Experiment 9:

Standing Waves

1. Introduction

Waves surround us. Your radio receives signals by means of electromagnetic waves and emits waves that sound (like music) when they hit your ear. Waves are visible on the surface of a body of water. The human body creates waves (like heartbeats) and diagnostics of the body's physical condition involve many different kinds of wave phenomena. The natural world is full of waves, and technology has multiplied their importance to us.

All waves share certain physical similarities. In this experiment, we gain experience with properties shared by many different kinds of waves.[+]

2. Theory

2.1 Waves

Waves are periodic disturbances propagating in space and time. We illustrate here properties of waves using waves in a stretched string and sound in an air column.

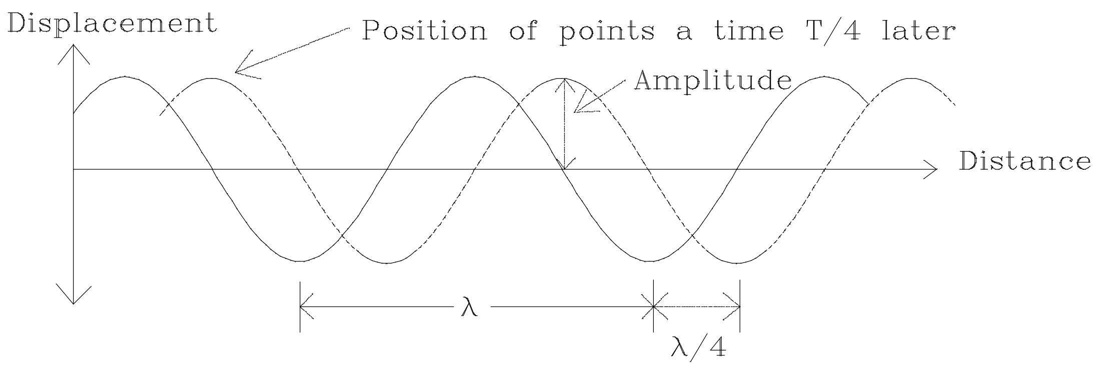
To be able to deal with waves we introduce a number of definitions:

- The period T is the time it takes until a wave repeats itself.
- The frequency f , defined as $f = 1/T$, measures the number of complete cycles the wave repeats in one second. The unit of frequency (1/seconds) is Hertz (Hz).
- The wavelength λ is the spatial separation between repeating points in a wave.
- The amplitude A of the wave is the maximum magnitude of the displacement.

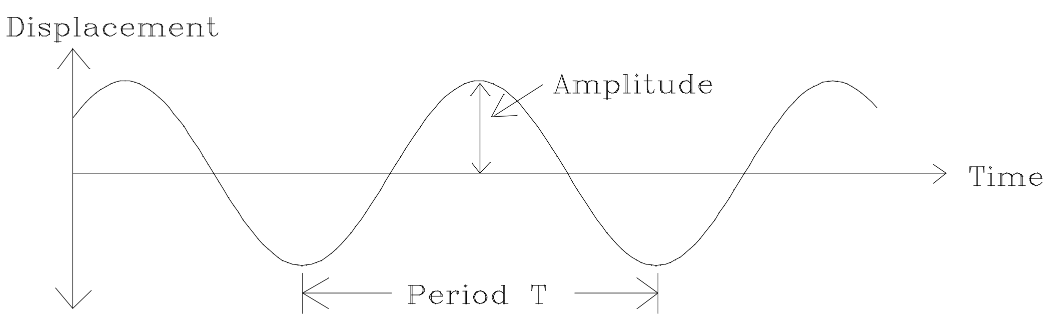
If a long, taut horizontal string is sharply pulled up at some point and released, this part of the string will vibrate up and down. Neighboring points will then follow the motion, and the original disturbance will propagate down the string as a "travelling wave". Since the individual particles vibrate in a direction perpendicular to the direction of propagation, the wave is called a transverse wave. If the disturbance is periodic, i.e., if the identical disturbance is repeated continuously, a wave train will move down the string. The following figure shows the disturbance along the string at a single instant of time. If the wave is moving to the right and a second picture is taken a quarter period T/4 later, all points on the wave will have moved an equal distance to the right, as shown by the dotted curve. The frequency, f, of the wave is defined as the number of times per second the disturbance is repeated. (Thus $f = 1 / T$). Note that the period of the motion is determined

[+] The discussion of waves is treated in the text in the chapters on Sound (Mechanical Waves) and Electromagnetic Waves. We will discuss these in lecture during the next semester. This experiment should familiarize you with the phenomenology of waves.

by the cause of the initializing disturbance. During each period T, the wave travels a distance of one wavelength λ; therefore the <u>velocity of the wave</u> is given by[*] $c = f\lambda$.



