New Developments in Mathematics, Computing, the Sciences, and New Views of Computation

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Abstract

This paper presents the idea that an exciting synergy is occurring in theoretical work in mathematics, computation, and natural sciences. The view of computation currently held by many is "Newtonian", but alternative views are emerging, brought about by the accessibility of massively parallel computers, research in very small computing elements, and by the adoption of a new understanding of nonlinear dynamics in experimental mathematics and the sciences. The adoption of these views of computation are analogous to the revolutions of thinking in physics. Two examples are given to illustrate how the synergy is causing a rethinking of the nature of computation.

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I. Introduction

The behavior of very large systems of computation and very small computers often reflects the behavior of large and small systems in the world. Consequently, it is not surprising that researchers in computer science are adopting techniques traditionally used in describing and analyzing behavior in the real world to the study of computation. Hillis has put forth the idea that such fundamentally new views of computation are needed so that problems scale with the capabilities of very large parallel computers and very small computing elements [Hillis 82]. Two emerging views of computation in particular serve this purpose: the *population statistics view* and the *quantum view.* The full development of these new views makes use of recent developments in mathematics and the sciences. One major benefit of applying these views to computation is that they allow non-synchronization between processes, and thus, can eliminate synchronization overheads.[1]

2. The Current "Newtonian" View of Computation and Emerging Modern Views

The Newtonian view divides computation into components that interact detenninistically, with strict precedence observed for communication between modules. In this view of computation, the need for maintaining *spatial precedence* (conununicating to the exact module) and *temporal precedence* (communicating when planned) is paramount.[2] Each actor in this computational universe is following its role in a master script, just as the pieces of the great clockwork universe of the classical era of physics did. Strict precedence ensures the same behavior for parallel and serial machines, except for a speedup whose extent is limited by the number of processors. This speedup is normally not obtained because of the sequencing needs of communication among the software modules.

The attempt to analyze physical systems with large numbers of actors (gases) by Maxwell, Boltzmann and others, led to a characterization of the macrostates of an ensemble system which is called thennodynamics. A similar attempt to deal with a large number of parallel computations is leading computer scientists to characterize the macrostates of some global computation from the population of computations. The individual computations are viewed as having stochastic behavior or as interacting stochastically. In the population statistics view, the ensemble statistics of the processes fonn the collective computation of interest.

ITbe historic example of using non.synchronization in computer science for speedup is chaotic asynchronous algorithms for computing the fixed point of matrices [Lubacbevsky 86], [Baudet 78], [Chazan 69].

[2]The implication for the new views of computation is that either spatial precedence or temporal precedence can be relaxed.

The quantum view of computation is just beginning to emerge as computer scientists think in tenns of smaller and smaller computing elements. At this level quantum effects occur and several researchers have proposed quantum computers of different types and abilities [Benioff 82], [Feynman 82], [Albert 83], [Deutsch 85]. Deutsch has argued that these quantum computers are more powerful than Turing machines in their ability to compute random numbers [Deutsch 85]..

3. Examples of the Synergy

Below are two surveys of recent work that illustrate the population statistics view and the quantum view of computation.

3.1. Population Statistics Models of Machine and Human Computation

It is compelling, faced with the overwhelming complexity of the neural network that makes up the mind, to adopt the population statistics view of computation to describe such a network. This approach has been called *statistical neurodynamics* [Amari 77].3 There has also been new work analyzing the nonlinear dynamics of individual nerve systems and neural networks and the nonlinear dynamics aspects of memory and pattern-based cognition [Hopfield 82], [Harth 83], [Guevara 83]. In systems such as neural networks, one adopts a capability-based view of computational processes since an element by element analysis of the internal state of a neural configuration that is doing the desired computation is hopelessly complicated. One can go far if one knows the theoretical limitations in the capabilities of different configurations, since one of the primary abilities of these organizations is to *evolve* into some state that displays the desired behavior. This outlook is described in the introduction of [Kohonen 78]:

