The Golden Rule of Agile Estimation: Fibonacci Story Points

Arjun Kudinoor

July 19, 2023

Abstract

In the agile methodology of project management, software development teams often use an idea called story points to quantify the effort it takes to complete user stories, like a feature or an application. In this note, different systems of assigning story points to stories are explored. These systems include linear, exponential, combinatoric, and Fibonacci-based scales of story points. These scales are benchmarked against a toy model of squares generated using the Fibonacci sequence. Finally, a connection between the Fibonacci-based story point system and the golden ratio is derived. The implications of this connection to our understanding of effort in stories are explained.

Contents

1 Introduction 2

2 A toy model for measuring effort 2

3 Story points and other measures of effort 4

3.1 A linear measure of effort 4

3.2 An exponential measure of effort 4

3.3 A combinatoric measure of effort 6

3.4 A Fibonacci measure of effort 8

4 References 10

1 Introduction

In the agile methodology of project management, software development teams often use an idea called **story points** to quantify the effort it takes to complete user stories, like a feature or an application. The factors involved in the estimation of effort include the complexity of the story, the amount of work, and the risk/impacts of failure [1], [2]. The process by which an individual or a team assigns story points to a story is simple: simply choose a minimal-effort story as a baseline and assign it a point value of 1. Then, if another story requires more effort than that, assign 2 story points to it. If there's a story that requires even more effort, assign more than 2 story points to it, and so on.

Remark 1.1. It is important to note that story points are an estimation of the effort required to complete a story, and are not a measure of the time it takes to complete a story [3].

There are multiple scales of story points one can use. Your scale can be linear — so points follow a scale like 1, 2, 3, 4, ... They can also be exponential, like measuring effort in powers of 2 as 1, 2, 4, 8, 16, ... Another option for potential scales of story points are ones that are additive (like Fibonacci, which is semi-additive). This note explores various scales of story points and their relationships to the types of stories they describe.

In Section 2, we will build a toy model to understand how we quantify effort in stories. Then, in Section 3, we will explore different scales of story points, benchmark them against our toy model, and explain how they relate to the stories they describe. In particular, we will examine linear, exponential, combinatoric, and Fibonacci scales of story points. In the end, we will draw a surprising connection between Fibonacci story points and the famed golden ratio.

2 A toy model for measuring effort

To get a better understanding of story points and the effort it takes to complete a story, we begin with a toy model of squares. Begin by drawing a single square of side length 1. To its right, draw another square of side length 1 that sits flush with the first square. Then, draw a square of side length 2 that sits on top of both these squares. After which, draw a square of side length $2 + 1 = 3$ to the left of these squares. Finally, draw a square of side length $3 + 2 = 5$ below these squares. Continue repeating this process, as shown in Figure 1.

We can quantify the effort required to draw each new square as its side length. This makes sense because drawing the perimeter of a square requires that we draw lines a total length of 4 times the square's side length. If we normalize the perimeter by dividing by 4, a constant, we get the side length of the square itself. Thus,

