Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Nonlinear Phillips Curve*

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Abstract

We propose a model with heterogeneous downward nominal wage rigidity for individual labor varieties arising from cross-sectional dispersion in nominal fairness standards and labor productivity. The model delivers a nonlinear wage Phillips curve linking current wage inflation with current unemployment that is relatively steep at high levels of inflation and relatively flat at low levels of inflation. The predicted nonlinear Phillips curve matches well the pattern of wage inflation and unemployment observed in the United States over the past 40 years. In particular, it accounts for the resilience of the labor market in the tightening cycle following the Covid-19 inflation spike and for the missing inflation in the recovery from the 2008 great contraction. For the pandemic era, the model predicts that in 2020 and 2021 the U.S. economy was hit by large supply shocks, but that the inflation spike of 2022 was primarily due to demand shocks. Although the model features occasionally binding constraints for individual labor types, there are no such constraints in the aggregate, making the model amenable to perturbation analysis.

Keywords: Downward nominal wage rigidity, nonlinear wage Phillips curve, unemployment, inflation.
JEL Classification: E24, E31, E32.

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1 Introduction

Two recent phenomena observed in the United States and elsewhere have sparked renewed interest in whether the wage Phillips curve could be nonlinear, exhibiting a relatively steep slope at high levels of inflation and a relatively flat slope at low levels of inflation. One of these phenomena is the resilience of the labor market in the midst of the monetary tightening cycle aimed at curbing the Covid-19 inflation spike. The other is the apparent missing inflation during the recovery from the high level of unemployment caused by the 2008 financial crisis. The question of whether the Phillips curve could be nonlinear was largely dormant during the great moderation, when both inflation and unemployment fluctuated in a relatively narrow window around their intended levels. But right at its inception, Phillips (1958) presented it as a nonlinear empirical relationship between unemployment and wage inflation. Understanding the nature of nonlinearity in the Phillips curve is important because it can shed light on the possibility that both the cost of stabilizing high inflation in terms of unemployment and the cost of reducing high unemployment in terms of inflation can be relatively low.

This paper proposes a model of a nonlinear wage Phillips curve due to heterogeneous downward nominal wage rigidity. Specifically, wage rigidity is assumed to vary in intensity across a continuum of labor varieties. The nominal wage of each labor variety is bounded below by the average wage prevailing in the previous period times a variety-specific scalar. In all respects other than the heterogeneity of downward nominal wage rigidity, the model economy is standard; households and firms operate in competitive markets and are rational and forward looking.

In equilibrium the model delivers a nonlinear wage Phillips curve. An increase in wage inflation raises the fraction of labor varieties that are not constrained by the wage lower bound. As a result, the fraction of the labor force suffering involuntary unemployment falls. These effects imply a negative relationship between current wage inflation and current unemployment. Importantly, the sensitivity of the implied relationship between unemployment and wage inflation changes at different levels of aggregate activity. For low levels of inflation, a large measure of workers is stuck at their wage lower bound. As a result, an increase in inflation, by lowering the real value of the wage lower bound, raises employment for a large measure of workers. Thus, equilibrium unemployment is relatively sensitive to changes in inflation. By contrast, for high levels of inflation, the mass of workers with a binding wage constraint is small, so an increase in inflation stimulates employment, but only for a small group of workers, rendering unemployment relatively insensitive to changes in inflation.

There is extensive econometric, survey, and experimental evidence documenting the presence of heterogeneous downward nominal wage rigidity, which we discuss in the next section. This evidence emphasizes both behavioral and technological reasons for heterogeneity in this type of nominal rigidity. On the behavioral side, the empirical literature has highlighted significant cross-sectional variation in fairness standards with respect to nominal wage cuts. The baseline formulation of the proposed model is inspired by this evidence. It features a lower bound on nominal wages that varies idiosyncratically across workers. On the technological side, idiosyncratic labor productivity has been shown to display significant cross-sectional variation. This is relevant for the purpose of the present investigation, because workers experiencing low productivity shocks are likely to require a fall in their real wage to be able to maintain full employment, and therefore are the ones most likely to be constrained by downward nominal wage rigidity. Further, there is econometric evidence from micro data suggesting that low productivity workers are more likely to accept wage cuts. Accordingly, we consider a variation of the baseline model featuring idiosyncratic labor productivity shocks and individual lower bounds on nominal wages that depend on each worker's productivity level.