> *It seems that the basic operation of associative memory. the storage of information together with the relations or links between the data items. and the selective recall of stored information relative to a piece of key or cue information presented. is not restricted to certain computer-technological implementations but can also be reflected in more general mathematically describable processes in certain physical or other systems, especially in their adaptive state changes.*
> *• {Kohonen 78J. Preface.*

3.1.1. Kanerva's Theory of Memory and Attractors

Kanerva (Kanerva 84,85] has developed a conceptually elegant theory of a population-statistics-based sparse distributed memory that is claimed to have strong neurophysiological parallels. Hofstadter [Hofstadter 85], in his chapter on attraetors, characterized Kanerva's theory of gross memory processing as follows:

> *In short, locldng-in - that is. convergent and self-stabilizing behavior - will surely pervade the ultimate explanation of most mysteries of the mind. One example is the question of memory retrieval. How do things that are only vaguely similar to each other stir up rumblings of recollection, and eventually trigger the retrieval of amazingly deep abstract resemblances? One theory. best formulated and articulated by cognitive scientist Pentti Kanerva of Stanford University. sees the initial input as a seed - a vector in a very high-dimensional space... The seed is fed into memory-retrieval mechanisms. which convert it into an output vector that is then led back in again. This cyclic process continues until it either converges on a stable fixed point • the desired memory trace • or is seen to be wandering erratically without any likelihood of locking in. tracing out a chaotic sequence of "points" in mind-space.*
> - *fHo/stadter* 85*J Chapter* 16 *page* 394-395.

3See [Levine 83] for a research review of earlier work of the population statistics view in this field.

3.1.2. Vision and Simulated Annealing

Marroquin [Marroquin 85] and Poggio [Poggio 85] have viewed early[4] visual processing in biological systems and machines as a problem of *inverse optics.* Techniques to solve such inverse problems have used population statIStics approaches [Marroquin 85],[5] [Camevali 85], as well as variational methods [poggio 85]. Marroquin and Poggio have analyzed these methods for parallel hardware. Geman and Geman [Geman 84], Camevali [Camevali 85], and Marroquin [Marroquin 85] have used *simulated annealing* as a statistical mechanics approach to vision.[6] Researchers are fmding useful analogies between the statistical mechanical analysis of the dynamics of ensemble physical systems, such as ferromagnetic materials, and the dynamics of computation in massively parallel machines and neural networks which accomplish early visual processing.

3.1.3. Non-Synchronous Networks and Nonlinear Dynamics

The effects of random communication time delays and non-synchronization in neural networks have also been studied. Hopfield [Hopfield 82] indicated that such an organization implemented on silicon would provide rapid solutions to some (unspecified) classes of computational problems. Peretto and Niez [Peretto 86] have studied the stochastic dynamics of non-synchronous neural networks and found that in these networks, short tenn memory cannot be supported with loops of recursive neural activity in structures without some modification means such as the Hebbian rule for modifying synaptic strengths. They also emphasize the close analogy between neural network models and current models of statistical mechanics such as the Ising spin glass models of ferromagnetic materials.[7]

One may wonder how much the work cited in the above paragraphs goes beyond the results in Minsky and Pappert's book on perceptrons [Minsky 69] which had a dampening effect on research of neuron-like networks within the anificial intelligence community. There are several differences between the work cited above and theirs. Minsky and Papel1 studied synchronous constructions and there is no evidence for such in real neural networks. The analysis of constructions with strong feedback that are now being considered also proved to be intractable with the analytic tools available at the time. A major missing tool was our present understanding

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4Early refers to early in the processing chain.

5It is interesting to note that Marroquin mentions the possibility of using quantum computers for the rapid reconsbUction of piecewise constant functions.

6Simulated annealing [Kirkpatrick 83] is a method of optimization that "cools down" a system by lowering an application specific system lttemperature" by a schedule in analogy with the annealing of metals. This gradual cooling allows the system to escape from local minima. Simulated annealing has been used for several optimization problems such as VLSI design, traveling salesman, and scheduling. Ackley et aI. [Ackley 85] have used this approach for more general computation in their so-called Boltzmann Machine. The Boltzmann machine is an example of a *connectionist* architecture. These architectures are massively parallel machines with simple computing elements. See the January-March 85 issue of Cognitive Science for several papers dealing with connectionist models of mind and machine.

