Preference for Commitment

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Preference Over Menus

- In order to discuss preference for commitment we need to be able to discuss preferences over menus
- Let C be a compact metric space
- $\Delta(C)$ set of all measures on the Borel σ-algebra of C (i.e. all lotteries
- Endow $\Delta(C)$ with topology of weak convergence
- Z all non empty compact subsets of $\Delta(C)$ (Hausdorff topology)
- Let $\succeq$ be a preference relation on Z
  - Interpretation: preference over menus from which you will later get to choose
- Let $\trianglerighteq$ be a preference relation on $\Delta(C)$
  - Interpretation: preferences when asked to choose from a menu

- For $x, y \in Z$ and $\alpha \in (0,1)$ define

$$\begin{aligned} & \alpha x + (1-\alpha) y \\ = \; & \{p = \alpha q + (1-\alpha) r | q \in x, r \in y, \} \end{aligned}$$

- E.g. if $x = \{\delta_a\}$, $y = \{\delta_b, \delta_c\}$ the

$$\begin{aligned} & \alpha x + (1-\alpha) y \\ = \; & \left\{ \begin{array}{c} \alpha a + (1-\alpha) b \\ \alpha a + (1-\alpha) c \end{array} \right\} \end{aligned}$$

Axiom 1 (Preference Relations) $\succeq$, $\trianglerighteq$ are complete preference relations

Axiom 2 (Independence) $x \succeq y$ implies
$$\alpha x + (1-\alpha)z \succeq \alpha y + (1-\alpha)z \ \forall\ x, y, z \in Z,$$
$$\alpha \in (0,1)$$

- Interpretation of independence: Standard Independence + Indifference to Timing of Uncertainty
  - Imagine we extended $\succeq$ to preferences over lotteries over menus
  - Independence would now say that, if we prefer choosing from x to choosing from y then we prefer choosing from x $\alpha\%$ of the time (and z $(1-\alpha)\%$ of the time) to choosing from y $\alpha\%$ of the time (and z $(1-\alpha)\%$ of the time) to the
  - Randomization occurs before choosing at second stage
  - In our definition of mixing, randomization occurs *after* second stage choice
  - There is an equivalence between choosing a contingent plan in the former case and a lottery over outcomes in the second case
  - So if you buy 'standard' independence and don't care about timing of resolution, you get Axiom 2

- Example

$$\frac{1}{2}x + \frac{1}{2}y$$

$$x = \{x_1, x_2\},\ y = \{y_1, y_2\}$$

- Gul-Pesendorfer: a menu of

$$\left\{ \begin{array}{c} \frac{1}{2}x_1 + \frac{1}{2}y_1 \\ \frac{1}{2}x_2 + \frac{1}{2}y_1 \\ \frac{1}{2}x_1 + \frac{1}{2}y_2 \\ \frac{1}{2}x_2 + \frac{1}{2}y_2 \end{array} \right\}$$

- Contingent plan: choose either x_1 or x_2 from x and either y_1 or y_2 from y
- Provides same menu of lotteries

Axiom 3 (Sophistication) $x \cup \{p\} \succ x \Leftrightarrow p \rhd q \; \forall \; q \in x$

Axiom 4 (Continuity) Three continuity conditions:

1. (Upper Semi Continuity): The sets $\{z \in Z | z \succeq x\}$ and $\{p \in \Delta(C) | p \unrhd q\}$ are closed for all x and q
2. (Lower vNM Continuity): $x \succ y \succ z$ implies $\alpha x + (1 - a)z \succ y$ for some $\alpha \in (0, 1)$
3. (Lower Singleton Continuity): The sets $\{p : \{q\} \succeq \{p\}\}$ are closed for every q

- The Standard Model of preference over menus

$$U(z) = \max_{p \in z} u(p)$$

for some linear, continuous utility $u : \Delta(C) \rightarrow \mathbb{R}$ such that

