Entanglement, Teleportation, and Quantum Computing

Arjun Kudinoor

Instructions to code along with me are at the QR Code or at
arjunkudinoor.com/quantum

You will need to create an IBM Quantum account at
quantum-computing.ibm.com

Society of Physics Student | Columbia University
October 27, 2022

TOPICS

1) Entanglement and the EPR Paradox
2) 2-Photon Entanglement Experiment
3) Entanglement on a Quantum Computer
4) Teleportation - An Application of Entanglement

ENTANGLEMENT

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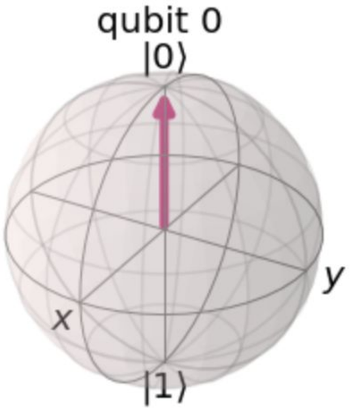
QUBIT ENTANGLEMENT

2-level Quantum State: $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$



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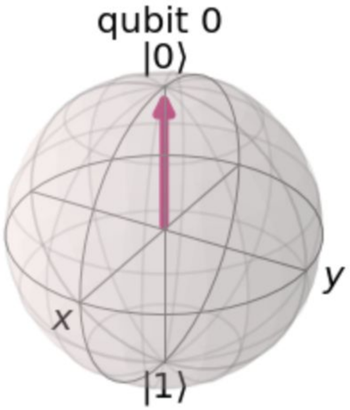

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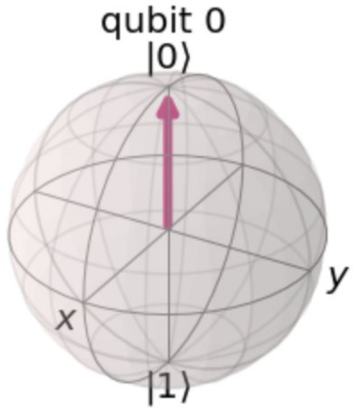

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A 2-qubit quantum state is entangled if it cannot be separated into two distinct qubit states

Example: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq |a\rangle \otimes |b\rangle$

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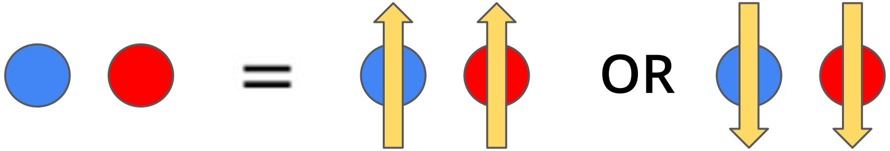
EPR PARADOX

1) PREPARE AN ENTANGLED PAIR OF PARTICLES



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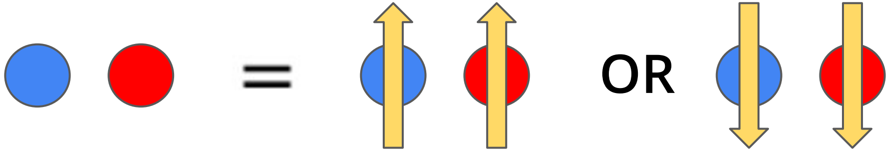

2) SEPARATE THEM BY A LARGE DISTANCE

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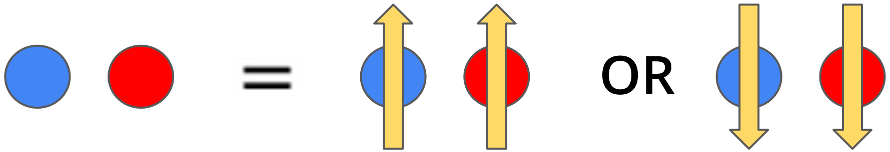

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3) MAKE A MEASUREMENT ON ONE PARTICLE

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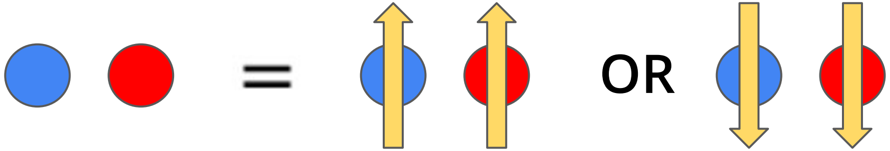

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EPR PARADOX

- Quantum Mechanics violates the principle of locality
- To reconcile this, Einstein, Podalsky, and Rosen theorize the existence of "hidden variables"
- Hidden variables predetermine the states of entangled particles before a measurement is made - the states are just hidden from us

BELL'S COUNTERARGUMENT

In 1964, John Bell published a paper in response to EPR's hidden-variable reconciliation of the EPR paradox. Bell's paper showed that the hidden variable description of quantum mechanics was incorrect and that quantum mechanics was inherently non-local. This was done by examining the correlation between measurements of entangled particles.

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A hidden variable theory would predict correlations with certain bounds. A violation of these bounds would demonstrate that quantum mechanics is non-local via entanglement.

2-PHOTON ENTANGLEMENT
THE EXPERIMENT

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2-PHOTON ENTANGLEMENT
THE EXPERIMENT

Nobel Prize, 2022
Aspect, Clauser, Zeilinger

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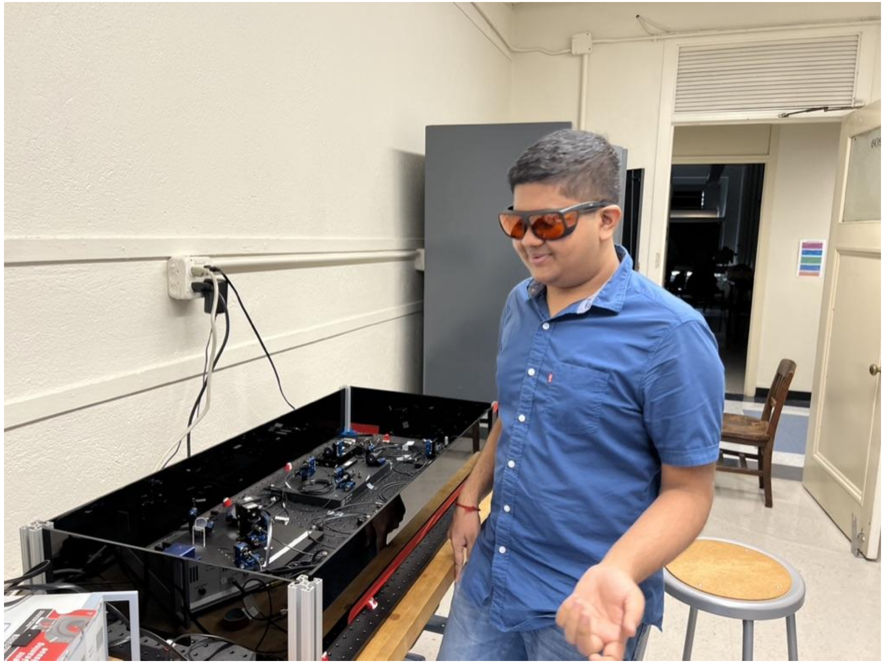
YOU CAN DO THIS AT COLUMBIA!