The figure below shows the vertical <u>displacement</u> of the string <u>versus time</u> <u>at a fixed location</u> along the string. The time interval between successive identical displacements of a given point is the period T of the wave. Remember that the wavelength of a travelling wave can be determined only when one observes the displacement as a function of distance at one instant in time. The period, on the other hand, is obtained when one observes the displacement of one point as a function of time.



There are in general two possible types of waves: transverse and longitudinal waves. If something in the wave oscillates up and down or left and right (so the oscillation takes place perpendicular to the direction of propagation of the wave), the wave is transverse.[+]

If a wave changes <u>along</u> the direction of propagation, it is called a longitudinal wave. (Such a wave clearly has only one polarization state.) Examples of longitudinal waves include sound waves, in which there are periodically changing regions of low and high air pressure <u>along</u> the direction of wave propagation.

[*] In all the cases here, *c* will be a constant. But in general *c* could be a function of the frequency. (This effect is called dispersion).

[+] The two independent possibilities for making a transverse wave are referred to as the two possible polarizations of the wave. (If the wave oscillates diagonally, it can be viewed as a combination of vertical and horizontal polarizations.)

A sound wave is normally initiated by a vibrating solid (such as a tuning fork), which alternately compresses and rarefies the air adjacent to it. The wave thus consists of pressure variations in the air, moving away from the fork. Since these pressure and density variations oscillate back and forth along the direction of wave propagation, the sound wave is a longitudinal travelling wave.

The definitions of *f*, λ , and T (made above for a transverse wave) hold equally for a longitudinal wave. The diagrams in the above figures also apply, so long as we understand "displacement" to mean the longitudinal (i.e., forward or backward) pressure or density variation of air from its undisturbed equilibrium value. The relationship between wave velocity and the other parameters is also still valid:

$$c=f\cdot\lambda$$

2.2 Standing Waves

So far we have considered only very simple wave disturbances. More complicated waves are created when two or more travelling disturbances are present simultaneously in the same medium. In general, any number of waves can be combined to give a more complicated wave.

A particularly interesting example occurs when two waves of equal λ, *f*, and A are travelling along a taut string in opposite directions. (This occurs for example if the wave encounters a barrier, which reflects the wave back in the original direction with the original wave still propagating.) At some particular points on the string, the two waves will always be out of phase, i.e., one wave will try to move the point up and other wave will try to move the point down. The result is that the two waves will cancel each other at this particular point, and so that point will remain stationary. This condition is known as destructive interference, and the points at which this occurs are called nodes.

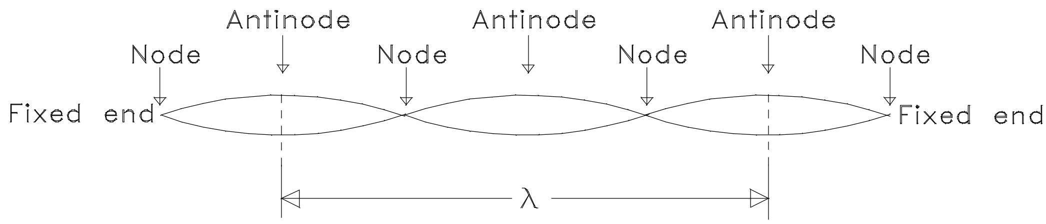
At some other points on the string, the two waves will move up and down together so that the amplitude of the disturbance at these points will be twice what it would be if only wave were present. This condition is known as constructive interference and the points at which this occurs are called antinodes.

As long as the λ, *f*, and A of the waves remain fixed, the positions of the nodes and antinodes will not change. The pattern produced in this circumstance is called a standing wave. The analytic description for the displacement versus position and time with one end fixed at x=0 is given by

$$\mathrm{A}\cdot\sin[\frac{2p}{l}\mathrm{x}]\cdot\cos[2p\ f\ \mathrm{t}].$$

Remember that a standing wave is produced by the interference of two waves travelling in opposite directions.

