

where

$$E[U(w_0 - p, w_1 + s(x)) \mid z] = \int_{x_L}^{x_H} U(w_0 - p, w_1 + s(x)) \cdot f(x \mid z) dx.$$

We consider three types of (common) signal structures.

<u>Proposition 6</u>

Suppose the agent has $U(c_0,c_1)$ and obtains a noisy signal $z = x + \varepsilon$ where ε is an independent random variable with $E[\varepsilon] = 0$, $Var(\varepsilon) = \sigma_\varepsilon^2$ and ε and x are stochastically independent. Any security that minimizes IS has $s(x)=x$ for all x where $EU(s(x), w_0, w_1) < U(w_0,w_1)$.

<u>Proof</u>

Denote p' as the agent's maximum willingness to pay when he is uninformed. Suppose $p=p'$. Upon observing z, the expected payoff of the security is $EU[s(x), w_0, w_1 \mid z]$. The buyer does not buy the security $s(x)$, if he observes $EU[s(x), w_0, w_1 \mid z] < p$. Since $E[z] = 0$, the same arguments as given in the proof of Proposition 2 show that debt gives rise to the smallest set of states z where information has value to the buyer and for any of these states $p - EU[s^D(x), w_0, w_1 \mid z] \leq p - EU[s(x), w_0, w_1 \mid z]$. This argument holds analogously for any p. **QED**

<u>Proposition 7</u>

Suppose the agent has $U(c_0,c_1)$ and obtains a signal z that induces $F(x|z)$ where the support of each $F(x|z)$ is a partition of the state space X. Any security that minimizes IS has $s(x)=x$ for all x where $EU(s(x), w_0, w_1) < U(w_0,w_1)$.

<u>Proof</u>

Denote p' as the agent's maximum willingness to pay when he is uninformed. Suppose $p=p'$. For a security $s(x)$, define $\{F^<\}$ as the set of posterior distributions where z induces $f_Z = f(x|z)$ such that $EU[s(x), w_0, w_1 \mid z] < p'$. Analogously, define $\{F^>\}$ to be the set of posterior distributions where z induces $f_Z = f(x|z)$ such that $EU[s(x), w_0, w_1 \mid z] \geq p'$. Given a debt contract, agent B does not buy it for the price p if he observes $f_Z \in \{F\}^<$. The value of information is $\pi_L^D = \sum_{Z \in \{F^{D<}\}} \left(p - EU[s^D(x), w_0, w_1 \mid z]\right) \cdot \lambda_Z$ where λ_Z is the probability for observing this subset of the partition. The value of information of another security *s(x)* is $\pi_L^S = \sum_{Z \in \{F^{S<}\}} \left(p - EU[s(x), w_0, w_1 \mid z]\right) \cdot \lambda_Z$. Since $s^D(x) = x \geq s(x)$ for all $x<p$, $\{F^{D<}\} \subseteq \{F^{S<}\}$ and $EU[s^D(x), w_0, w_1 \mid z] \geq EU[s(x), w_0, w_1 \mid z]$ for all $f_Z \in \{F^{D<}\}$. Thus $\pi_L^D \leq \pi_L^S$ for all *s*. Similarly, any *s(x)* that minimizes π_R must have slope 1 otherwise, information value can be reduced. **QED**