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ENTANGLEMENT SETUP

In QuTools, our first step is to create a pair of entangled photons.

We perform entanglement on the polarization property of our photons with the aid of two spontaneous parametric down-conversion crystals.

$$|H\rangle_p \rightarrow |V\rangle_1|V\rangle_2$$
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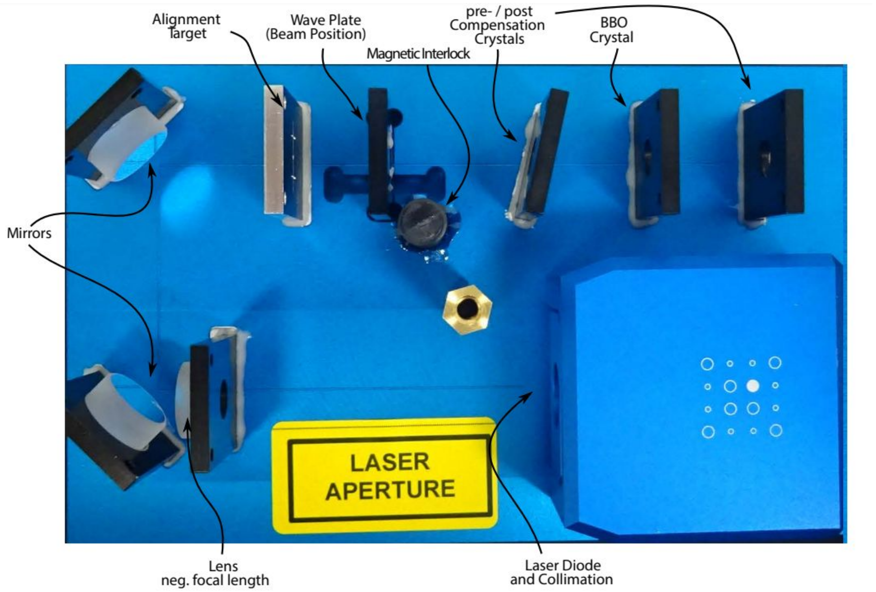

Figure 1: Inner details of laser pump assembly (quED manual)

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Sending in pump light in an equal superposition:

$$\frac{1}{\sqrt{2}}(|H\rangle_p + |V\rangle_p) \rightarrow \frac{1}{\sqrt{2}}(|V\rangle_1 |V\rangle_2 + |H\rangle_1 |H\rangle_2)$$

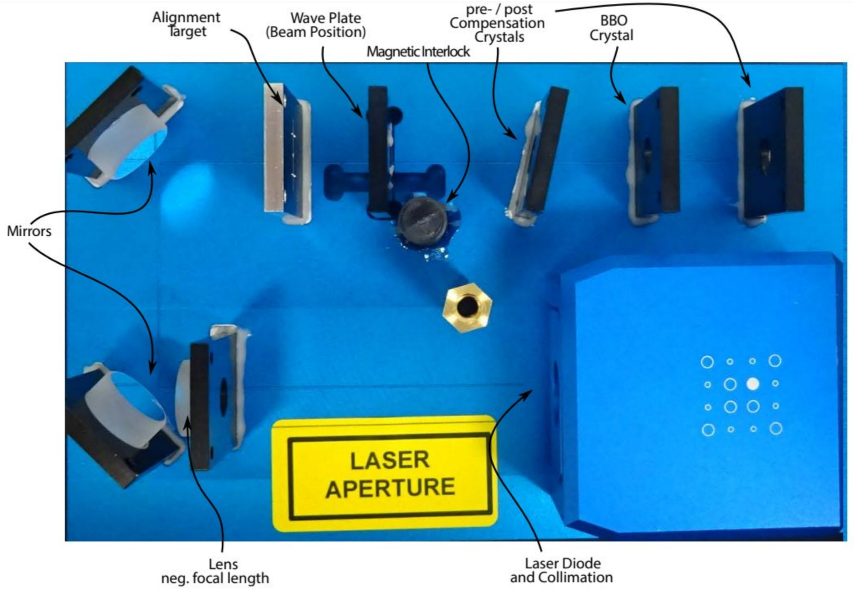

Figure 1: Inner details of laser pump assembly (quED manual)

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POLARIZATION CORRELATION

We now measure the photons' polarization in different (rotated) bases $\langle \pm_\alpha |$

The probability of measuring the polarization in this basis is

$$P_{\pm\pm}(\alpha, \beta) = \left| \langle \pm_\alpha | \langle \pm_\beta | | \psi \rangle \right|^2$$

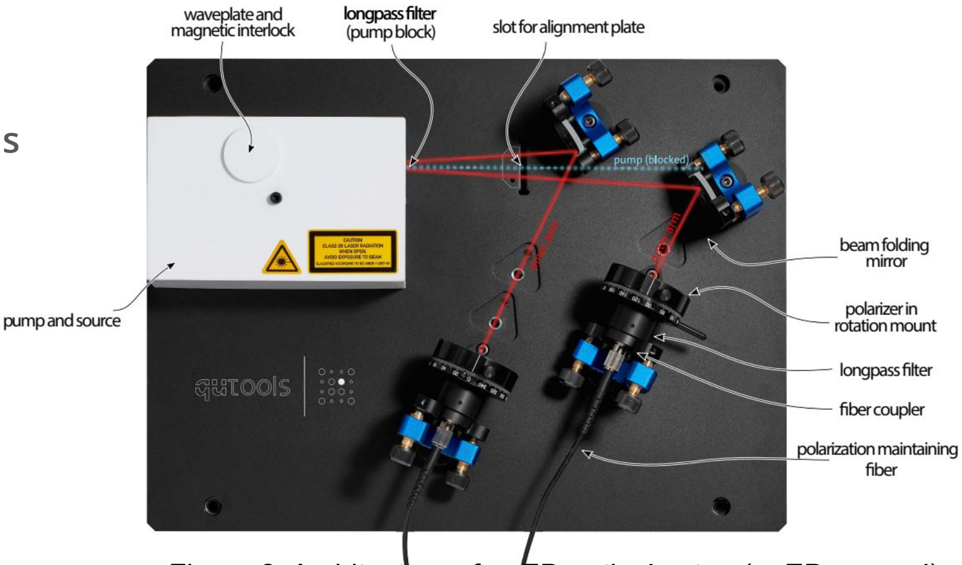

Figure 2: Architecture of quED optical setup (quED manual)