  - U represents $\succeq$
  - u represents $\trianglerighteq$

- Equivalent to axioms 1-4 and

$$x \succeq y \Rightarrow x \cup y \sim x$$

- Preference over menus given by

$$U(x) = \max_{p \in x} [u(p) + v(p)] - \max_{q \in x} v(q)$$

- u : 'long run' utility
  - Choice over singleton choice sets
- v : 'temptation' utility
  - Can lead to preference for smaller choice sets
- Interpretation:
  - Choose p to maximize $u(p) + v(p)$
  - Suffer temptation cost $v(p) - v(q)$

- Consider p, q, such that

$$\begin{aligned} u(p) &> u(q) \\ u(q)+v(q) &> u(p)+v(p) \end{aligned}$$

- Then

$$\begin{aligned} U(\{p\}) &= u(p) \\ U(\{p,q\}) &= u(q)+v(q)-v(q)=u(q) \\ U(\{q\}\} &= u(q) \end{aligned}$$

- Interpretation: give in to temptation and choose q
- 'Weak set betweenness'

$$\{p\} \succ \{p,q\} \sim \{q\}$$

Why Preference for Smaller Choice Sets?

Avoid 'Willpower Costs'

- Consider p, q, such that

$$\begin{aligned} u(p) &> u(q) \\ v(q) &> v(p) \\ u(p)+v(p) &> u(q)+v(q) \end{aligned}$$

- Then

$$\begin{aligned} U(\{p\}) &= u(p) \\ U(\{p,q\}) &= u(p)+v(p)-v(q) \\ U(\{q\}\} &= u(q) \end{aligned}$$

- Interpretation: fight temptation, but this is costly
- 'Strict set betweenness'

$$\{p\} \succ \{p,q\} \succ \{q\}$$

- We say that q tempts p if $\{p\} \succ \{p, q\}$
  - This implies that $v(q) > v(p)$
- We say that a decision maker exhibits self control at y if there exists x, z such that $x \cup z = y$ and

$$\{x\} \succ \{y\} \succ \{z\}$$

- $\{x\} \succ \{y\}$ implies there exists something in z which is tempting relative to items in x
- $\{y\} \succ \{z\}$ implies tempting item not chosen

Limiting Case: No Willpower

- Imagine that differences in v are large relative to differences in u
- In the limit, model reduces to

$$U(x) = \max_{p \in x} u(p) \text{ s.t. } v(p) \geq v(q) \; \forall \; q \in x$$

- This is the 'Strolz' model
- Implies not strict set betweenness, and not self control
- $\beta - \delta$ model is of this class

Axiomatic Characterization of GP Model

- Set Betweenness: for any x, y s.t $x \succeq y$

$$x \succeq x \cup y \succeq y$$

- Necessesity:
  - $x \succeq y$ implies that

$$u(p^x) + v(p^x) - v(q^x) \geq u(p^y) + v(p^y) - v(q^y)$$

    where $p^i = \arg\max_{p \in i} u(p) + v(p)$ and $q^i = \arg\max_{q \in i} v(q)$
  - NTS $x \succeq x \cup y$
  - Two cases:

$$\begin{aligned} u(p^x) + v(p^x) &\geq u(p^y) + v(p^y) \\ v(q^x) &\leq v(q^y) \end{aligned}$$

Axiomatic Characterization of GP Model

- Case 1: $u(p^x) + v(p^x) \geq u(p^y) + v(p^y)$

$$\begin{aligned}
u(p^x) + v(p^x) &\geq u(p^y) + v(p^y) \Rightarrow \\
u(p^x) + v(p^x) &= u(p^{x \cup y}) + v(p^{x \cup y}) \Rightarrow \\
u(p^x) + v(p^x) - v(q^x) &\geq u(p^{x \cup y}) + v(p^{x \cup y}) - v(q^{x \cup y})
\end{aligned}$$

- Case 2: $v(q^x) \leq v(q^y)$ (assume also that $u(p^x) + v(p^x) < u(p^y) + v(p^y)$)

$$\begin{aligned}
u(p^y) + v(p^y) &= u(p^{x \cup y}) + v(p^{x \cup y}) \\
v(q^{x \cup y}) &= v(q^y) \Rightarrow \\
u(p^{x \cup y}) + v(p^{x \cup y}) - v(q^{x \cup y}) &= u(p^y) + v(p^y) - v(q^y) \\
&\leq u(p^x) + v(p^x) - v(q^x)
\end{aligned}$$