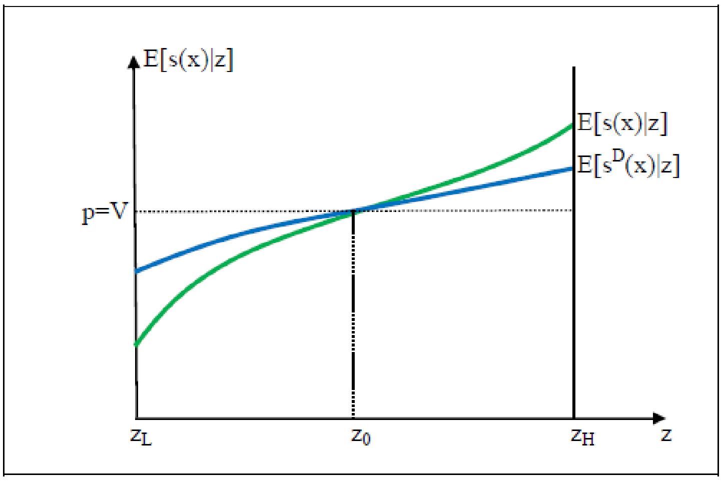
<u>Proposition 8</u>

Suppose the risk neutral agent receives a signal *z* which induces *F(x|z)* where *x* and *z* are affiliated. And suppose *s(x)* is non-decreasing. Debt has minimal IS.

<u>Proof</u>

Suppose $E[s(x)]=E[s^D(x)]=V$ and the signal *z* is continuous and has density *g(z)* and the prior z_0 is an element in the set of posteriors. So $E[s(x)|z_0]=E[s^D(x)|z_0]=V$. Lemma 1 in DeMarzo, Kremer and Skrzypacz(2005) shows that $E[s(x)|z]$ intersects $E[s^D(x)|z]$ once from below in the *z* space. See Figure 10 where the blue curve is debt and the green curve an arbitrary security. It is obvious, that for $\pi_L^D = \int_Z \max[p - E[s^D(x) \mid z], 0] f_z(x) dx < \int_Z \max[p - E[s(x) \mid z], 0] f_z(x) dx = \pi_L^S$ and for $p>V$, $\pi_R^D < \pi_R^S$. **QED**

Figure 10



These propositions show that given an underlying collateral *x*, the most senior security *s(x)* has the minimal IS and this observation holds under standard assumptions about information structures and utility functions.

C. Stochastic Contrasts and Security Insurance

This subsection discusses stochastic contracts and relaxing the assumption of limited liabilities. Consider the following general class of stochastic contracts. The payoff *s(x)* is a random variable that maps each realization *x* to an element *s(x)* in the interval $[x_L, x]$ according to the distribution H_x where the set $\{H_x\}$.

Quasi-debt has a strictly smaller IS than any stochastic contract. Note, even if *s(x)* is stochastic, the stochastic repayment *s(x)* must be backed by the outcome of the underlying *X*. Suppose $p=E[s(x)]$. If a potential buyer knows that $x<p$, he knows $s(x)<p$ for any stochastic realization *s(x)*. So information about *X* has value to a potential buyer even if *s(x)* is stochastic and determined *at t=1*. More importantly, stochastic repayments increase the value of information since with positive probability the agent obtains $s(x)<x$ for all $x<p$. Thus the IS of a stochastic contract is strictly higher than quasi-debt.

The limited liability assumption is not crucial. Suppose there is security insurance. For any *x* where $s(x)<p$, the seller can repay *m(x)* where $m(x)>x$. Then debt with $s(x)=[m(x), D]$ has minimal IS. The key point is seniority of repayment.[12]

7. Portfolio Choice, Asset Pricing and Information Sensitivity

In this section we discuss the final applications of IS. We do not assume information acquisition and adverse selection but focus on another important interpretation of IS in a setting of symmetric information. As mentioned, for a security buyer, π_L measures the expected monetary loss of the security in low payoff states and π_R measures the expected monetary gains in high payoff states. See Figure 1. In Section A we show that the IS and variance of a security are not rank-correlated. In Section B we derive a class of utility functions where IS is a sufficient statistic for the optimal portfolio choice and expected utility maximization. Section C shows that there is a tradeoff between IS and expected return. Section D shows that IS is a pricing factor.

[12] An alternative interpretation is the following. If there is unlimited liability this is equivalent to a shift of *f(x)* to the "right", i.e. there is less or no probability mass on the left tail. The IS of equity is always strictly positive.