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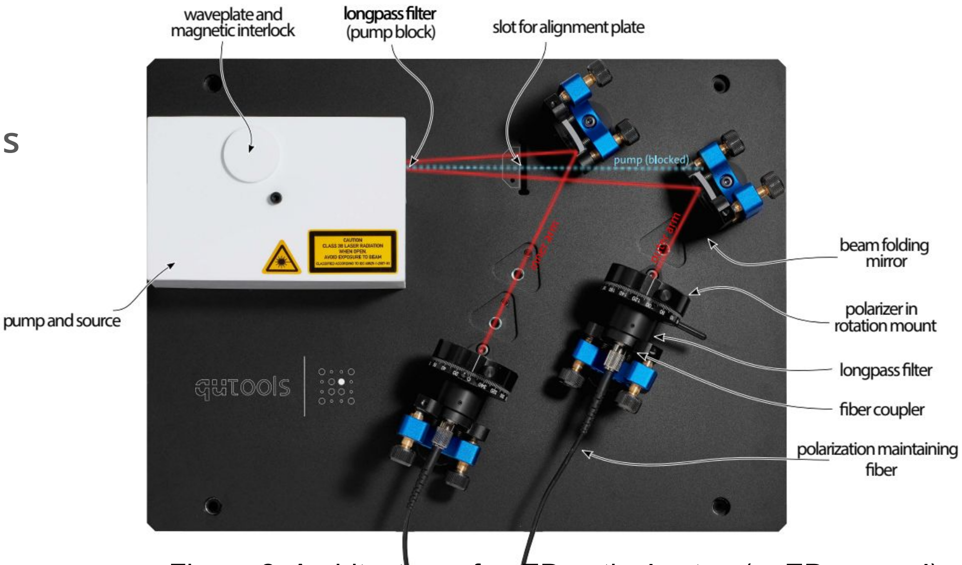
We may use this probability to define a correlation

$$\begin{aligned} E(\alpha, \beta) &= P_{++} + P_{--} - P_{+-} - P_{-+} \\ &= \cos(2(\alpha - \beta)) \quad \text{[theory]} \end{aligned}$$



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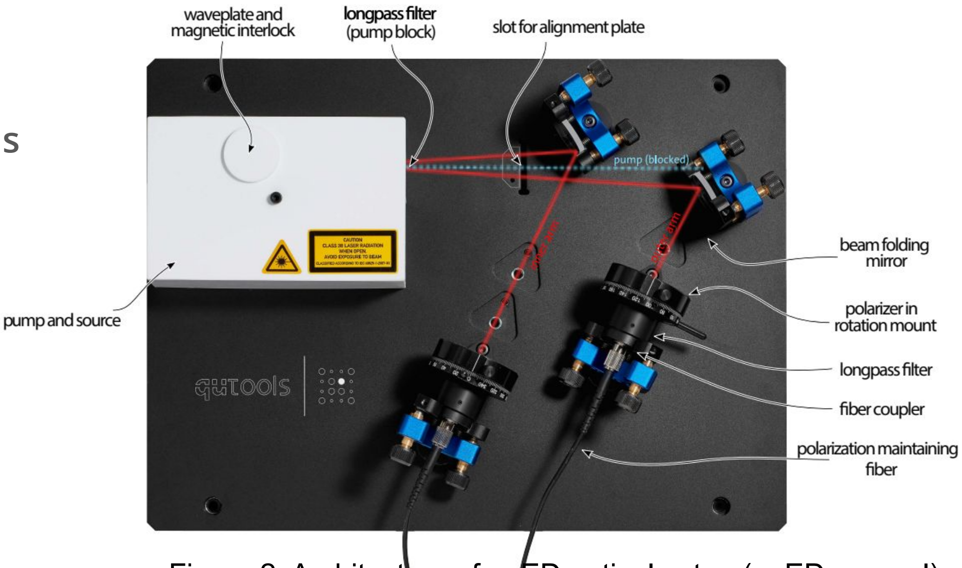
$$E(\alpha, \beta) = P_{++} + P_{--} - P_{+-} - P_{-+}$$
$$= \cos(2(\alpha - \beta)) \quad \text{[theory]}$$

We can use this to define a second measure of correlation

$$S(\alpha, \alpha', \beta, \beta') = E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta')$$
$$= \cos(2(\alpha - \beta)) - \cos(2(\alpha - \beta')) + \cos(2(\alpha' - \beta)) + \cos(2(\alpha' - \beta'))$$



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THE PUNCHLINE

A local theory of quantum mechanics - one with EPR's hidden variables - predicts bounds on the correlation S. In particular,

$$|S| \leq 2$$

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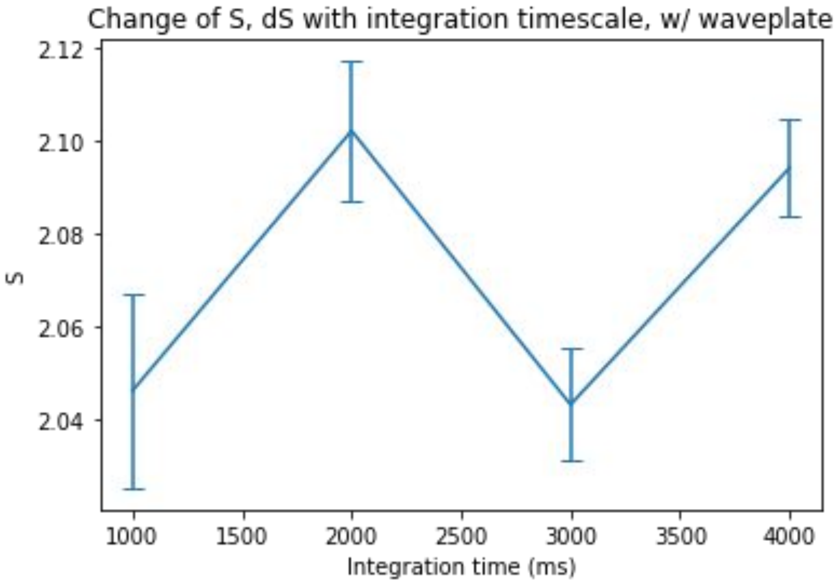
$$|S| \leq 2$$

IF we measure correlations S larger than 2 in our experiment, then Bell's inequality is violated, and entanglement is non-local...