A string with both ends fixed can be excited with standing waves as shown in the figure below. The fixed ends of the string cannot move, so the string has nodes at these points. It is evident that the distance between the node at a fixed end and the first antinode is $\lambda/4$;



the distance between successive nodes (or antinodes) is $\lambda/2$. (See figure above.) The positions on the string where $A \cdot \sin[2\pi/\lambda \cdot x] = 0$ correspond to a node, so whenever $2\pi/\lambda \cdot x$ is a multiple of π, i.e.

$$\frac{2\pi}{\lambda} x = n \cdot \pi ,$$

(x is a multiple of $\lambda/2$) we find a node. Midway between two nodes we will always find an antinode, a point where the string oscillates maximally.

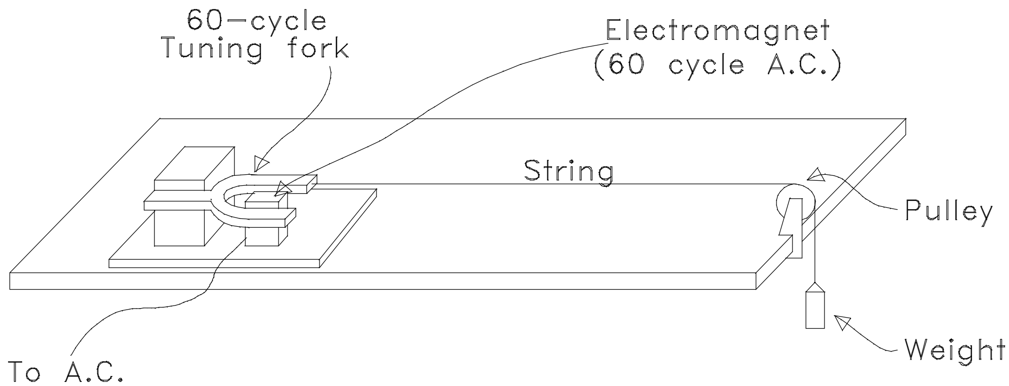
Sound waves can be created in a tube of gas as standing waves. In this part of the experiment, a tuning fork vibrates at a frequency f over the open end of a tube containing air. The tube is closed, or sealed, at the other end by the water. The motion of the fork causes longitudinal sound waves to travel down the tube. The sound wave is reflected back up from the closed end of the tube so that the incident and reflected waves interfere with each other. The air at the closed end of the tube is not free to move, so if a standing wave is to be produced in the tube, a node must exist at the closed end. The air in the open end of the tube is free to move; so when a standing wave is produced in the tube, the open end will be an antinode.

Since the distance between a node and the nearest antinode in a standing wave pattern is $\lambda/4$, it should be evident that the shortest tube in which a standing wave can be established has a length of $\lambda/4$. A standing wave with wavelength λ can be established in longer tubes. All that is required is that a node exist at the closed end and an antinode at the open end--i.e., that the length of the air column is an odd multiple of $\lambda/4$. When a standing wave is produced in the tube, a resonance condition is established and the intensity of the sound will increase.

3. Experiments

3.1 Standing waves on a String

The first part of the experiment deals with transverse waves on a string. A long horizontal string is attached to the tine of a driven tuning fork that vibrates at $f = 60$ Hz. The other end is fixed at a point where it passes over a pulley. You can change the distance between the pulley and the tuning fork by shifting the base of the tuning fork and you can hang weights M on the end of the string (which produces a string tension T=M·g). The apparatus is pictured in the figure below.



Your task is to measure the resulting wavelengths of standing waves for various values of the tension. How is this done? In the previous section, it was stated that the distance between two nodes in a standing wave is $\lambda/2$. So simply measure the distance between adjacent nodes and double it to get the wavelength.