Axiomatic Characterization of GP Model

Theorem

$\succeq$ satisfies Axioms 1, 2, 4 and set betweenness if and only if it has a Strolz representation or a G-P representation

Theorem

The proper relation $\succeq$ and $\trianglerighteq$ satisfy Axioms 1-4 and set betweenness if and only if

- *$\succeq$ has a Stroltz representation and $p \trianglerighteq q$ if and only if $v(p) > v(q)$ or $v(p) = v(q)$ and $u(p) \geq u(q)$*
- *or $\succeq$ has a G-P representation and $u(p) + v(p)$ represents $\trianglerighteq$*

Sketch of Proof that Axioms Imply Representation

- **Lemma 1:** Axioms 1, 2, 4 imply a linear $U : Z \to \mathbb{R}$ that represents $\succeq$ and is continuous on singleton sets
    - This is standard, and makes use of the mixture space axioms

Sketch of Proof that Axioms Imply Representation

- **Lemma 2:** Show that

$$\begin{aligned} U(x) &= \max_{p\in x}\min_{q\in x} U(\{p,q\}) \\ &= \min_{q\in x}\max_{p\in x} U(\{p,q\}) \end{aligned}$$

  - Utility depends only on 'chosen element', and 'most tempting element

- **Proof: Let** $\bar{u} = \max_{p\in x}\min_{q\in x} U(\{p,q\}) = U(\{p^*,q^*\})$
- Note that $U(\{p^*,q\}) \geq \bar{u} \ \forall \ q \in A$
- Set betweenness implies $U(A) = U(\cup_{q\in A}\{p^*,q\}) \geq \bar{u}$
- Also, for every $p \in A$, $\exists \ q_p \in A$ such that $U(\{p,q_p\}) \leq \bar{u}$
- By set betweenness $U(A) = U(\cup_{p\in A}\{p,q_p\}) \leq \bar{u}$

Sketch of Proof that Axioms Imply Representation

- **Lemma 3:** Show that

$$\begin{aligned} U(\{x\}) &> U(\{x,y\}) > U(\{y\}) \\ U(\{a\}) &> U(\{a,b\}) > U(\{b\}) \end{aligned}$$

  implies

$$\begin{aligned} & U(\alpha\{x,y\} + (1-\alpha)\{a,b\}) \\ = \; & U(\{\alpha x + (1-\alpha)a), \alpha y + (1-\alpha)b)\}) \end{aligned}$$

  - This comes straight from super independence and the fact that $\alpha x + (1-\alpha)a)$ is the best and $\alpha y + (1-\alpha)b)$ the most tempting element

Sketch of Proof that Axioms Imply Representation

- Define

$$
\begin{aligned}
u(p) &= U(\{p\}) \\
v(s;p,q,\delta) &= \frac{U(\{p,q\}) - U(\{p,(1-\delta)q+\delta s\})}{\delta}
\end{aligned}
$$

  - u is the long run utility
  - v is a measure of how tempting p is relative to q and r

Sketch of Proof that Axioms Imply Representation

- **Lemma 4:** Show that,

$$U(\{p\}) > U(\{p, (1-\delta)r + \delta s\}) > U(\{(1-\delta)r + \delta s\})$$

for all $s \in \Delta(C)$, then

1. $U(\{p\}) > U(\{p, s\}) > U(s) \Rightarrow v(s; p, q, \delta) = U(\{p, q\}) - U(\{p, s\})$
2. $v(p; p, q, \delta) = U(\{p, q\}) - U(\{p\})$

- Follows from Lemma 3

Sketch of Proof that Axioms Imply Representation

- **Lemma 5:** Show that, if

$$U(\{p\}) \geq U(\{p, q\}) \geq U(\{q\})$$

and for some r and δ

$$U(\{p\}) > U(\{p, (1-\delta)r + \delta s\}) > U(\{(1-\delta)r + \delta s\})$$

for all $s \in \Delta(C)$, then

$$\begin{aligned} & U(\{p, q\}) \\ = \quad & \max_{w \in \{p,q\}} [u(w) + v(w; p, r, \delta)] - \max_{z \in \{p,q\}} [v(z; p, r, \delta)] \end{aligned}$$