The identification of the parameters of the model uses moments stemming from both macro and micro data. The calibrated model predicts a wage Phillips curve that is smooth but significantly nonlinear. For example, it implies that reducing inflation from 6 to 5 percent raises unemployment by 0.3 percentage points, while reducing inflation from 2 to 1 percent raises unemployment by 3 percentage points. Overall, the predicted Phillips curve captures relatively well the nonlinear relationship between unemployment and nominal wage growth observed in the U.S. economy over the past four decades. In particular, it provides theoretical support for the relatively low cost in terms of employment resulting from the stabilization efforts in the aftermath of the Covid-19 inflation spike as well as for the missing inflation observed during the recovery from the 2008 financial crisis. For the pandemic era, the model predicts that in 2020 and 2021 the U.S. economy was hit by large adverse supply shocks, but that the inflation spike of 2022 was primarily due to demand shocks.

The contemporaneous relationship between unemployment and wage inflation implied by the present model is in line with Phillips' empirical formulation, but departs from the Phillips curve induced by the new-Keynesian model. Specifically, the new-Keynesian model implies a forward-looking Phillips curve that relates unemployment not only to current wage inflation but also to future expected wage inflation. The reason why the new-Keynesian model generates an expectations-augmented wage Phillips curve is that it assumes that workers have market power. This assumption together with the assumption of nominal wage rigidity implies that the wage setting decision is forward looking, as today's nominal wage choice impacts the entire expected future path of the worker's real wage. The assumption that workers have market power can be justified in economies with a strong presence of labor unions, but is less tenable in economies, like the United States, in which secularly a small fraction of the labor force is unionized. For this reason in the present paper we do away with the assumption that workers have market power.

In spite of the aforementioned differences with the new-Keynesian framework, for regular fluctuations of inflation around the intended target, under plausible calibrations, the proposed model delivers equilibrium dynamics that are quantitatively similar to those associated with the standard new-Keynesian model with wage rigidity. An implication of this result is that the assumption that workers have market power does not appear to play a crucial role, at least for standard calibrations of the model considered in the related literature. In sum, the proposed model globally delivers a nonlinear Phillips curve—which is important for understanding unusual events like the pandemic inflation and the great contraction—and locally preserves the dynamic properties of the new-Keynesian model—which is important for understanding normal fluctuations like those observed during the great moderation.

Finally, the paper makes a methodological contribution. One impediment that has limited a more widespread adoption of models with downward nominal wage rigidity in monetary analysis in spite of their empirical appeal, is the difficulty to approximate their equilibrium conditions due to the presence of occasionally binding constraints. This is most relevant for medium scale models used for policy analysis. This paper contributes to overcoming this impediment. Unlike standard models with homogeneous downward nominal wage rigidity, the proposed model is amenable to perturbation analysis, which is the standard method used to approximate and estimate equilibrium dynamics. Although in the present formulation there are occasionally binding constraints at the level of individual labor varieties, in the aggregate the equilibrium conditions do not feature such restrictions, thereby allowing for the differentiation of the aggregate equilibrium conditions around the deterministic steady-state.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the baseline model. Section 4 shows that the model implies a wage Phillips curve that is globally nonlinear. Section 5 extends the baseline model to allow for endogenous labor supply, and section 6 introduces idiosyncratic productivity shocks. Section 7 shows that for standard calibrations in a neighborhood around the steady state the equilibrium dynamics implied by the model with heterogeneous downward nominal wage rigidity are similar to those of the new-Keynesian model of wage rigidity. Section 8 concludes.

