Experiment 8:

Projectile Motion and Conservation of Energy

1. Introduction

In this experiment, we use the trajectory equations of a body in two-dimensional free fall to predict where a projectile hits. The initial (launch) velocity of the projectile is determined by applying the conservation of energy for the projectile dropping through a long inclined tube. We compare the predicted location after free fall with a measurement. This lab should demonstrate the predictive power of applying physical principles correctly, show that predictions correspond to something in the "real world", and provide insight about deciding what is important in making a measurement.

Remark:
You must prepare some derivations at home; otherwise, you may have trouble finishing the lab in the time given.

2. Physical Principles

2.1 Conservation of Energy

One of the most fundamental principles of physics requires that total energy be conserved in all physical processes. However, there are many different kinds of energy, and energy can be transformed from one kind to another! But the principle is invariably true: so long as all types of energy are taken into account, the total energy before something happens is the same as afterwards.
What kinds of energy are there? They include potential energy, kinetic energy, rotational energy, heat, chemical energy, and mass*. We will deal primarily with the first three types, and with a small loss due to friction (which usually ends up as heat).

The kinetic energy of a point particle moving with velocity v, is given by $E_{Kin}= ½\ mv^2$. Because the rolling ball used in this experiment is not a point object, we need to take into account its rotational motion. This is discussed in the next section.

The potential energy for an object near the earth surface is given by $E_{Pot}=mgh$, where h is the height above an arbitrarily chosen level. This means that when an object drops by a vertical height, $Dh=h_1-h_2$, the loss in potential energy is $mgDh$.

Some energy is always lost due to friction. If we label the friction energy loss W, it follows that

$$\textit{Gain in K.E.} = (\textit{P. E. lost}) - W.$$

* This is the content of Einstein's famous formula, $E=mc^2$!

2.2 Estimating the Friction Loss

If there were no friction, all the loss in potential energy (*mgDh*), as the ball rolls from the release point to the launch point, would be converted to kinetic energy. We need a technique to determine the energy lost to friction. Here, we determine the vertical distance (*Dh'*) that the ball traverses through the track in the case that the ball just comes to rest at the lower end of the track when released from the top. Then *mgDh'* must equal the energy lost to friction. If we assume that the same amount of energy is lost to friction when the track is tilted more steeply, we can use *W=mgDh'* for all tilts. It follows that

$$\begin{aligned} Gain\ in\ K.E. &= (P.\ E.\ lost)\ - W \\ &= mg\Delta h - mg\Delta h' \\ &= mg(\Delta h - \Delta h') \end{aligned}$$

Remark:
You have to measure the friction separately for each ball!

2.3 Kinetic Energy including Rotation

In this experiment, we deal with a rolling ball. We need to account for the rolling motion, so that there are two contributions to the kinetic energy:
Kinetic Energy = Kinetic Energy of center of mass + Energy of rotation
This relationship takes the analytic form

$$E_{Kin} = \tfrac{1}{2}\, m \cdot v^2 + \tfrac{1}{2}\, I \cdot w^2,$$

where m is the total mass of the object, v is the velocity of the center of mass, I is the moment of inertia ($I = 2/5\ m \cdot R^2$ for a sphere), and w is the angular velocity of the rotation. (For a rolling ball which is not sliding, $w=v/R$). So the total kinetic energy of the rolling sphere is

$$\begin{aligned} E_{Kin} &= \frac{1}{2} m \cdot v^2 + \frac{1}{2} \cdot (\frac{2}{5} m \cdot R^2) \cdot (\frac{v}{R})^2 \\ E_{Kin} &= \frac{1}{2} m \cdot v^2 + \frac{1}{5} m \cdot v^2 \\ E_{Kin} &= \frac{7}{10} m \cdot v^2. \end{aligned}$$

As can be seen from the final result, the kinetic energy of a rolling ball is slightly larger than that of a point object travelling at the same speed.[*]