Sketch of Proof that Axioms Imply Representation

- **Proof** (assuming)

$$U(\{p\}) > U(\{p,q\}) > U(\{q\})$$

  - By previous lemma

$$\begin{aligned} v(q;p,r,\delta) &= U(\{p,r\}) - U(\{p,q\}) \\ &\geq U(\{p,r\}) - U(\{p\}) \\ &= v(p;p,r,\delta) \end{aligned}$$

and so

$$\begin{aligned} & u(p) + v(p;p,r,\delta) - v(q;p,r,\delta) \\ = \; & U(\{p\}) + U(\{p,r\}) - U(\{p,q\}) - U(\{p,r\}) + U(\{p\}) \\ = \; & U\left(\{p,q\}\right) \qquad (1) \end{aligned}$$

Sketch of Proof that Axioms Imply Representation

- Finally, pick p, q such that

$$U(\{p\}) > U(\{p, q\}) > U(\{q\})$$

(if such exists) and pick δ such that

$$U(\{p\}) > U(\{p, (1-\delta)q + \delta s\}) > U(\{(1-\delta)q + \delta s\})$$

for all s (which we can do by continuity)

- Define $v(s)$ as $v(s; p, q, \delta)$, and show that $v(s; p, q, \delta)$ doesn't depend on the specifics of the last three parameters.
- Lemma 5 therefore gives

$$U(\{p, q\}) = \max_{w \in \{p,q\}} [u(w) + v(w)] - \max_{z \in \{p,q\}} [v(z)]$$

- Lemma 2 then extends this result to an arbitrary set A

Discussion: Linearity

- Imagine

$$\{p\} \succ \{p, q\} \succ \{q\} \succ \{q, r\} \succ \{r\}$$

- Implies

$$\begin{aligned} u(p) &> u(q) > u(r) \\ v(r) &> v(q) > v(p) \\ u(p) + v(p) &> u(q) + v(q) > u(r) + v(r) \end{aligned}$$

- Which in turn implies

$$\{p\} \succ \{p, r\} \succ \{r\}$$

- 'Self Control is Linear'

Discussion: What is Willpower?

- It seems that the following statement is meaningful:
  - Person A has the same long run preferences as person B
  - Person A has the same temptation as person B
  - Person A has more willpower than person B
- Yet this is not possible in the GP more.
- Alternative: Masatlioglu, Nakajima and Ozdenoren [2013]

$$\begin{aligned} U(z) &= \max_{p\in z} u(p) \\ \text{subject to } \max_{q\in z} v(q) - v(p) &\leq w \end{aligned}$$

Discussion: Strict Set Betweenness and Random Strolz

- Does $\{p\} \succ \{p, q\} \succ \{q\}$ imply self control?
- Imagine that you are a Strolz guy with $u(p) > u(q)$, but are not sure that you will be tempted
- Half the time

$$v(p) = v(q)$$

  half the time

$$v(p) < v(q)$$

- Implies

$$\begin{aligned} U(\{p\}) &= u(p) \\ U(\{p, q\}) &= \frac{u(p) + u(q)}{2} \\ U(\{q\}) &= u(q) \end{aligned}$$

- Strict set betweenness without self control

Discussion: Optimism

- Say with probability ε won't be tempted so

$$\hat{U}(z) = (1-\varepsilon)U(z) + \varepsilon \max_{p \in z} u(p)$$

- Can lead to violations of set betweenness.
- Let $g = gym$, $j = jog$, $t = tv$

$$\begin{aligned} u(g) &> u(j) > u(t) \\ v(g) &< v(j) < v(t) \\ u(j) + v(j) &> u(t) + v(t) > u(g) + v(g) \end{aligned}$$

- For ε small

$$\{t, j\} \succ \{t, g\}$$

as

$$\begin{aligned} U(\{t, j\}) &= u(j) + v(j) - v(t) \\ U(\{t, g\}) &= u(t) \end{aligned}$$

- but

$$\{t, j, g\} \succ \{t, j\}$$

as with probability ε no temptation and will go to the gym