Review of Basic Algebra

1 Exponents

Typical representation of an exponent is:

a^n the n^{th} power of a

where:
a base of an exponent
n power of an exponent

1.1 Properties of Exponents:

let $a \in \mathbb{R}$ then:

$a^n = a \times a \times a \times a ...$ a multiplied by itself n times

$a^1 = a$

$a^0 = 1, \quad a \neq 0$

$a^{-1} = \frac{1}{a}$

$a^{-n} = \frac{1}{a^n}$

$a^n \cdot a^m = a^{n+m}$

$\frac{a^n}{a^m} = a^n \div a^m = a^{n-m}$

$(a^n)^m = a^{n \cdot m}$

$(a \cdot b)^n = a^n \cdot b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

1.2 Problems:

1. Show that: $(-a)^2 \neq -a^2$
2. Simplify: $\frac{x^{12} \cdot x^{-3}}{x^2 \cdot x^{-3} \cdot x^4}$
3. Simplify $\left(\frac{a}{b^m}\right)^{-n}$

2 Roots

2.1 Square Root

A square root$\sqrt{a}$ is a number when multiplied with itself gives a. A square root can be expressed in exponential for as follows $a^{\frac{1}{2}}$.

Note:

$a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1 = a$

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

2.2 N^{th} Root

An n^{th} root $\sqrt[n]{a}$ of a number a is a number b, such that $b^n = a$.

Some operations with radicals:

$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}, \qquad a \geq 0,\, b \geq 0$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \qquad a \geq 0,\, b \geq 0$

$\sqrt[n]{a^m} = (\sqrt[n]{a})^m = (a^{\frac{1}{n}})^m = a^{\frac{m}{n}}$

Note: Once a number has been changed from radical form to exponentiated form, the rules of exponents still apply (see above)

Problems:

1. $\sqrt{9 \cdot 49}$
2. $\sqrt[3]{\frac{8}{27}}$
3. Rationalize the denominator:
   (a) $\frac{(a-1)}{\sqrt{a-1}}$
   (b) $\frac{a^2}{a^3\sqrt{a}}$

3 Basic Rules of Algebra

3.1 basic rules

$$a + b = b + a$$

$$(a + b) = (b + a)$$

$$a + 0 = a$$

$$a + (-a) = 0$$

$$ab = ba$$

$$(ab)c = a(bc)$$

$$1 \cdot a = a$$

$$a \cdot a^{-1} = 1 \qquad \text{(for all } a \neq 0\text{)}$$

$$(-a)b = a(-b)$$

$$(-a)(-b) = ab$$

$$-(a + b) = -a - b$$

$$a(b + c) = ab + ac$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

3.2 Special Identities:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a-b)(a+b) = a^2 - b^2$$

3.3 Problems:

1. Using the rules of algebra prove that the special identities from section 3.2 (above) are true.
2. Expand the following:
   (a) $(x^2+5)^2$
   (b) $(\sqrt{a-1}+\sqrt{a}-1)^2$
3. Simplify the following:
   (a) $(ab-3b)(a+3b)$
   (b) $(xy-4y^2)(x^2y+4xy^2-x^3)$

4 Fractions

4.1 Reduced Form

We can reduce the form a fraction by factoring the numerator and denominator and then canceling common factors between them (by dividing by the same factor provided it is not zero).

e.g. $\frac{16a^2b^3}{4ab^3} = 4a$

4.2 Signs and Fractions

$-\frac{a}{b} = \frac{-a}{b}$

$\frac{-a}{-b} = \frac{a}{b}$

4.3 Addition

$a + \frac{b}{c} = \frac{a \cdot c + b}{c}$

$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + c \cdot b}{b \cdot d}$

4.4 Multiplication

$a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

4.5 Problems:

1. $\frac{4}{2x} + \frac{4}{x} + \frac{8}{2x}$
2. $\frac{1}{1-a} + \frac{1}{2} + \frac{1}{1+a}$
3. $5x^2y \cdot \frac{4x}{20xy^2}$

5 Equations With a Single Unknown

The easiest way to solve an equation with a single unknown is to:

1. Isolate all the terms containing the unknown on one side of the equals sign and all the terms that do not on the other.
2. Combine all terms containing the unknown, and all the terms that do not contain the unknown respectively
3. Calculate the unknown

5.1 Problems:

1. $(x + 4) - (x - 2) = 6x$
2. $(x + 2)^2 - 5x = (x - 1)^2$
3. $\frac{x-1}{x+1} + 17 = \frac{4}{x+1}$