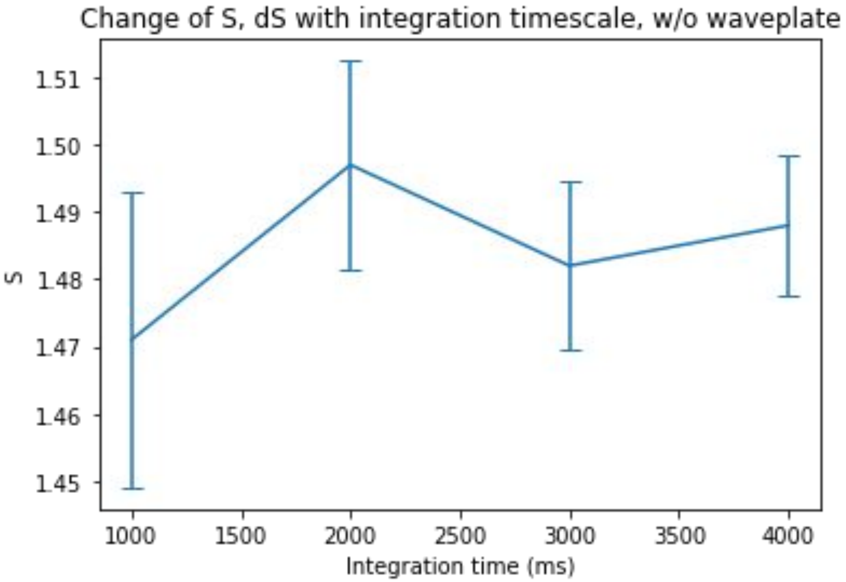
RESULTS

Plots of S, with error (Is it >2 for entangled?!)

Entangled Photons



Non-Entangled Photons



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EXPERIMENT SUMMARY

- Entangle photons displayed values of S>2 consistently
- Unentangled photons displayed values of S<2
- Errors in S (and lack of smoothness in correlation plot) caused by background noise from the environment, and laser misalignment

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Quantum Mechanics is non-local

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ENTANGLEMENT ON A QUANTUM COMPUTER

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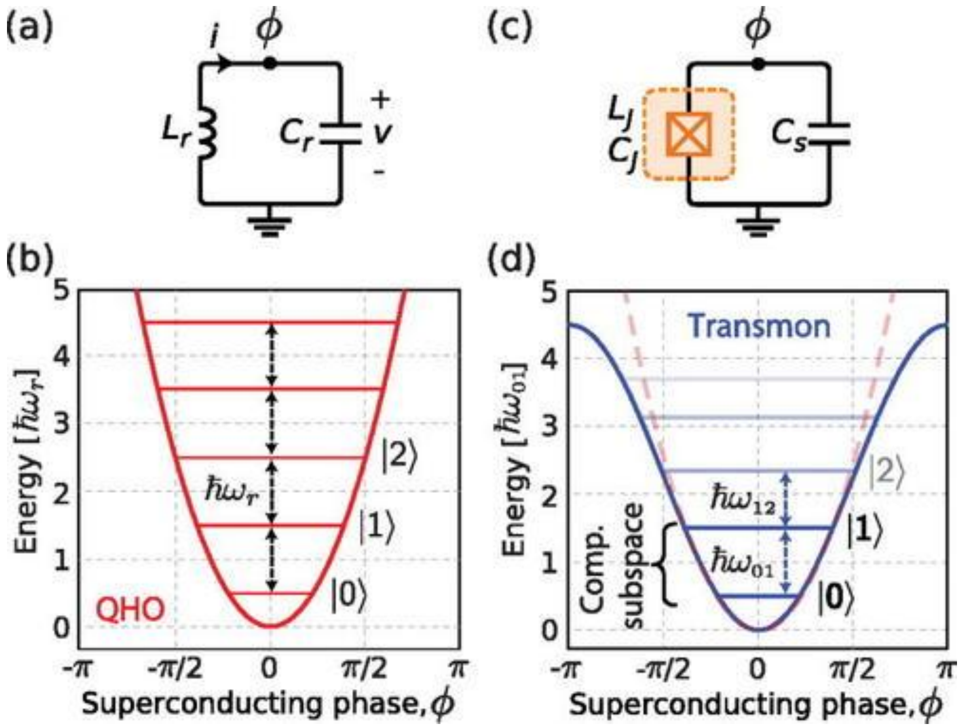
PHYSICAL IMPLEMENTATION

We will encode quantum circuits on IBM's cloud quantum computing platform, Qiskit.

IBM's circuits are based on superconducting qubits called "transmons". They use the first two levels of a quantum anharmonic oscillator.



Krantz, Philip, et al. "A quantum engineer's guide to superconducting qubits." Applied Physics Reviews 6.2 (2019): 021318.