$$\text{effort}(\square_n) = s_n,$$

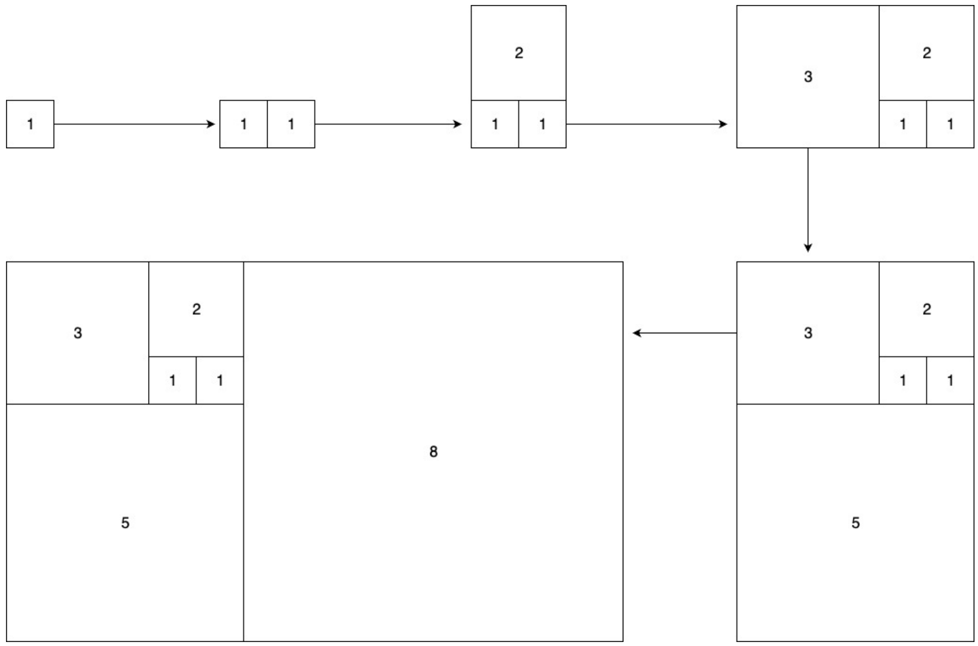

Figure 1: The value within each square denotes its side length.

where s_n is the side length of the n-th square $\square_n$, and n begins at 0. Specifically, both $\square_0$ and $\square_1$ have side length 1. We very quickly recognize that this effort function is simply the Fibonacci sequence.

Definition 2.1. Define the **Fibonacci sequence** $\{F_n\}$ recursively by

$$F_n = F_{n-1} + F_{n-2},$$

where $F_0 = F_1 = 1$. This sequence has values $F_0 = 1$, $F_1 = 1$, $F_2 = 2$, $F_3 = 3$, $F_4 = 5$, $F_5 = 8$, $F_6 = 13$, ...

Proposition 2.1. $\text{effort}(\square_n) = F_n$ for all $n \geq 0$.

Proof. We prove this by induction. Our base cases are $n = 0$ and $n = 1$, which immediately works:

$$\text{effort}(\square_0) = s_0 = 1 = F_0 \text{ and } \text{effort}(\square_1) = s_1 = 1 = F_1.$$

Now assume by way of induction that the proposition is true for some $n - 1$ and $n \geq 0$. Then, we calculate

$$\text{effort}(\square_{n+1}) = s_{n+1} = s_n + s_{n-1},$$

where the last equality follows by construction. By our inductive assumption, $s_n = \text{effort}(\square_n) = F_n$ and $s_{n-1} = \text{effort}(\square_{n-1}) = F_{n-1}$. Thus,

$$\text{effort}(\square_{n+1}) = s_n + s_{n-1} = F_n + F_{n-1} = F_{n+1}. \quad \square$$

Thus, the Fibonacci sequence is a good measure of the effort required to draw each square.

3 Story points and other measures of effort

Now that we have a good sense of how to quantify effort in our toy model of squares, we can explore other ways of quantifying effort for more general stories. In this section, we examine the relationship between stories and various scales of story points — in particular, linear, exponential, combinatoric, and Fibonacci scales.

3.1 A linear measure of effort

We return to our toy model of drawing squares as in the previous section. Suppose our story is to draw the draw the square $\square_n$ in the diagram, after all preceding n squares have been drawn. Suppose we quantify the effort for our story scales linearly with n. Most simply, suppose

$$\text{effort}(\square_n) = n.$$

This underestimates the actual effort required to draw a square because the perimeter of the square $\square_n$ does not scale linearly with n. Instead, the perimeter of squares increase with increasing n. So, effort cannot scale linearly with n.

Similarly, effort in more general and complex stories does not usually scale linearly with the complexity, amount of work, and risks involved in the story. Thus, a linear scale for story points is flawed.

