Microeconomic Analysis

Mark Dean

Homework 1

Due Thursday 12th September

Question 1 Some things to clear up from class

1. Come up with a choice correspondence that satisfies property α but not property β, and another that satisfies β but not α
2. Prove that a choice correspondence satisfies properties α and β if and only if it satisfies WARP
3. Show that if a choice function has a utility representation then it must satisfy α and β

Question 2 Say that, rather than starting with the preference relation 'at least as good as' $\succeq$ as a primitive, and using this to define 'strictly preferred to' $\succ$, we go the other way round.

1. If you start with the binary relation $\succ$, interpreted as 'strictly preferred to', how would you generate the relation 'at least as good as'?
2. Assume that $\succ$ satisfies the following two properties
   - Asymmetry: if $x \succ y$ then not $y \succ x$
   - Negative Transitivity: For all x, y and z, if $x \succ y$ then either $x \succ z$ or $z \succ y$ (or both)

   Show that under these conditions, the relationship you constructed in part (1) will be complete and transitive.

Question 3 Utility maximization is not the only choice procedure that are consistent with α and β. There are also other choice procedures that will satisfy these conditions and so are indistinguishable from rational choice. Consider the following decision making procedures. Prove whether or not they will result in choices that satisfy α and β

1. A decision maker (DM) is choosing between books from a set B. They have a utility function $u : B \to \mathbb{R}$, and a 'threshold utility' level u^*. In any choice set, they search through the books alphabetically by title, and choose the first book that has utility level u that is equal to or above u^*. If they have not found any such book by the time they reach the end of the choice set, they will choose the book with the highest utility (to make things simpler you can assume that there is no indifference - i.e. no two books have the same utilities)
2. A DM assigns a utility number to each alternative and chooses the alternative with the lowest utility
3. The DM ranks the alternatives according to a utility function, and in any choice set chooses all median elements

Question 4 In class we defined the $\succeq$ −maximal element of a set as

$$Max(A|\succeq) = \{x \in A | x \succeq y \text{ for all } y \in A\}$$

1. Show that if A is finite and $\succeq$ is a preferences relation then $Max(A|\succeq)$ is non-empty
2. Here is another definition: for a binary relation B, define

   $$M(A|B) = \{x \in A | yBx \text{ for no } y \in A\}$$

   Show that if B is equal to $\succ$ for some preference relation $\succeq$ then $Max(A|\succeq) = M(A|\succ)$
3. If $\succeq$ is transitive and reflexive, but not necessarily complete, and we still say: $x \succ y$ if $x \succeq y$ but not $y \succeq x$ is it still the case that $Max(A|\succeq) = M(A|\succ)$? If not, what is the relationship between the two sets?
4. We say that a binary relation $\succ$ is **acyclic** if there exists no set $x_1, ..., x_n$ such that

   $$x_1 \succ x_2 \succ ... \succ x_n \succ x_1$$

   Define

   $$\hat{C}(A) = M(A|B)$$

   for all $A \in 2^X/\varnothing$. Show that $\hat{C}$ is a choice correspondence if and only if B is acyclic

Question 5 Consider two binary relations, $\succeq$ and $\succeq^*$ on some finite set X. Suppose that $\succeq^*$ is transitive, antisymmetric and complete and $\succeq$ is transitive, antisymmetric and reflexive, but not necessarily complete.

Consider the following choice correspondence c on X: for all $A \subseteq X$,

$$c(A) = M(M(\succ |A)| \succ^*)$$

where $\succ$ and $\succ^*$ and respectively the asymmetric parts of $\succeq$ and $\succeq^*$.

1. Show that c is a well defined choice function (i.e. that in any choice set c will contain exactly one element)
2. Does c satisfy property α?
3. What properties of $\succeq$ guarantee that c satisfies property α for all $\succeq^*$?
4. Here are two properties of choice.
   - **WEAK WARP:** If x is chosen from $\{x, y\}$ and is also chosen from $\{x, y, z_1, ..., z_n\}$, then y is not chosen from any set consisting of x, y and some subset of $\{z_1, .., z_n\}$. i.e.

$$\begin{aligned} \{x, y\} &\subset S \subset T \\ C(\{x, y\}) &= C(T) = x \\ &\Rightarrow y \neq C(S) \end{aligned}$$

   - **EXPANSION:** if x is chosen from each of two sets is also chosen from their union. i.e.

$$\begin{aligned} x &= C(S) = C(T) \\ &\Rightarrow x = C(S \cup T) \end{aligned}$$

   (a) Show that property α implies weak WARP and expansion, but not visa versa

   (b) Show that c will satisfy weak WARP and expansion