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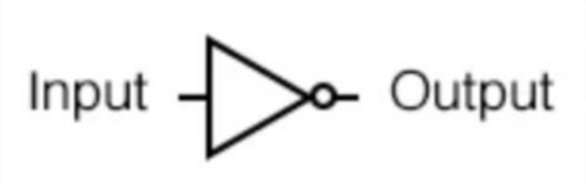
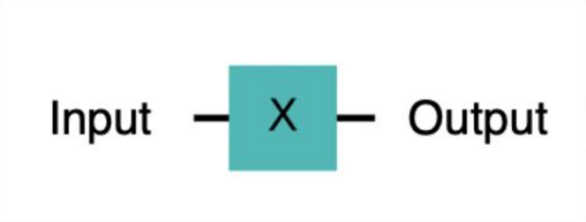
QUANTUM GATES

Classical Bits	Qubits
Boolean logic (0 or 1)	$\lvert\psi\rangle = c_0\lvert 0\rangle + c_1\lvert 1\rangle$
NOT GATE	X GATE
	

INPUT	OUTPUT
A	NOT A
0	1
1	0

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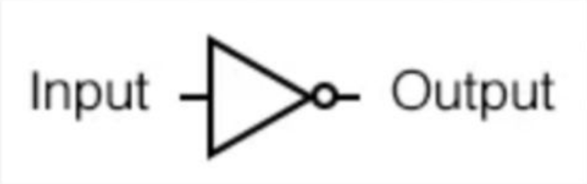

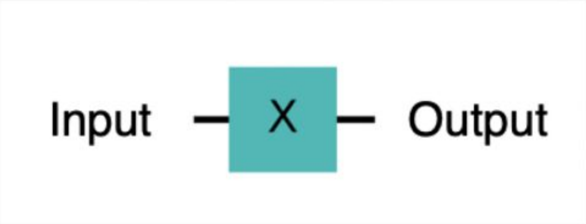

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$$\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

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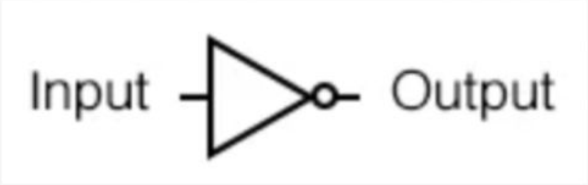	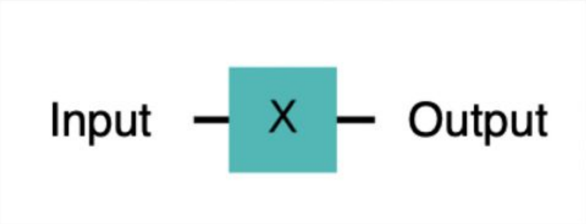

INPUT	OUTPUT
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$$\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

These are the Pauli Matrices!

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GATES RELEVANT TO ENTANGLEMENT

THE HADAMARD GATE (SINGLE QUBIT)

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|0\rangle \rightarrow |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|1\rangle \rightarrow |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

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THE CNOT GATE (TWO QUBITS)

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\text{CNOT}_{01}|ij\rangle = \text{CNOT}\,(|i\rangle \otimes |j\rangle) = |i\rangle \otimes |(i+j)\,\text{mod}(2)\rangle$$

Before		After	
Control	**Target**	**Control**	**Target**
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

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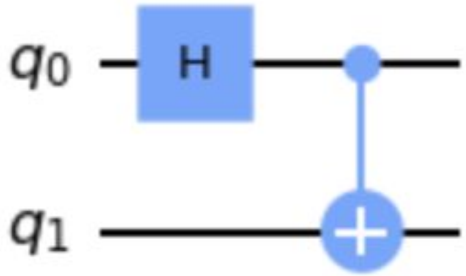
ENTANGLEMENT CIRCUIT

TWO-QUBIT STATE: $|q_0 q_1\rangle$



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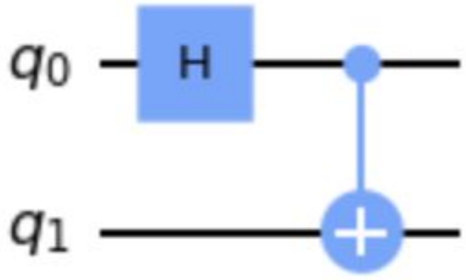
ENTANGLEMENT CIRCUIT

TWO-QUBIT STATE: $|q_0 q_1\rangle$

$$\mathrm{CNOT}_{01} H_0 |00\rangle = \mathrm{CNOT}_{01} | +0\rangle$$



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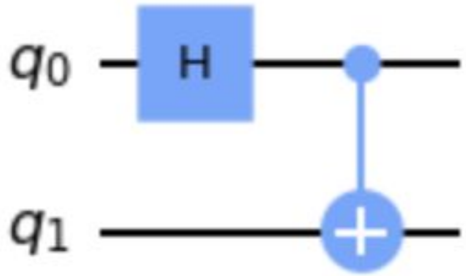

$$\begin{aligned}\mathrm{CNOT}_{01} H_0 |00\rangle &= \mathrm{CNOT}_{01} |+0\rangle \\ &= \frac{1}{\sqrt{2}} \mathrm{CNOT}_{01} (|00\rangle + |10\rangle)\end{aligned}$$

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ENTANGLEMENT CIRCUIT

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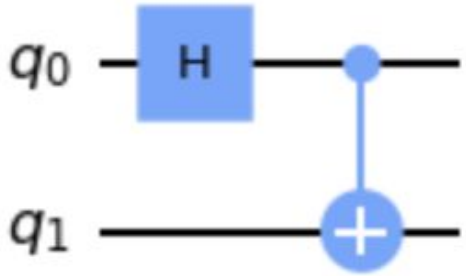

$$\text{CNOT}_{01} H_0 |00\rangle = \text{CNOT}_{01} |+0\rangle$$
$$= \frac{1}{\sqrt{2}} \text{CNOT}_{01} (|00\rangle + |10\rangle)$$
$$= \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

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LET'S ENCODE IT ON QISKIT!

TELEPORTATION FOR THE MASSES

(QUANTUM TELEPORTATION WITHOUT EQUATIONS!)