Linear scale stories

More generally, stories the scale linearly (and independently) with complexity, amount of work, and risk would be best described using a story point scale that is linear.

Remark 3.1. A linear scale for story points is generally discouraged because most real-world stories do not scale linearly with complexity, work, and risk. Additionally, a story point scale of 1, 2, 3, 4, ... does not differentiate effort well enough between consecutive numbers. For example, two members on a team might rate a story with points 4 and 5. These points are so close that it is difficult to differentiate between them. Furthermore, the debate between which value is a better estimate of effort prolongs a consensus. In fact, this petty debate alone would add to the effort of the story, defeating the purpose of the debate.

3.2 An exponential measure of effort

Suppose we quantify the effort required to draw the n-th square as the exponential of some number — say 2 — by the power n, i.e.

$$\text{effort}(\square_n) = 2^n.$$

This takes into account the increasing size of squares. However, it overestimates the effort required. To understand how this is the case, we introduce another quantity called the consecutive effort ratio.

Definition 3.1. Let $\square_{n-1}$ and $\square_n$ be consecutive squares. Define the **consecutive effort ratio**

$$k_n(\square) = \frac{\text{effort}(\square_n)}{\text{effort}(\square_{n-1})}.$$

Here $\square = \{\square_n\}$ is the set of squares. The consecutive effort ratio measures how many times more effort it takes to draw a square when compared to the effort of drawing the previous one.

If we take an exponential measure of effort, $\text{effort}(\square_n) = 2^n$, then for all n,

$$k_n = \frac{\text{effort}(\square_n)}{\text{effort}(\square_{n-1})} = \frac{2^n}{2^{n-1}} = 2.$$

This means it takes twice as much effort to draw a square when compared to the previous one. An increase in effort by a factor of 2 seems excessive here. A reasonable question to ask is: Can we assign story points to our toy model of squares that follows an exponential measure of effort? Mathematically, for what number $\phi \in \mathbb{R}$ will

$$\text{effort}(\square_n) = \phi^n$$

be a good estimation of effort to draw the n-th square? We will answer this question in Remark 3.4 of Subsection 3.4.

Exponential scale stories
An exponential scale (or one that is scales even faster) for story points would work best for stories that scale exponentially with complexity, amount of work, and risk. Our toy model of squares does not do this, and so an exponential scale would overestimate the change in effort between successive stories.

Remark 3.2. A story point system that scales faster than an exponential one has the property that $k_n \to \infty$ as $n \to \infty$. This is because an exponential scale of story points is the fastest scale for which the consecutive effort ratios k_n converge — they converge immediately because they are always constant.

Let $\mathcal{S} = \{\mathcal{S}_n\}$ be an ordered set of stories sorted in increasing order of effort. Suppose we want to design a story point scale for which the consecutive effort ratios for $\mathcal{S}$ do not grow without bound, but rather converge. That is, we want $k_n \to k$ for some constant k. We can further restrict what this constant should be — in particular, $k \geq 1$. This is because if $k_n \leq 1$, that would wrongly imply that the effort required to complete the more complex, more demanding, and riskier stories takes less effort when compared to the effort it takes to complete the story immediately preceding it in difficulty. This restriction allows us to define the following rule.

Definition 3.2. We call this the **rule of converging effort ratios**: Although effort($\mathcal{S}_n$) > effort($\mathcal{S}_{n-1}$), the consecutive effort ratios between stories must converge, i.e.

$$k_n \to k,$$

where $k \geq 1$ is a constant. In other words, this rule says that the marginal growth in effort between successive stories should converge.

There are two types of story sets that obey this rule. One is where k_n decreases monotonically towards a constant value. The other is where k_n does not decrease monotonically, but nevertheless converges. We examine examples of each of these story point scales in the following subsections.