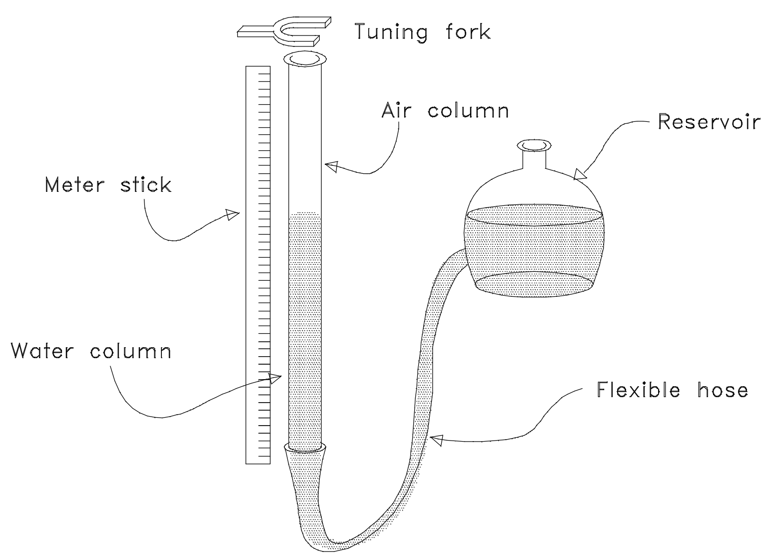
Also calculate the velocity with which the wave propagates on the string, using the relation between frequency and wavelength. One can also analytically calculate the velocity of propagation of waves on a string from the physical properties of the string:

$$c = \sqrt{\frac{T}{m}},$$

where T is the tension, and μ is the mass per unit length of the string. You can compare your experimental results with this prediction.

3.2 Standing Sound Waves

In the second part, we measure the wavelength of standing longitudinal waves.



Sound waves (longitudinal waves) are set up in a long open tube, which is partly filled with water, as shown in the figure to the right. We change the height of the water column by lifting or lowering a water reservoir. Produce a sound wave using a tuning fork (you will have two different ones, one with

512 Hz and one with 1024 Hz). Strike the tuning fork and hold it at the top of the glass tube. As you change the level of water in the tube, you find some water levels at which the sound from the tuning fork becomes much louder. This occurs whenever you have created a standing sound wave in the glass tube. By measuring the distances between water levels that achieve successive sound maxima, you can determine the wavelengths of the soundwaves.
As before, from the wavelengths of the standing waves and the frequencies, you can compute the velocity of propagation of the sound waves for each case. This should equal the speed of sound in air.

Remark:
It is often easiest if you lower the water level continuously and hit the tuning fork somewhat hard. Please be gentle; don't damage any equipment, because there are more generations of students to come.

4. Step by Step List

4.1 Standing waves on a String

- Measure the mass and length of the string. Calculate μ.
- Attach the string to the screw on the tuning fork and place it over the pulley.
- Put a mass of 100g on the end of the string and choose the distance between pulley and tuning fork such that you get a standing pattern of nodes and antinodes.
- Measure the distance between adjacent nodes (including uncertainty!) and calculate the wavelength. You get the best results if you measure nodes in the middle of the string and if you average over several measurements.
- Calculate the experimental and the theoretical value of c and compare them. Are they equal within uncertainty?
- Do these measurements for 3 different weights!
- What are the main sources of error in this experiment?