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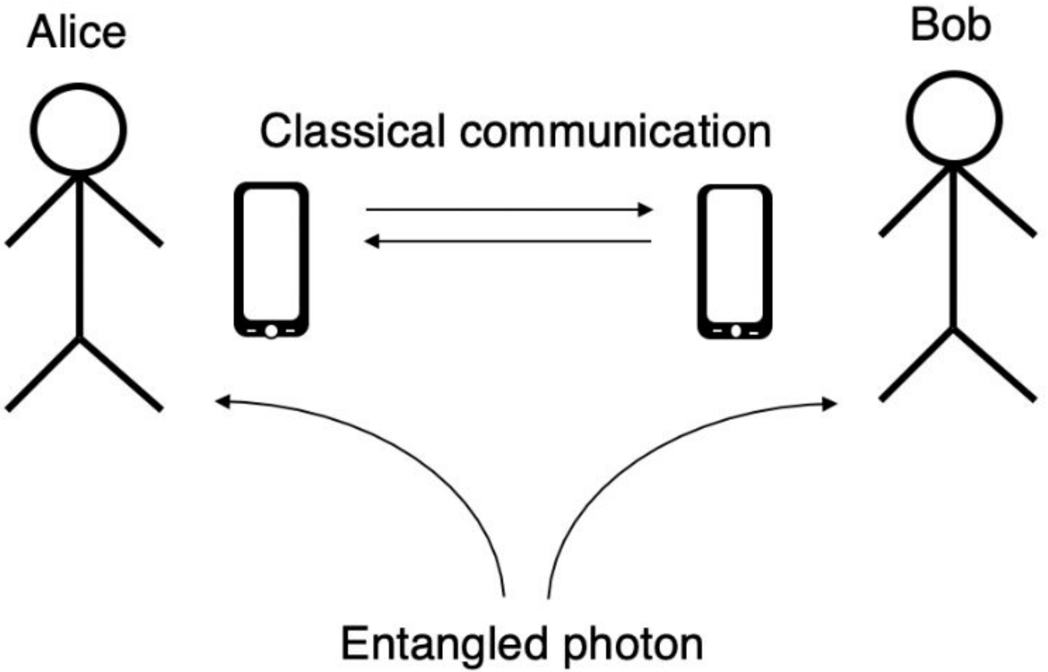
QUANTUM TELEPORTATION (REALLY INFORMATION TRANSFER)



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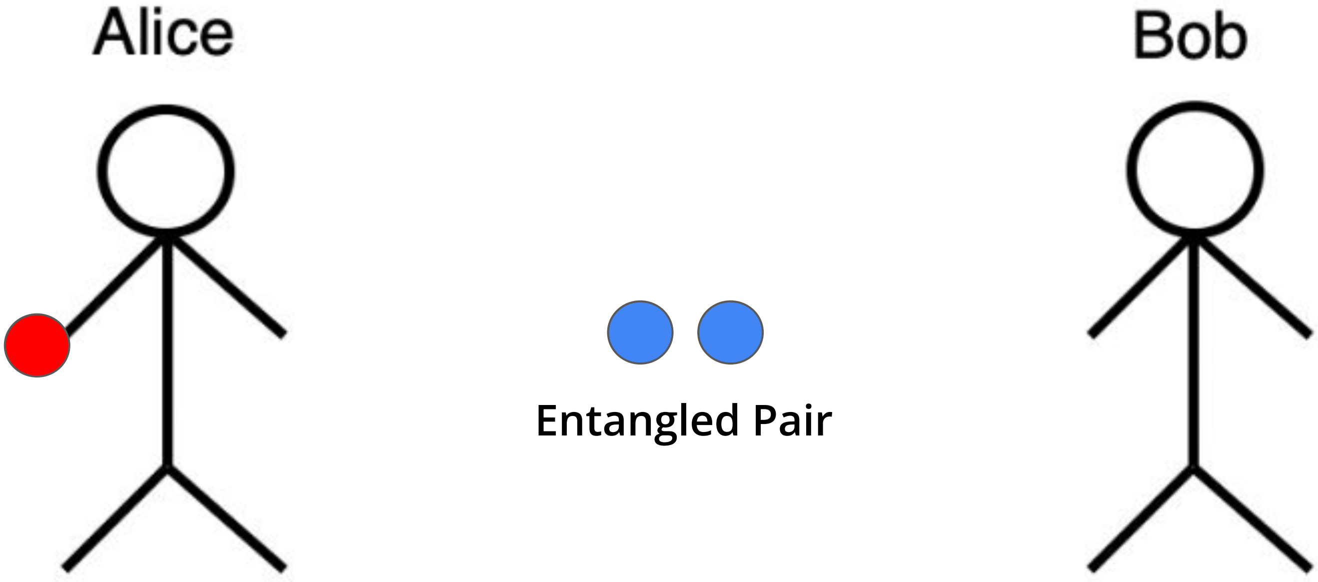
STATE PREPARATION



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SHARED ENTANGLED STATE

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ALICE ENTANGLES HER QUBITS

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(INCOMPLETE) INFORMATION IS TRANSFERRED TO BOB VIA ENTANGLEMENT

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ALICE MAKES A MEASUREMENT AND RELAYS IT TO BOB

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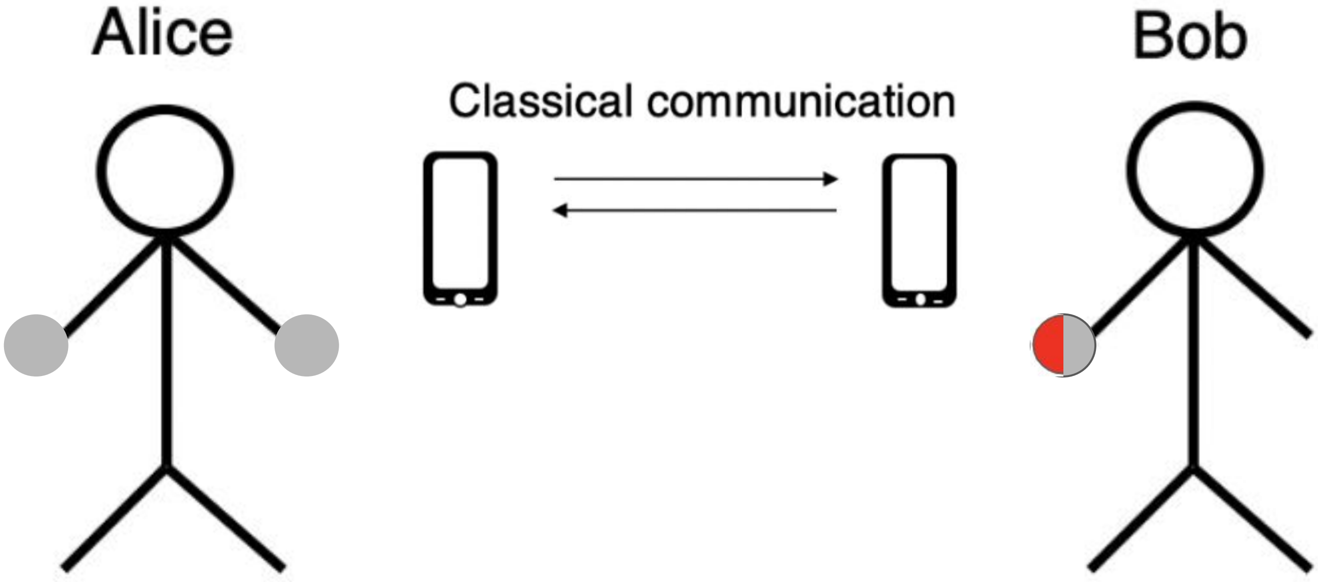
ALICE MAKES A MEASUREMENT AND RELAYS IT TO BOB