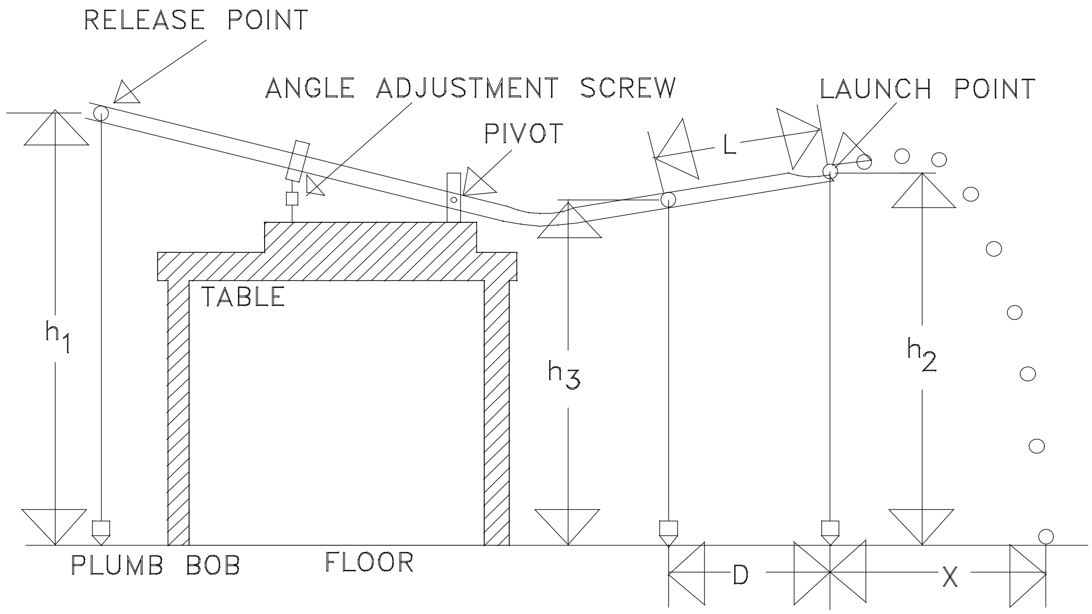
[*] If you have not yet had rotations in lecture, you may not follow this argument in detail. But the last equation indeed does express the kinetic energy of the rolling ball. Be sure to use it.

2.2 Parabolic Trajectory

The motion of a mass launched into free fall with initial velocity, *v*, at an angle *j* relative to the horizontal, can be treated easiest by evaluating the horizontal (or *x*) and vertical (or *y*) position in terms of the time (*t*) as two independent motions. This problem, in which explicit expressions for *y(t)* and *x(t)* are obtained, has been treated in your text, in lecture, and you have done several related homework problems. (We neglect air resistance while the ball is in free fall.)

<u>3. Experiment</u>

The figure shows the apparatus to be used in this experiment. A ball is released into a tube at the release point, rolls through the tube and emerges into space at the launch point. You measure the parameters shown explicitly in the figure, and the additional parameter, *Dh',* which is used to estimate friction losses.



3.1 Prediction of Position

Before you do the experiment, you should derive <u>at home</u> a set of equations, which provide a prediction for the *x*-position where the ball hits the ground (*y=0*). This expression should depend <u>only</u> on measured parameters shown in the figure: h_1, h_2, h_3, *D*, *L*, as well as the value of *Dh'*. You will need to substitute these quantities rather than parameters we do not directly measure, like *v* or *j*.

Rather than derive a single complicated formula for *x* in terms of symbols for all the preliminary measurements, it will be more convenient to calculate, in sequence, several intermediate quantities and then combine these to find *x*. In other words, it is advisable to break up the entire problem into several smaller ones:

1. Find v, the magnitude of the launching velocity, by using the conservation of total mechanical energy (with subtraction of the estimate of the energy lost to friction).
2. Find v_x and v_y, the horizontal and vertical components of v, by referring to the geometry of the final section of the track.
3. Find t, the time the ball is in the air, by considering the vertical motion involving v_y and h_2 alone.
4. Finally, find x.

Steps 3 and 4 may be combined by using the trajectory equation $y(x)$, obtained by eliminating the time in the equations for $y(t)$ and $x(t)$.

3.2 Comparing Prediction with Observation

For any configuration of the tube, you should be able to predict the x position (where it hits the floor). In each case, compare the measured position with the prediction.

In the first part, place an extended object (like a quarter) centered on the predicted position and see if the projectile hits it. In the second part of the experiment, make measurements of the hit position relative to that predicted. These data will permit a measure of the spread, or uncertainty, from the reproducibility of the results.