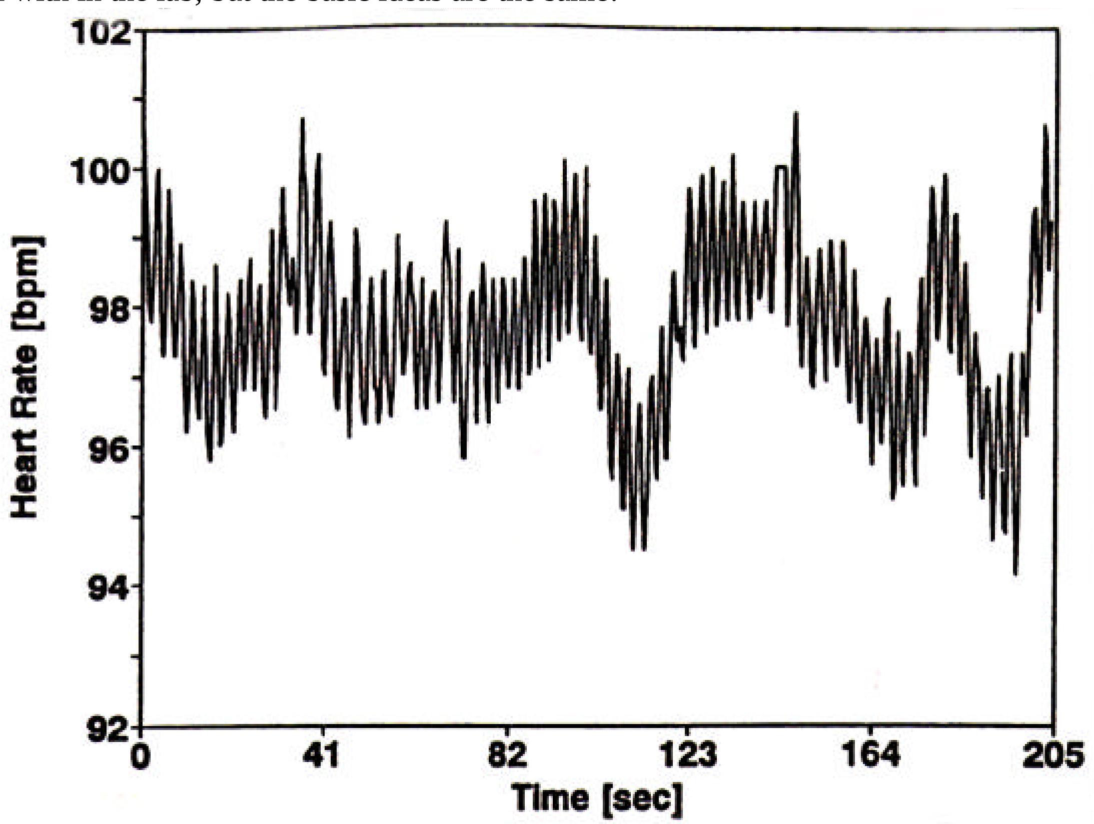
4.2 Standing Sound Waves

- Measure the wavelength for both tuning forks. There should be small string-rings on the glass tube that you can use to mark the levels at which you get resonance (standing waves). To improve your data try to average over several measured values. Also don't forget to include uncertainties.
- Calculate the speed of sound for both frequencies. Do you get the same value within uncertainty?
- Is your value for the speed of sound close to the standard value of 340 m/s? What could be reasons that you get a different value?
- Was one of the tuning forks easier to hear than the other? If yes, do you have an idea why?
- Give the main sources of error and make suggestions of how the setup could be improved!