3.3 A combinatoric measure of effort

A monotonically decreasing story point scale can be constructed as follows. Taking inspiration from [4], suppose we have an ordered set of stories $\mathcal{S} = \{\mathcal{S}_{n>1}\}$ that are described using a story point scale with

$$\text{effort}(\mathcal{S}_n) = \frac{n(n-1)}{2}.$$

Complexity and combinatorics

This scale is modeled after a simple problem in graph theory. More specifically, the number of connections between n nodes in a graph that are each connected to all other nodes is

$$\binom{n}{2} = \frac{n!}{(n-2)!2!} = \frac{n(n-1)}{2}.$$

Equivalently, this is the combinations of pairs you can form between n items. Connections between nodes of a graph are good heuristics for understanding the complexity of systems with multiple dependent components. Hence, combinatoric scales of story points, like 1, 3, 6, 10, 15, ..., would nicely reflect combinatoric scales of complexity in real-world stories.

Proposition 3.1. The sequence $\{k_n(\mathcal{S})\}$ for this set of stories is monotonically decreasing and converges to 1.

Proof. We first show that $\{k_n\}$ is monotonically decreasing.

$$\begin{aligned} k_{n+1} &= \frac{\text{effort}(\mathcal{S}_{n+1})}{\text{effort}(\mathcal{S}_n)} = \frac{n+1}{n-1} \\ &< \frac{n+1}{n-2} = \frac{n}{n-2} + \frac{1}{n-2} \\ &< \frac{n}{n-2} = \frac{\text{effort}(\mathcal{S}_n)}{\text{effort}(\mathcal{S}_{n-1})} = k_n \end{aligned}$$

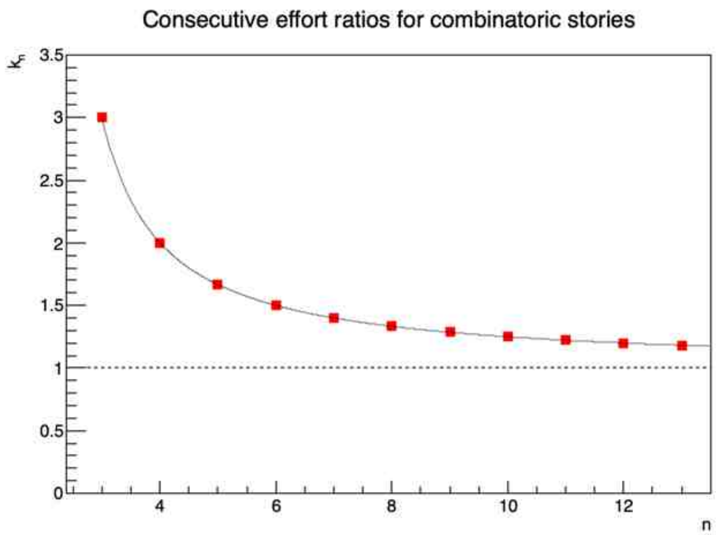

Figure 2: Consecutive effort ratios k_n for a set of stories that follows a combinatoric story point system of effort$(\mathcal{S}_n) = \frac{n(n-1)}{2}$.

Thus, $k_{n+1} < k_n$ for all $n > 1$, and $\{k_n\}$ is monotonically decreasing. To determine convergence, we observe the limiting behavior of k_n as follows.

$$\lim_{n\to\infty}(k_n) = \lim_{n\to\infty}\left(\frac{\text{effort}(\mathcal{S}_n)}{\text{effort}(\mathcal{S}_{n-1})}\right) = \lim_{n\to\infty}\left(\frac{n}{n-2}\right) = 1.$$

Thus, $k_n \to 1$. □

One can also see that $\{k_n\}$ decreases monotonically towards 1 graphically, as in Figure 2.

Remark 3.3. Since the combinatoric scale we have investigated here is monotonically decreasing, such a story point scale is useful when the marginal growth in effort for completing successively difficult stories decreases consistently.