A. Information Sensitivity and Variance

In this section we consider an economy with $\varpi_1,..,\varpi_\Omega$ states and N tradable assets in the standard state space framework. Without loss of generality, we normalize all assets to have the same expected payoff. (Otherwise we can scale down the asset payoff by a factor.) Also, suppose the price p of the portfolio equals expected payoff. In order to find a portfolio $(\phi_1,..,\phi_N)$ with the minimal IS, we solve:

$$\min_{(\phi_1,..,\phi_N)} \text{IS} = \sum_{i=1}^{\Omega} \max\left[\sum_{n=1}^{N} \phi_n x_{ni} - p, 0\right] \cdot prob(\varpi_i)$$

with

$$\sum_{i=1}^{N} \phi_i = 1$$

and where $\sum_{n=1}^{N} \phi_n x_{ni}$ is the payoff of the portfolio in state ϖ_i. We allow for short selling. The portfolio with the minimal variance solves

$$\min_{(\phi_1,..,\phi_N)} \text{Var} = \sum_{i=1}^{\Omega} \left(\sum_{n=1}^{N} \phi_n x_{ni} - p\right)^2 \cdot prob(\varpi_i)$$

where p is the expected payoff (mean). The solutions to both optimization problems are identical if there exists a portfolio with *Var=0*. Then IS=0. We provide two numerical examples to highlight that the IS and the variance of two securities are not rank-correlated and the portfolio with minimal IS and the one with minimal variance can be very different.

<u>Example1</u>

Suppose there are two assts and three states where the states arise with probabilities 0.25, 0.5 and 0.25, respectively. The payoffs of assets A and B in the three states are given as follows:

	ϖ_1	ϖ_2	ϖ_3	E[X]	Var(X)	Skew(X)	IS(X)
Asset A	1.70	2.50	3.30	2.50	0.320	0	0.20
Asset B	2.10	2.25	3.40	2.50	0.274	0.158	0.225

Among all portfolios with expected payoff 2.5, (i.e. $\Sigma\phi_i = 1$) the portfolio $(\phi_A, \phi_B) = (0.185, 0.815)$ with payoff (2.026, 2.296, 3.382) has the minimal variance of 0.271, a portfolio IS of 0.220 and skewness of 0.141. In contrast, the portfolio with the minimal IS only consists of asset A.

The intuition is the following. In states 2, asset A pays off 2.50 and asset B pays off 2.25. By combining assets A and B the portfolio payoff is different from p=E[X]=2.50. Since state 2 has a high probability this increases the portfolio IS. Note, in state 2, asset A has a payoff of 2.50 and does not contribute to IS.

For $\Sigma\phi_i = 1$ with $\phi_i \geq 0$, $IAS_{PF} = \Sigma\phi_i \cdot IAS_i$. In general, the portfolio IS is not the weighted sum of the IS of individual assets. For example, the portfolio $(\phi_A, \phi_B) = (11, -10)$ has an IS of 1.25 but the weighted sum of individual IS is negative. Zero is the lower bound of the IS.

<u>Example 2</u>

Suppose there are four states with equal probability and three assets with payoffs given as follows:

	ϖ_1	ϖ_2	ϖ_3	ϖ_4	E[X]	Var(X)	Skew(X)	IS(X)
Asset A	2.0	2.0	3.0	3.0	2.50	0.250	0	0.25
Asset B	2.1	2.2	2.2	3.5	2.50	0.335	0.221	0.25
Asset C	1.7	2.5	2.5	3.3	2.50	0.320	0	0.20

Suppose there is no asset C. The portfolio with the minimal variance is $(\phi_A, \phi_B) = (0.68, 0.32)$ and has an IS=0.226 and Var=0.226. The portfolio with the minimal IS is $(\phi_A, \phi_B) = (0.375, 0.625)$ and has an IS=0.203 and Var=0.248. If we add asset C to this economy, then the portfolio with the minimal IS invests 100% in asset C and has an IS=0.2 and Var=0.32.