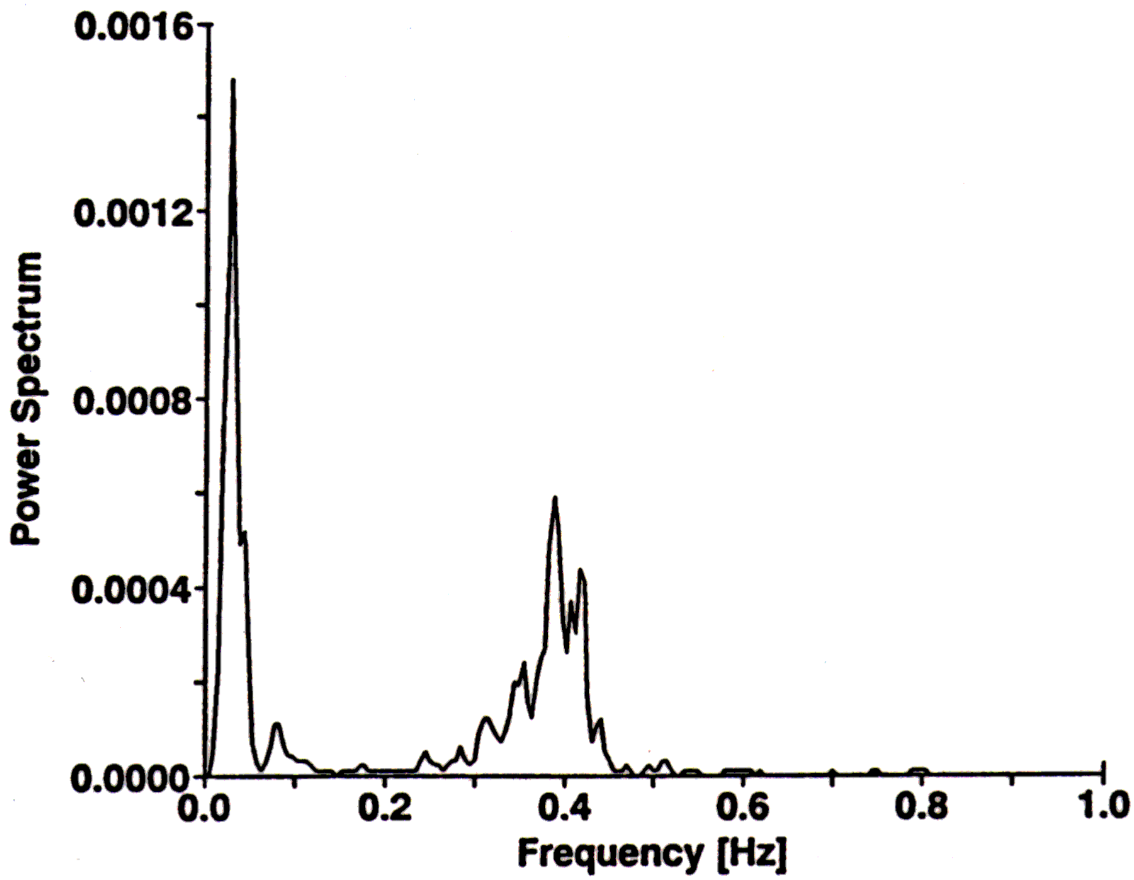
5. Applications

Biological systems that create periodic pulses (like a beating heart) are optimized for a range of operating conditions. The variations in the operating parameters often provide an important diagnostic tool. For example, the period of beats in a human heart can vary; one may ask to what extent and how often the heartbeat rate changes. To identify patterns of heart rate, one makes use of a mathematical procedure called Fourier analysis. Besides the dominant frequency of the heartbeat (about 1 Hz), there are other frequencies that correspond to changes in the heart rate; Fourier analysis uses the heart rate to make a diagram of the amplitudes of the lower frequencies present in the heart rate.

This may at first sound a little bit complicated, but the basic idea is simple. In the experiment, we find only specific standing waves for each system. With the system parameters fixed, only certain frequencies are possible: most frequencies cannot achieve a standing wave. Obviously these frequencies are special and characterize the system! If you look now at a complicated oscillation of the system and take the Fourier spectrum, you find that only these special frequencies contribute. Other frequencies don't contribute at all. A beating human heart is, of course, much more complicated than the systems we deal with in the lab, but the basic ideas are the same!



This picture shows the heart rate of a healthy adult over time. As one can easily see, the heart rate changes over time. The frequencies with which the heart rate changes can most easily be seen in the Fourier spectrum (sometimes called power spectrum) in the next figure.



One immediately sees a prominent change in the heart rate with a frequency of about 0.4 Hz. Therefore, roughly every 2.5s the heart rate of this person changes. This frequency corresponds to the many little spikes we saw in the first diagram. (Another characteristic frequency is at about 0.02 Hz, corresponding to a change about every 50s. We will not deal with this part here, even though it contains valuable information.)

Where does the change every 2.5s come from? It turns out that the person took a breath about every 2.5s. The heart then speeds up slightly to pump more blood through the vessels of the lung, where the blood is oxygenated. Subsequently, it slows down again. (This particular frequency is called the respiratory frequency.)

The process of speeding up and slowing down is governed by the autonomic nervous system. Some diseases (e.g. diabetes) can damage the autonomic nervous system and therefore stop this adaptation of the heart rate. For example, as diabetes reaches its final stage the peak in the Fourier spectrum at the respiratory frequency vanishes.

This method has the nice feature that it is non-invasive, relatively simple, and shows dynamic processes rather than snapshot pictures.

Reference:

Amos D. Korczyn: Handbook of Autonomic Nervous System Dysfunction

6. Lab Preparation Problems

Waves:

1. Is a wave on the water surface a longitudinal or transverse wave?

2. The wavelength of visible light is between 400-800nm. What is the frequency of visible light?

Standing Waves:

3. A transformer is humming at a frequency of 60 Hz and produces a standing wave. What is the distance between adjacent nodes? What is the distance between adjacent antinodes?

4. You have a string and produce waves on it with 50 Hz. The wavelength you measure is 7 cm. What is the speed of the wave on this string?

5. You put a mass of 400g on the string of experiment 1. (The string is 50 cm long and weights 12.5g) What distance between adjacent nodes do you then expect for a frequency of 100Hz. (Use $g = 10\ m/s^2$)

6. With a 660 Hz tuning fork you measure a distance of 25±2 cm between adjacent nodes. Is the value of $c = 340$ m/s within the uncertainty of your measured value?

Explanations:

7. If you blow air along the top of an open soda bottle you can excite a standing wave in the bottle and you hear a sound. Explain what happens if you put some water into the bottle and then perform the same experiment!