3.3 Checking Formula

If you want to check if your derived formula for the position where the projectile hits the ground is correct, you can use the following data:

h_1	=	124.3 cm	h_3	=	110.0 cm
h_1'	=	122.7 cm	D	=	27.7 cm
h_2	=	119.3 cm	L	=	29.2 cm
h_2'	=	120.5 cm			

The position the projectile hits the ground is then at

$$x = 30.5 \text{ cm.}$$

4. Specifics of the Experiments

Remark:

If you want to clean the tube beforehand, there should be a swab on a string available.

4.1 Prediction of Position

- Bring the sheet with your derivation of the formulae for x in terms of the measured parameters. You should prepare this before coming to the lab! It should be attached to the report when you are finished.
- Choose your first ball as the heavy metal ball.
- Adjust the screw such that the ball, when released at the release point, just makes it to the launch point before reversing direction. Record h_1' and h_2'.
- Increase h_1 with the adjustment screw so that the ball will be launched. Make sure that that h_1-h_2 is at least twice as big as h_1'-h_2'.
- Measure all the required quantities and predict where the ball will hit the floor. Place a coin at that position.
- Release the ball and see if the ball hits the coin.
- Repeat the experiment for the same ball with a different height h_1. (Check or measure all relevant quantities.)
- Comment on your results!
- Repeat the same steps for the plastic ball, then the aluminum ball.
- The difference $Dh' = h_1' - h_2'$ provides a measure of the energy lost to friction as the ball traverses the tube. Order the measurements of the balls from highest friction to lowest friction. Explain why you might have expected this order!
- Which ball would you have expected to fly the furthest horizontal distance from the same release point? Why?
- What are the major sources of error? How far off would your results be if you had not corrected for the friction losses in the tube? From the comparison of your results with the predictions, how much effect might the neglected air resistance in the free-fall trajectory have contributed?

4.2 Quantitative Measurements and Uncertainty

- Place a sheet of white paper on the floor centered at the predicted location and place a piece of carbon paper on top of it. Tape them to the floor.
- Before proceeding, make a guess of how large the spread of results will be!
- Roll a single ball about twenty times using the same setup; you should obtain a number of points marked on the white paper. These should be spread around the expected value. Do this experiment with different papers for the heavy ball and for the plastic ball.
- Describe and compare the two spreads. (How large is the spread? Is it uniform in all directions? Should it be? Is the spread the same for both balls? Should it be? Is the spread about as big as you expected?)
- If the spread much bigger along the direction of the trajectory than perpendicular to it, how would you interpret this?
- Make a few suggestions for how you could have improved the first part of the experiment so that you would always hit a smaller area, like that of a dime!

5. Lab Preparation Examples

Trajectories:

1. What is your predicted value for x, assuming you measure the following values:

$h_1' = 105$ cm	$h_2' = 90$ cm	
$h_1 = 120$ cm	$h_2 = 80$ cm	$h_3 = 65$ cm
$L = 25$ cm	$D = 20$ cm	

2. If $h_1' = 125$ cm and $h_2' = 100$ cm, what percentage of the potential energy is lost to friction?

3. You shoot a bullet with velocity v and an angle j to the horizontal direction and observe where it hits the ground. You now increase the angle with which a bullet is shot. Does it reach further or not? Does the answer depend on what the original angle was?

Spread:

4. How do you expect the spread of measurements to appear? (Qualitatively, not quantitatively!)

5. Assume you cannot control the release point very well. (In the experiment we control this quite well.) Suppose you sometimes release the ball further up the tube and sometimes further down. How do you think this would effect the spread?

6. Assume you perform the lab outdoors, where there is a strong and unsteady wind blowing along the length of the launch tube. How will your spread look in this case?

Air Resistance:

7. We need to measure the friction in the tube in this experiment. But we do neglect the air resistance of the ball after it is launched. How big an effect do you estimate this to be?

8. Assume you want to make a parachute jump. You know that you can change the air resistance by giving the air a bigger or smaller cross section to resist. But you are curious about what influence your body weight has on the maximum speed you can reach: You know that the driving force pulling you down to earth is $m \cdot g$. Furthermore you also know that the air resistance is a function that increases with increasing velocity (and area). (If you are not convinced of this, drive your car at different speeds and put your hand out of the window!) Finally you remember that the maximum speed is the speed when there is no more acceleration (i.e. the force pulling you down and the force pulling you up are equal). With all this knowledge, try to explain why a heavy body can reach a higher maximum velocity than a light one (given that they have the same shape and size). (If you want to use an equation for the air resistance you can e.g. use $F(v) = \text{const.} \cdot v^2$ which is true in some limit.)