Additionally, since k_n converges to the lowest possible constant — namely 1 — this story point scale yields ideal increases in effort with more difficult stories. More explicitly, in the large-n limit, there is almost no marginal increase in effort between stories.

Combinatoric scale stories
Thus, stories that are described using this combinatoric scale have decreasing growth in marginal effort, to the point where the marginal effort between extremely difficult stories is negligible.

What scales would describe stories that have larger marginal efforts between them? For these stories, the effort required to complete more complex, more demanding, and riskier stories increases even in the large-n limit. Our next task is to design a story point system whose consecutive effort ratios converge to a value greater than 1. A Fibonacci sequence-based story point system will be an ideal and rich candidate.

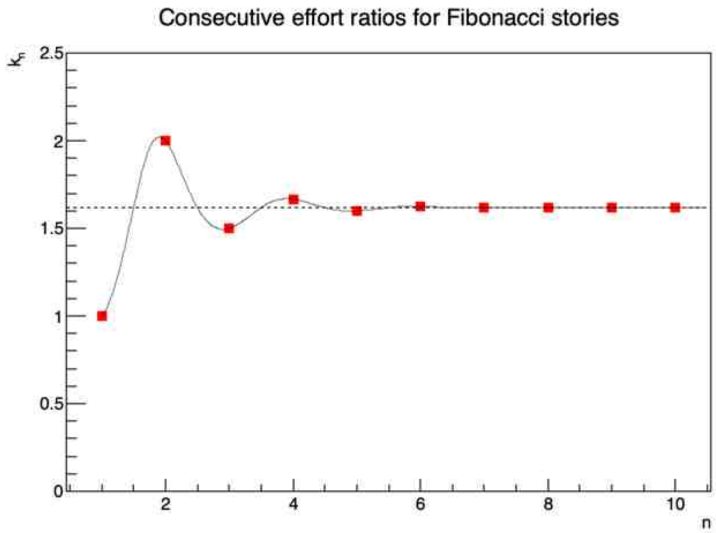

Figure 3: Consecutive effort ratios k_n for a set of stories that follows a Fibonacci story point system of effort($\mathcal{S}_n$) $= F_n$.

3.4 A Fibonacci measure of effort

Suppose, as in our toy model of squares, we have an ordered set of stories $\mathcal{S} = \{\mathcal{S}_n\}$ that are described using a story point system that is Fibonacci, i.e.

$$\text{effort}(\mathcal{S}_n) = F_n,$$

where F_n is the n-th number in the Fibonacci sequence. A Fibonacci-based scale is the most popular scale for evaluating story points in agile methodology today [5].

Such a story point system does not have a monotonically decreasing sequence of consecutive effort ratios, as can be seen in Figure 3. Instead, the consecutive effort ratios oscillate between a value of around 1.618. Mathematics enthusiasts might recognize this number as the GOLDEN RATIO!

Theorem 3.1. For an ordered set of story that follow the Fibonacci-based story point system above, their consecutive effort ratios converge to the golden ratio, i.e.

$$k_n \to \frac{1+\sqrt{5}}{2}.$$

Proof. The following proof will rely on the iterative nature of the Fibonacci sequence. Suppose we work in the large n-limit, i.e. we will assume that we can reduce n by integers arbitrarily without getting close to 0. We begin with the n-th consecutive effort ratio.

$$k_n = \frac{\text{effort}(\mathcal{S}_n)}{\text{effort}(\mathcal{S}_{n-1})} = \frac{F_n}{F_{n-1}} = \frac{F_{n-1} + F_{n-2}}{F_{n-1}},$$

where in the last equality we have used the fact that $F_n = F_{n-1} + F_{n-2}$. Simplifying and rearranging terms,

$$k_n = 1 + \frac{F_{n-2}}{F_{n-1}} = 1 + \frac{1}{\frac{F_{n-1}}{F_{n-2}}} = 1 + \frac{1}{\frac{F_{n-2}+F_{n-3}}{F_{n-2}}},$$