The two examples highlight that IS and variance are not rank-correlated even if two random variables have zero skewness. In example 2, the portfolio variance increases as the IS of the portfolio decreases. In that example, an agent who minimizes the expected loss in low payoff states or has a reference point utility as defined below, buys asset C. An agent who has a maxmin utility function chooses asset B. An agent who has a mean-variance utility function chooses to hold the portfolio with the minimal variance since all assets have the same expected return.

B. Information Sensitivity and Expected Utility Maximization

In this section we provide a class of utility functions where IS is a sufficient statistic for the optimal portfolio choice and expected utility maximization. Consider an economy with *N* assets with joint distribution *F*. The state space can be discrete or continuous. In the latter case, the IS of a portfolio $(\phi_1,..,\phi_N)$ is given by:

$$\text{IS} = \int \max\left[\sum_{n=1}^{N} \phi_n x_n - \mu, 0\right] dF(\cdot)$$

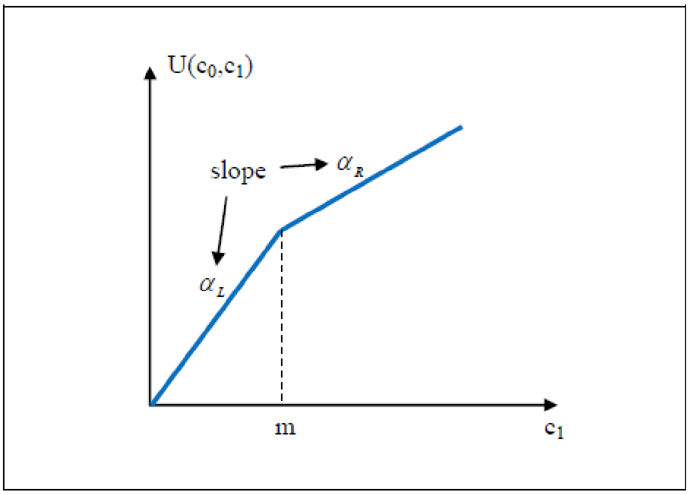
where μ is the expected payoff of the portfolio and also its price.

<u>Definition (Linear reference point utility function)</u>

$U = c_0 + \alpha_L \cdot \min[c_1, m] + \alpha_R \cdot \max[c_1 - m, 0]$ is called a linear reference point utility function where $\alpha_L, \alpha_R, m > 0$ and m is the amount invested and the reference point.

We assume that the reference point is the amount the agent invests. See Figure 11. In the return space the reference point is the zero (net) return. We give some interpretations of the reference point utility function for three important cases.

Figure 11



(i) Suppose $\alpha_L > \alpha_R = 1$. See Figure 11. The first *m* units of *t=1* goods generates a marginal value of $\alpha_L > 1$. If the agent consumes more than *m* units of *t=1* goods, then any additional amount has a marginal utility of 1. We can interpret *m* as the dollar amount that the agent plans to spend for consumption (drinks and food) at *t=1* (say in a restaurant). If he obtains more than *m*, then additional money gives rise to a marginal valuation of 1. In terms of banking, we can interpret the agent has a bank that needs cash to pay off its liability *m*. Any amount up to *m* reduces potential bankruptcy and potential expected bankruptcy renegotiation costs. So the first *m* units of dollar have a higher marginal valuation. If the agent is an entrepreneur (firm), then *m* can be interpreted as the optimal investment level. If he obtains less than *m* dollars, he has to scale back the size of a profitable project. Therefore, the first m dollars have the highest valuation. $\alpha_L > \alpha_R = 1$ captures a kind of loss aversion.

(ii) Suppose $\alpha_R > \alpha_L = 1$. Now in order to derive a high marginal valuation the agent needs to have more than *m* units of *t=1* goods. In a consumption setting, suppose the agent can only dine at an expensive restaurant if he has at least *m* dollars. Otherwise, the agent cannot afford it. In a corporate finance setting, *m* denotes the minimum investment amount. A firm can only invest in a profitable project if it has an investment amount of *m*. Otherwise it cannot invest in the project.