7These models view atoms as having two states: up or down. Atoms can interact with a local neighborhood of other atoms and also with a global magnetic field. In such a system there are two macroscopic states: the ferromagnetic state and antiferromagnetic state. Interesting behavior is present in these systems in their phase transitions and wOOn nonunifonn fields are used.

of nonlinear dynamics including chaotic behavior. Harth's [Hanh 83] paper is on the analysis of nonlinear dynamics of higher brain functions, and evidence for chaotic behavior emerging from neural networks. Hanh has also related the infonnation generating structure of neural networks to that of the excellent work of Shaw [Shaw 81] on how infonnation is generated and moved from the microscale to the macroscale by nonlinear systems. Guevara et ale [Guevara 83] have studied chaotic behavior at the neuronal level.

This brief overview indicates that a new understanding of nonlinear dynamics is helping researchers to go beyond the work on perceptrons in the 1960's.8 We have also seen that people are drawing useful parallels between neural network dynamics and the dynamics of ensemble physical systems, such as spin glasses. Researchers are going beyond synchronous constructions and considering asynchronous constructions because of the new analytical tools available. Progress in modeling neural networks is also influencing connectionist models of machine computation of artificial intelligence and motivating work on random differential equations [Geman 79] and Markov Random Fields (MRFs) [Geman 84].

3.2. Quantum Mechanics and Quantum Computers

Erber and Puttennan [Erber 85] have proposed an experiment to detennine whether quantum systems are pseudorandom or random. They entertain the possibility of a physical basis for interpreting the randomness of quantum mechanical variables in tenns of the mixing which is generated by the detenninistic iteration of some appropriate function. This line of reasoning and the potential to test it is the result of research in algorithmic complexity theory [Chaitin 75], the ability to isolate single atoms [Dehmelt 83], and nonlinear systems and their simulation [Erber 83].

3.2.1. The Quantum Computer; Too Powerful?

Part of the motivation for suggesting that quantum mechanical behavior may be due to a chaotic nonlinear system is based on the startling paper by Deutsch (Deutsch 85] on Quantum Theory and the Church-Turing principle. Deutsch attached the computability of a Turing machine to physical reality in what he calls the Church-Turing principle:

> *Every finitely realizable physical system can be perfectly simulated by a universal model computing machine operating by finite means.*
> *-1.2 in [Deutsch 85J*

He argues that the more familiar fonnalization, known as the Turing principle, is vague:

> *Every "function which would be naturally regarded as computable" can be computed by the universal Turing machine.*
> *-11 in [Deutsch 85]*

compared to physical principles such as the third law of thennodynamics:

> *No finite process can reduce the entropy or temperature ofa finitely realizable physical system to zero.*
> *-1.3 in {Deutsch 85J*

He proposes to reinterpret the phrase "function which would be naturally regarded as computable" as "function which may be in principle be computed by a real physical system".

8IEEE Trans. on Systems Man and Cybernetics September-October 1983 is a special issue devoted to new theories of neural dynamics and computation.

This elevates the epistemological status of the Turing principle to a physical principle - as stated by the Church-Turing principle. Deutsch's thinking has been shaped by C. H. Bennett's interest in the physical limitations of computation [Bennett 85].[9]

Deutsch also shows that a quantum mechanical computer is more powerful than a Turing machine in its ability to generate arbitrary length random sequences. The tie between random sequences and the complexity of the algoritluns to compute them comes from the work on algorithmic complexity theory by Chaitin, Kolmogorov, and Solomonoff [Chaitin 75]. Algoritlunic complexity theory has been a very useful development, and along with chaotic systems, has led to a new understanding of the relationship of determinism and randomness in nature.[10] This theory says that a random sequence is one where the number of bits used to describe how to generate it (say by a program) is about the same as the number of bits used to represent the sequence explicitly.

We see from this that:

> *In this respect, quantum mechanics implies that even the simplest physical systems are equivalent to algorithms and computers of unbounded complexity.*
> • *[Erber85}*

The question arises; Maybe this is unrealistic? Hence the motivation to fmd a new interpretation of quantum mechanics based on pseudorandom processes.