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BOB APPLIES CONDITIONAL GATES ON HIS QUBIT

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BOB RECOVERS ALICE'S ORIGINAL QUBIT

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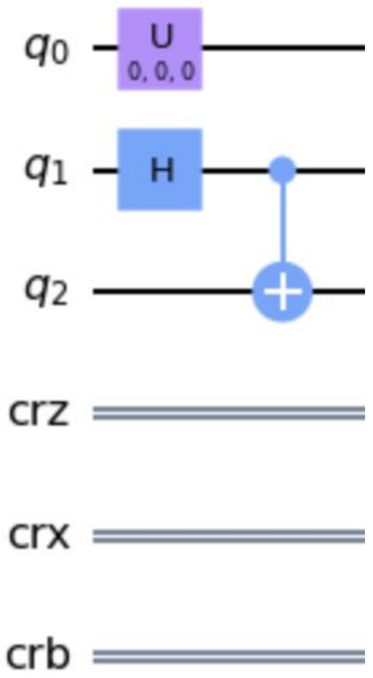
WHAT JUST HAPPENED?

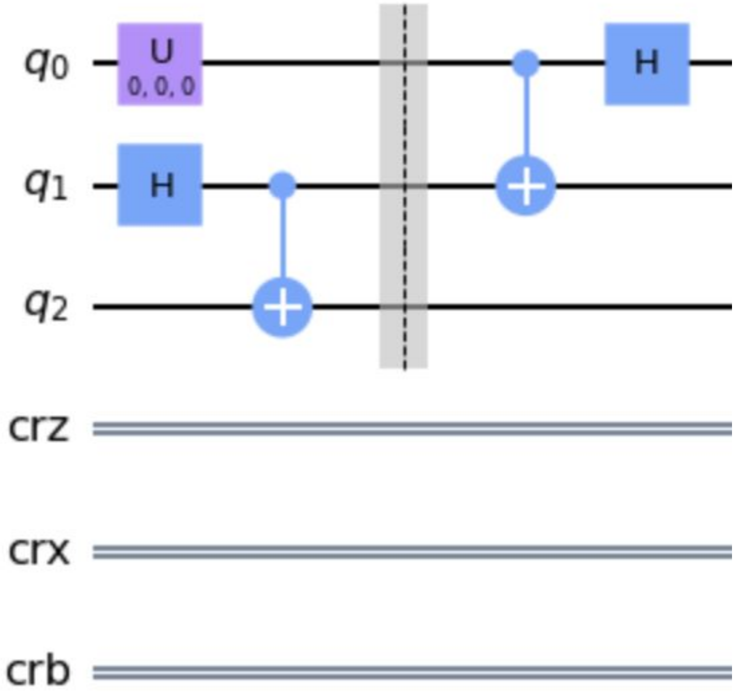
- Alice transferred the quantum information in her qubit to Bob
- Neither Alice nor Bob needed to know what the original state was
- Quantum information about states was never known by any party
- Classical information transfer was necessary - compatible with special relativity
- Alice's original state was completely transferred, NOT cloned onto Bob's qubit

TELEPORTATION CIRCUIT



State Preparation

TELEPORTATION CIRCUIT



Alice Entangles her qubit
with one entangled qubit

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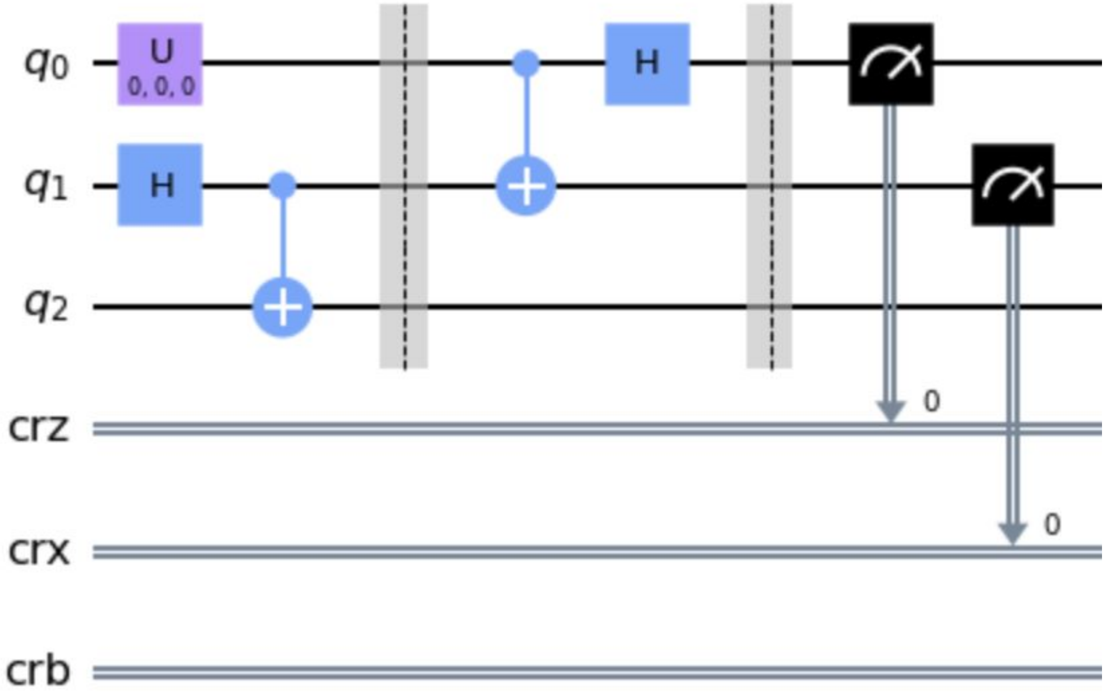
TELEPORTATION CIRCUIT