(iii) If $\alpha_L = \alpha_R = \alpha$, the utility function is linear and all payoffs generate the same marginal valuation at t=1. So $U = c_0 + \alpha c_1$ which is the utility function introduced in the baseline model in Section 2. As we will see, in that case IS does not matter for the optimal portfolio choice.

The ratio $\frac{\alpha_L}{\alpha_R}$ has the following economic interpretation. If $\frac{\alpha_L}{\alpha_R} > 1$, the utility function is concave and the agent cares more about low payoff states or downside risks and the left tail of the payoff distribution. $\frac{\alpha_L}{\alpha_R} < 1$ implies a convex utility function and the agent cares more about high payoff states or upside potentials and the right tail of the payoff distribution.

Proposition 9: Consider a set of *N* assets where all assets have the same price *m*. Suppose the agent with linear utility and reference point *m* can choose (only) one asset. Then $\pi_L(i)$ and $\pi_R(i)$ are sufficient statistics for expected utility maximization.

Proof: From Corollary 1.1 and Lemma 1, the expected consumption by choosing asset *X* at *t=1*, is given by:

$$
\begin{aligned}
EU &= E[\alpha_L \min[x,m] + \alpha_R \max[x-m,0]] \\
&= \alpha_L E[m - \max[m-x,0]] + \alpha_R E[\max[x-m,0]] \\
&= \alpha_L (m-\pi_L) + \alpha_R \pi_R
\end{aligned}
$$

QED

An asset with price equal expected payoff or an expected net return of zero has $\pi_L = \pi_R$. If the expected return is larger than zero, then $\pi_R > \pi_L$.

Corollary 9.1: Consider a set of *N* assets where all assets have the same expected payoff *m*. Suppose the agent with linear utility and reference point *m* can choose (only) one asset. If $\frac{\alpha_L}{\alpha_R} > 1$ ($\frac{\alpha_L}{\alpha_R} < 1$), then the agent chooses the asset with the minimal (maximal) IS. If $\frac{\alpha_L}{\alpha_R} = 1$, the agent is indifferent between all assets.

Proof: $EU = \alpha_L (m-\pi_L) + \alpha_R \pi_R = \alpha_L (m-\pi_L) + \alpha_R \pi_L = \alpha_L m + (\alpha_R - \alpha_L)\pi$ where *m* is the expected payoff and π the IS of asset X. For $\frac{\alpha_L}{\alpha_R} > 1$ ($\frac{\alpha_L}{\alpha_R} < 1$) the asset with minimal (maximal) IS maximizes expected utility. If $\frac{\alpha_L}{\alpha_R} = 1$, IS does not matter. **QED**.

In Example 1 above, the expected utility is $EU(A) = 2.5\alpha_L + 0.2(\alpha_R - \alpha_L)$ and $EU(B) = 2.5\alpha_L + 0.25(\alpha_R - \alpha_L)$. If $\frac{\alpha_L}{\alpha_R} > 1$, then EU(A)>EU(B).

Proposition 10: Suppose the agent has a linear reference point utility function with $\alpha_L, \alpha_H \geq 1$ and *m* units of *t=0* goods. Suppose the agent can hold a portfolio consisting of *N* assets where the price of each asset is *p=E[s(x)]*. Expected utility maximization is equivalent to (i) the minimization of the IS of the portfolio if $\frac{\alpha_L}{\alpha_R} > 1$; and (ii) the maximization of the IS of the portfolio if $\frac{\alpha_L}{\alpha_R} < 1$.