2 Related Literature

The empirical relevance of downward nominal wage rigidity has been extensively documented by, among others, Card and Hyslop (1997), Kahn (1997), Altonji and Devereux (2000), Gottschalk (2005), Barattieri, Basu, and Gottschalk (2014), Daly and Hobijn (2014), Schmitt-Grohé and Uribe (2016), Jo (2022), and Grigsby, Hurst, and Yildirmaz (2021).

Cross-sectional heterogeneity in the degree of downward nominal wage rigidity, the friction that gives rise to a convex wage Phillips curve in the model proposed in this paper, has been documented using administrative, survey, and experimental data. Using data on nominal wage changes from 1991 to 1997 from a Swiss representative labor force survey and a representative administrative database, Fehr and Goette (2005) estimate significant dispersion in the cost of cutting nominal wages across individual workers. This finding is consistent with experimental results by Fehr and Falk (1999) and Fehr and Gächter (2000) showing that respondents vary with respect to their nominal fairness standards. In turn, Bewley (1999) finds that fairness considerations are the main reason why firms are reluctant to cut wages during recessions. Specifically, Bewley conducts a survey of more than 300 firm managers in the northeastern United States during the recession of the early 1990s and finds that the most common answer of why firms were reluctant to cut wages was that wage cuts undermine morale, which disrupts productivity in the workplace. There is also evidence of heterogeneous downward nominal wage rigidity at the layoff margin. Davis and Krolikowski (2024) conduct a survey of new unemployment insurance recipients in Illinois over the period September 2018 to July 2019, during which the labor market was relatively tight. They find that survey respondents varied significantly in the nominal wage cut that they would have been willing to accept to maintain their last job, even when controlling for individual characteristics including race, gender, education, and job tenure. Heterogeneity in nominal wage rigidity stemming from the fact that the timing of wage changes varies across firms and sectors and is determined independently of the state of the business cycle has been documented using large representative administrative data sets (see Murray, 2021, for the United States; Faia and Pezone, 2023, and Fanfani, 2023, for Italy; and Adamopoulou, Díez-Catalán, and Villanueva, 2024, for Spain). All of these studies demonstrate that heterogeneity is substantial and has significant real effects. To the best of our knowledge, the present paper is the first one to introduce heterogeneous downward nominal wage rigidity into a general equilibrium setting.

This paper is also related to a large literature on the role of nominal wage rigidity for macroeconomic adjustment. In the context of the new-Keynesian framework, sticky wages à la Calvo was introduced by Erceg, Henderson, and Levin (2000). The derivation of a linear wage Phillips curve associated with that model is presented in Casares (2010) and Galí (2011). Kim and Ruge-Murcia (2009) study a model with nominal wage rigidity à la Rotemberg but with an asymmetric wage adjustment cost function. They estimate the parameters of this cost function and find that wage cuts are more costly than wage increases. Elsby (2009) studies downward nominal wage rigidity in the context of a model in which firms have monopsony power in the labor market and Benigno and Ricci (2011) in a model in which workers have monopoly power. There is also a literature combining labor search frictions and nominal rigidities including Faia (2008), Gertler, Sala, and Trigari (2008), and Dupraz, Nakamura, and Steinsson (2022). Unlike the present study, these papers are not concerned with the global nonlinearity of the Phillips curve.