3.2.2. The Ability to Isolate Single Atoms

The ability to test the hypothesis of mixing transformations are the underlying dynamics of quantum mechanics depends on the ability to isolate single atoms. Tests which mix the statistical behavior of a collection of atoms with the statistical behavior of individual atoms make it hard to recover any pseudorandom signature. Erber and Puttennan [Erber 85] suggest a double resonance technique to measure the on/off time of an isolated ion's fluorescence. Based on Erber's prior work in characterizing the properties of pseudorandom sequences that are generated by computers. and assuming that an isolated ion behaves like a computer in that it it has access only to a finite number of states. they derive a relation that predicts both the number of cycles after which one would expect the system to fall into a tenninalloop and how large that loop would be as a function of the number of states. The work by Erber and cohorts may lead us to adopt quantum mechanics as a *mesoscopic* description of nature, if the suggested experiment confums pseudo-random behavior at the quantum level.

A reason for the nonlinearity of the underlying mechanism is yet to be proposed. However. it is interesting to speculate that the nonlinearity arises from random time skews in the propagation of fields at the scale of quantum mechanical systems resulting from the fractal sponge of

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[9] A new line of reasoning is developing; since computation bas to be carried out on something physically realizable, some questions of computability may have interesting pbysical consequences (and .vice versa). as is currently being shown. Omohundro [Omohundro 84] mentions the question of the possibility of a closed fonn solution of a set of Nonlinear PDEs that describe a family of cellular automata as being related to the unsolvability of the halting problem.

[10] For a thought provoking discussion on this subject see [Ford 83].

Wheeler space-time.[11] I suggest that, in any case, a study of chaotic systems due to random time delays in the iterates of a mixing transfonn might lead to interesting results. Possibly one might adopt a mesoscopic view of computation that shares many of the principles of quantum mechanics. This mesoscopic scale would cover small clusters of computation interacting asynchronously.. In limited experiments I have done with asynchronous simulation, accomplished by relaxing communication precedence, some quantum effects do arise.[12]

4. Conclusion

From the examples surveyed, we have seen that the birth and maturation of computers and computer science has added a new member to the traditional team of mathematics and natural philosophy. Developments in one field have been beneficial to the others, and there are many examples in addition to those discussed above. These include:

- The relationship between cellular automata, fractals, and nonlinear dynamics. Wolfram [Wolfram 84] has been using cellular automata to model physical processes that generates materials with fractal geometry. Omohundro [Omohundro 84] has modeled cellular automata as nonlinear differential equations. Orbach [Orbach 86] has studied the dynamics of fractal networks.
- A more detailed study of simulated annealing and genetic algorithms[13] applied to vision processing, boltzmann machines, traveling salesman, scheduling, other optimization problems and the relationship between equilibrium systems and optimization [Kirkpatrick 85].
- Statistical Mechanics applied to the study of distributed systems [Yemini 83]. This offers a macroscopic description of network dynamics that one does not get with traditional methods of analysis such as queueing theory.
- The thennodynamics of computation [Bennett 85]. People have studied ways to make computation reversible [Fredkin 82] and to embed irreversible computation in a reversible cellular automata [Toffoli 77].
- The use of computers to study nonlinear systems [Campbell 85]. Much of what we know of nonlinear dynamics comes from studies on the computer.

As these examples illustrate, computer science has begun and will continue to feed research in the natural sciences and mathematics. Computer science in tum is adopting new views of computation based on how natural science has described very large ensembles and very small individuals. As was shown, these new views have used recent results in mathematics.

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[11] For a readable introduction on the notion of a space time sponge see [DeWitt 83].

[12] One powerful idea to be exploited in such an organization of computation is the fact that different population statistics can either wash out or carry mesoscopic quantum effects at the macro level' in analogous ways' that quantum effects are either averaged-out as in everyday materials or carried upward to the macroscale as in superconductors and superfluids.

[13] Geoetic algorithms take inspiration from genetic mechanisms in biological systems and are used for optimization type problems. [Holland 78]

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