Proof: Denote $y = \sum_n^N \phi_n x_n$ as the portfolio (random variable). The price of (any) portfolio is *p=E[y]*. It is easy to see that it is optimal for the agent to buy a portfolio with *E[y]=m*, i.e. invest all his *t=0* endowment. Proposition 8 shows $EU = E[\alpha_L \min[y,m] + \alpha_R \max[y-m,0]] = \alpha_L m + (\alpha_R - \alpha_L)\pi$ where *m* is the expected payoff and π the IS of the portfolio. It is easy to see that in case (i) among all portfolios with the same expected payoff, the portfolio with the minimal IS maximizes *EU*. In case (ii) the portfolio with the maximal IS maximizes *EU*. **QED**

Proposition 10 shows that if $\frac{\alpha_L}{\alpha_R} > 1$, a save asset (portfolio) is the most desirable asset since the agent cares more about low payoff states or downside risks. If $\frac{\alpha_L}{\alpha_R} < 1$, the agent is more interested in high payoff states and thus prefers a portfolio that minimizes the probability of obtaining payoffs with $y = \sum_n^N \phi_n x_n$ <*m*. He prefers a kind of digital options with positive payoff starting at level *m*. If $\frac{\alpha_L}{\alpha_R} = 1$, then IS does not matter for expected utility maximization.

C. A Trade-off Between Expected Return and Information Sensitivity

In this section we consider the following thought experiment. Suppose the agent has an endowment of *m* units of *t=0* goods, a linear reference point utility function and can buy a portfolio with expected payoff *m* at *t=1*. We ask what is the price the agent is willing to pay for that portfolio at *t=0*. Fixing an expected payoff *m*, we still have $\pi_L = \pi_R$, denoted by π. Denote the expected gross return of portfolio *y* by $R \equiv \frac{E[y]}{p}$. The kink of the utility function is at *R*=1.

Proposition 11: An agent with a linear reference point utility function faces the following trade-off between expected return and IS. Along an indifference curve: (i) If $\frac{\alpha_L}{\alpha_R} > 1$, then $\frac{dR}{d\pi} > 0$. (ii) If $\frac{\alpha_L}{\alpha_R} < 1$, then $\frac{dR}{d\pi} < 0$. (iii) If $\frac{\alpha_L}{\alpha_R} = 1$, then $\frac{dR}{d\pi} = 0$.

Proof:

The expected utility at *t=1* is $EU = \alpha_L(m - \pi_L) + \alpha_R \pi_R$. The agent is indifferent between consuming *m* at t=0 and owning the security *x*, if $EU(x) = \alpha_L(m - \pi_L) + \alpha_R \pi_R \overset{!}{=} \alpha_L m = EU(m)$. This requires that $\alpha_R \pi_R - \alpha_L \pi_L = 0$ or $\pi_R = \frac{\alpha_L}{\alpha_R}\pi_L$. Along an indifference curve, $\frac{d\pi_R}{d\pi_L} > 1$, since $\frac{\alpha_L}{\alpha_R} > 1$. Lemma 1 shows that if the expected payoff of *x* equals the price (i.e. E[x]=p), then $\pi_L = \pi_R$. If $\pi_R > \pi_L$ then *E[x]>p*. Define *R=E[x]-p*. Since $\frac{d\pi_R}{d\pi_L} > 1$, this implies that $\frac{dR}{d\pi_L} > 0$. Note, the expected net return is

$r = \frac{E[x]-p}{p} = \frac{R}{p}$. So $\frac{dr}{d\pi_L} > 0$ since the price p needs to decrease in order to make the investor indifferent. **QED**

Proposition 11 shows that expected return and IS are "complements" if $\frac{\alpha_L}{\alpha_R} > 1$ (i.e. when downside risk is perceived as more important). In other words, along an indifference curve, if an agent bears more IS the agent requires a higher expected return. On the other hand, if $\frac{\alpha_L}{\alpha_R} < 1$ and the agent cares more about potential upside gains, then an agent is willing to accept less expected return for holding a portfolio with higher IS. If $\frac{\alpha_L}{\alpha_R} = 1$, then IS does not matter for expected utility maximization and the agent only cares about expected return.