Alice makes a measurement and relays it to Bob

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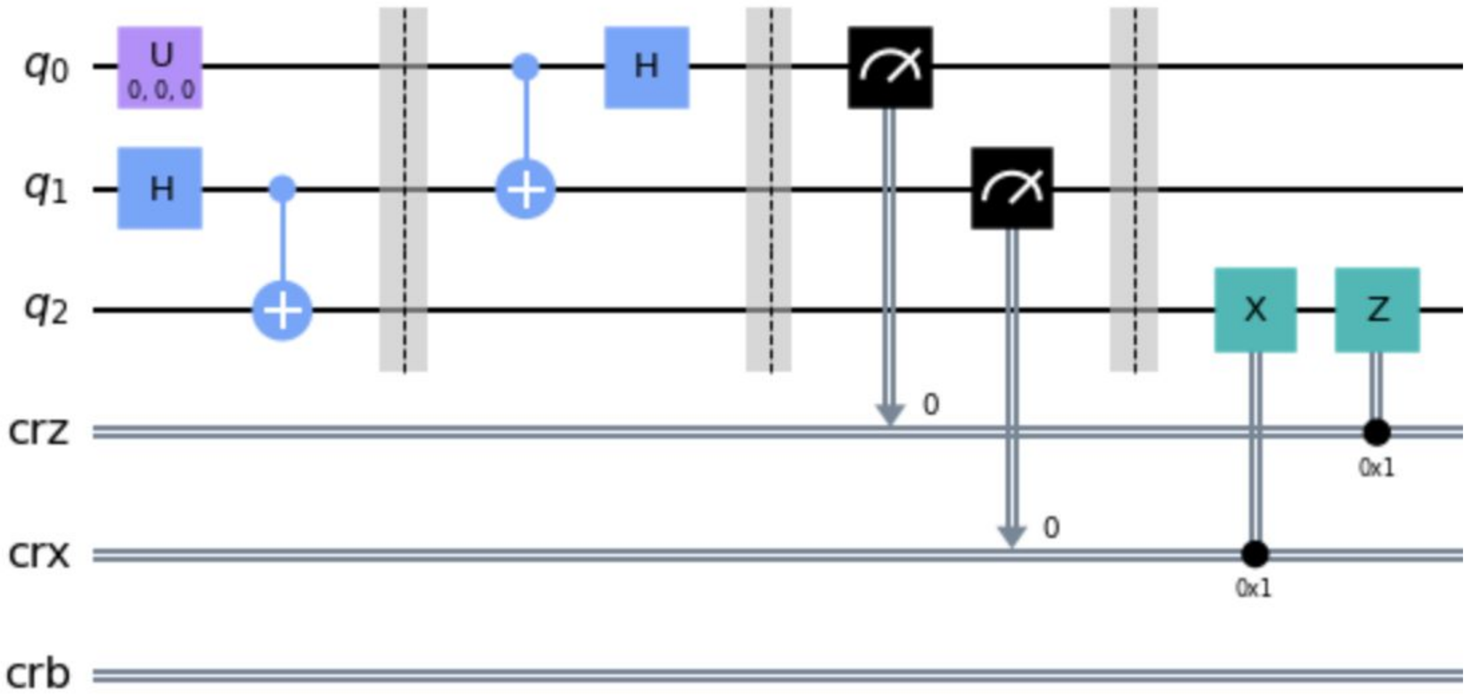
TELEPORTATION CIRCUIT



Bob applies conditional gates on his qubit

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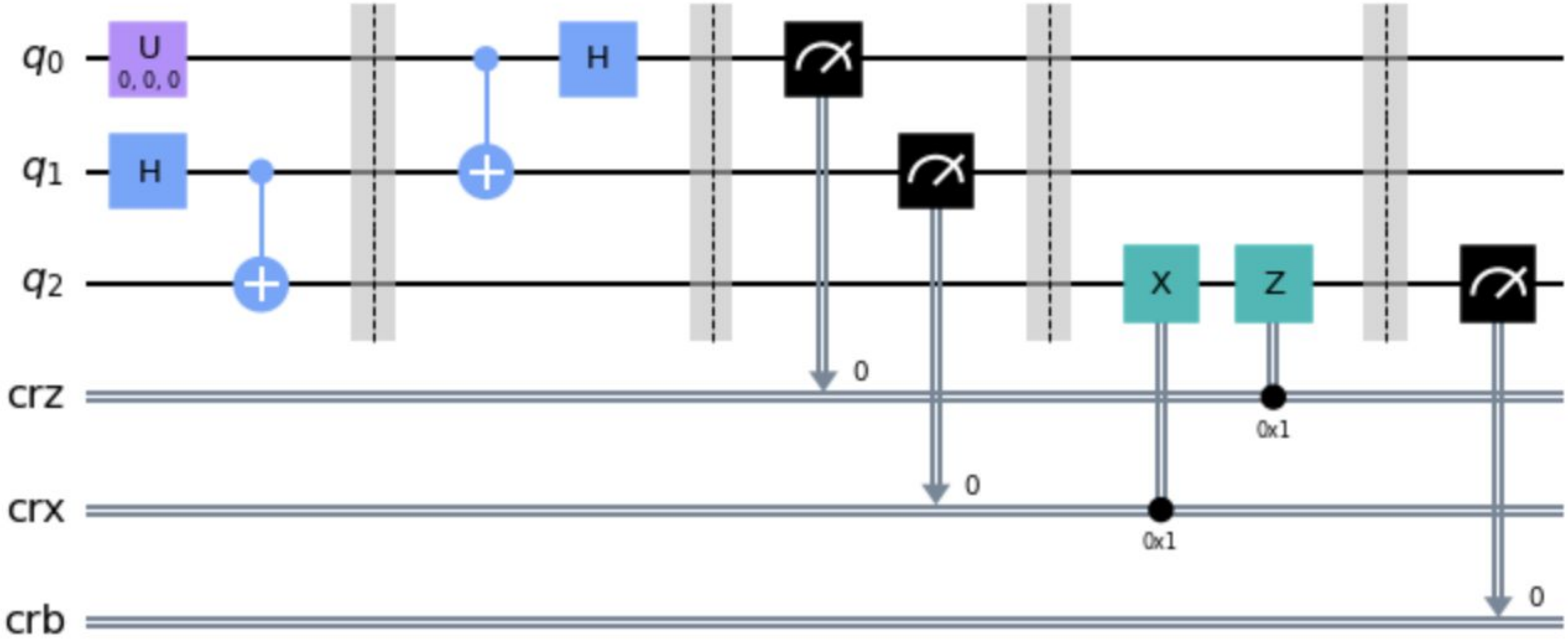
TELEPORTATION CIRCUIT



We measure the final state to check if the protocol worked

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LET'S TEST IT ON QISKIT!

Thank You!

Questions?