where in the last equality we have used the fact that $F_{n-1} = F_{n-2} + F_{n-3}$. We repeat the steps above in a similar fashion to get

$$k_n = 1 + \frac{1}{1 + \frac{F_{n-3}}{F_{n-2}}} = 1 + \frac{1}{1 + \frac{1}{\frac{F_{n-2}}{F_{n-3}}}} = 1 + \frac{1}{1 + \frac{1}{\frac{F_{n-3}+F_{n-4}}{F_{n-3}}}} = 1 + \frac{1}{1 + \frac{1}{1+\frac{F_{n-4}}{F_{n-3}}}}.$$

Since we are in the $n \to \infty$ limit, we can repeat this process ad infinitum to get

$$k_{n\to\infty} = 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+\dots}}}.$$

For simplicity, define $\phi = k_{n\to\infty}$. Then, we notice that ϕ is nested within the first denominator of the infinite fraction on the right hand side of the above equation. This observation yields

$$\phi = 1 + \frac{1}{\phi}.$$

Solving for ϕ gives us a quadratic equation $\phi^2 = \phi + 1$, whose solutions are

$$\phi = \frac{1 \pm \sqrt{5}}{2}.$$

Since $k_n \geq 0$ for all $n \geq 1$, we choose the positive solution for ϕ. Therefore, as $n \to \infty$,

$$k_n \to \phi = \frac{1 + \sqrt{5}}{2}.$$

□

Remark 3.4. We note that the sequence of consecutive effort ratios converge quite fast to ϕ in Figure 3. So, even in the medium-n limit, k_n seems to be roughly constant. Constant consecutive effort ratios are associated with exponential story point systems. Therefore, the answer to the question asked in Subsection 3.2 of whether we can describe the effort to draw squares in our toy model using an exponential scale is yes. The exponential effort function would be

$$\text{effort}(\square_n) = \phi^n,$$

where ϕ is the golden ratio. Since such a function would involve decimal values it would be unwieldy in practice. Furthermore, the spacing between points in such a golden ratio-based exponential scale would be similar to the spacing between points in the Fibonacci story point system. So, such a decimal-ridden exponential point system provides no additional value over the simpler, integer-valued, Fibonacci story point system.

Fibonacci scale stories

The Fibonacci scale for determining story points is the richest we have observed so far. The stories that follow this scale demonstrate the following:

1. A more-than-linear increase in effort when complexity, amount of work, or risk is increased,
2. An inconsistent change in marginal effort between stories because k_n is not monotonic but rather oscillates around $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$,
3. A constant nonzero marginal growth in effort between stories in the high complexity, high work, and high risk (large-n) limit because $k_n \to \phi > 1$,

The third point implies a **golden rule of agile estimation**: The consecutive effort ratios between stories converge to the golden ratio when measuring stories using a Fibonacci story point system. More simply, a most difficult story takes a golden ratio times more effort to complete than the story one-level below it in difficulty.

4 References

[1] Dan Radigan. *Story points and estimation*. URL: https://www.atlassian.com/agile/project-management/estimation#:~:text=Story%20points%20are%20units%20of,work%2C%20and%20risk%20or%20uncertainty.

[2] Mike Cohn. *Story Points Estimate Effort Not Just Complexity*. June 2010. URL: https://www.mountaingoatsoftware.com/blog/its-effort-not-complexity.

[3] Mike Cohn. *Don't Equate Story Points to Hours*. Nov. 2022. URL: https://www.mountaingoatsoftware.com/blog/dont-equate-story-points-to-hours.

[4] Clever Neologism. *Why would teams use the Fibonacci sequence for story points?* Apr. 2014. URL: https://pm.stackexchange.com/a/11369.

[5] Lucid content team. *Using the Fibonacci scale in Agile estimation*. URL: https://www.lucidchart.com/blog/fibonacci-scale-for-agile-estimation.