D. Asset Pricing and Information Sensitivity

In this section we derive an equilibrium asset pricing model with a representative agent who has a linear reference point utility function and wealth *m*. There are one riskless asset (i.e. IS=0) and *N* risky asset where *F* is the joint distribution of the *N* risky assets and F_i is the marginal distribution of asset *i*. All assets have the same (normalized) expected payoff *m*. The gross expected utility for holding the riskless asset is $EU(x_{rf}) = m + m\alpha_L - p_{rf}$ and for the other assets, $EU(x_i) = m + m\alpha_L + \pi_i(\alpha_R - \alpha_L) - p_i$. The agent is indifferent between consumption at t=0 and buying the riskless if $p_{rf} = \alpha_L m$. The agent is indifferent between holding the riskless asset and asset *i* if $EU(x_{rf})=EU(x_i)$, i.e. $p_i = p_{rf} + \pi_i(\alpha_R - \alpha_L)$. The agent is indifferent between holding asset *i* and *j* if $EU(x_i)=EU(x_j)$, i.e. $p_j = (\pi_j - \pi_i)(\alpha_R - \alpha_L) + p_i$. Substitution yields $p_j = p_{rf} + \pi_j(\alpha_R - \alpha_L)$. In an economy with *N* risky assets, we need to calculate the contribution of asset *i* to the IS of the market portfolio of risky assets. For a two asset economy, this leads to the next Proposition.

<u>Proposition 12:</u> Consider a representative agent economy with two assets (one riskless (rf) and one risky (rk) asset, the market portfolio) and where the agent has a linear reference point utility. For an expected consumption of one unit of t=1 good, the equilibrium prices are given as follows: $p_{rf} = \dfrac{1}{\alpha_L}$ and $p_{rk} = p_{rf} + \pi(\alpha_R - \alpha_L)$ where π is the IS of the risky asset.

This proposition has some potential implications for time varying asset pricing. If the marginal investor has a linear reference point utility function and $\frac{\alpha_L}{\alpha_R}$ depends on the state of the economy, then equilibrium expected return (premium) depends on the state of the economy. We think that in an economic recession $\frac{\alpha_L}{\alpha_R} > 1$ since in distressed situations agents typically care more about "liquidity", i.e. payoffs in low payoff states. Therefore, an asset with low IS has a lower expected return or higher price. In contrast during an economic boom period agents are less cash constraint and less concerned about the left tail risk but are more interested in potential upside gains. In this case an agent is willing to pay a higher price for a higher IS asset and thus accepts a lower expected return.

If we add information acquisition and potential adverse selection, we may get a richer set of implications. However, this would require the assumption of a market microstructure model and whether and how private information is aggregated.

8. Conclusion

A financial security is characterized by the distribution of its payoff return and stochastic moments. We introduce a new characteristic of a security, its "information sensitivity" (IS). IS has two components, one component measures a security's expected monetary loss in low payoff states relative to its price and the other component measures the expected monetary profit in high payoff states. We characterize IS and illustrate its usefulness with several applications.

We show that IS captures an agent's incentive to acquire information in a trading context. Using this measure, we solve a security design problem where an uninformed buyer wants to buy a security and faces a seller who can acquire costly information about the final payoff of the security. We show that debt is an optimal security for an uninformed buyer to purchase and there is never endogenous adverse selection in equilibrium even if the information cost is vanishingly small. The buyer either reduces the amount of debt to purchase or bribe the seller not to acquire information by paying a price higher than the expected payoff of debt.

Using IS, we derive the optimality of securitization. In addition, we use IS to analyze optimal portfolio choice and asset pricing and show that it is a sufficient statistic for expected utility maximization and IS is a pricing factor if the agents have a liner reference point utility.

Several open questions remain. E.g. is IS a useful measure to solve a broader class of problems in optimal contracting setting with endogenous information and is it useful in mechanism design with endogenous type space? Is IS a good approximation for expected utility maximization if an agent has general concave utility functions? How can we empirically calculate IS? Further extensions and analyses in these directions look promising, especially that of empirical nature.

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