There is a literature that has examined the effects of nominal wage rigidity more globally. The starting point is the empirical estimate by Phillips (1958) of a negative nonlinear relation between wage inflation and unemployment. Phillips hypothesized that the observed nonlinearity might be the consequence of downward nominal wage rigidity, but he did not provide a theoretical model. Harding, Lindé, and Trabandt (2022, 2023) show, using nonlinear numerical approximation methods, that the new-Keynesian model of Smets and Wouters (2007), which features a Kimball aggregator for both intermediate goods and labor varieties, when reparameterized to allow for stronger real rigidities implies a convex relationship between the contemporaneous equilibrium values of the output gap and inflation. Specifically, in the Smets and Wouters model the strength of real rigidities is increasing in the curvature of the Kimball aggregator for intermediate goods. Smets and Wouters use a value of 10 for this curvature parameter, whereas Harding, Lindé, and Trabandt, based on a reestimation of the model, use a value of 64.5. A difference with the present study is that the Smets and Wouters model assumes bilateral wage rigidity, whereas the present study assumes that wages are downwardly rigid, which has been shown to be more empirically compelling. Elsewhere (Schmitt-Grohé and Uribe, 2016, 2017) we have investigated the implications of downward nominal wage rigidity for macroeconomic adjustment in dynamic general equilibrium models of open and closed economies. In contrast to the present formulation, these earlier studies maintain a homogeneous lower bound on nominal wages. This class of models yields a limiting case of nonlinearity, characterized by an L-shaped Phillips curve, horizontal at all positive levels of unemployment and vertical at zero unemployment. This type of Phillips curve does not fully align with the observed convex but relatively smooth relation between unemployment and wage inflation (see Figure 2 below). A further distinction between the heterogeneous and homogeneous versions of the downward nominal wage rigidity model is that the latter is not amenable to perturbation analysis due to the occasionally binding constraint in its aggregated equilibrium conditions. In contemporaneous work, Benigno and

Eggertsson (2023) add downward nominal wage rigidity to a new-Keynesian model with labor search frictions. Wages are assumed to be flexible when the labor market is tight and downwardly rigid when it is not. This assumption introduces a kink in a piece-wise linearized Phillips curve relating price inflation to labor market tightness (and other variables). By contrast, the model proposed here predicts a nonlinear, convex relationship between wage growth and unemployment but without a kink, which is more consistent with existing empirical estimates. For example, Bernanke and Blanchard (2023) find no evidence of kinks in the relationship between wage growth and market slack when unemployment is linked to wage growth in a smooth and convex fashion.[1] Also, the assumption of wage flexibility when the labor market is tight, maintained in Benigno and Eggertsson (2023), is not entirely consistent with the evidence in Davis and Krolikowski (2024) discussed above, indicating that even in tight labor markets nominal wages display a high degree of downward rigidity. The model with heterogeneous downward nominal wage rigidity proposed here is consistent with this evidence, because it predicts that even when the labor market is tight, unemployment is the consequence of downward nominal wage rigidity. A further difference with the work of Benigno and Eggertsson (2023) is that the Phillips curve predicted by their formulation with a single kink can capture a steepening of the Phillips curve but not a flattening, and therefore does not address the missing inflation observed in the recovery from the great recession.

Finally, econometric estimates of linear wage Phillips curves are provided by Galí (2011) and Galí and Gambetti (2019). The low inflation exit from the great recession of 2008 and the labor market resilience during the post-Covid-19 disinflation spurred empirical work on non-linearities in the Phillips curve. Leduc and Wilson (2017) relate the missing inflation post great recession to a flattening of the Phillips curve, and Crust, Lansing, and Petrosky-Nadeau (2023) interpret the missing unemployment post Covid-19 as a steepening of the Phillips curve. Cerrato and Gitti (2022) using data from U.S. metropolitan statistical areas find that post Covid-19 the slope of regional Phillips curves was three times larger than pre Covid-19. Using the same data, Gitti (2024) finds nonlinearities in regional Phillips curves linking price inflation and labor market tightness. The heterogeneous downward nominal wage rigidity model proposed in the present paper provides a uniform theoretical framework for explaining both the missing inflation and the missing unemployment episodes documented in these empirical studies.

[1]See in particular Bernanke and Blanchard (2023) Section IV, and especially the regression results in Table A10 and the discussion in footnote 27.

3 The Model

The model features firms that use a variety of differentiated labor inputs. Nominal wages are downwardly rigid and the degree of rigidity varies across labor varieties.

3.1 Firms

Firms are price takers. They use labor to produce a final good. Profits are given by

$$P_t a_t F(h_t) - W_t h_t,$$

where P_t denotes the product price level, h_t denotes labor, W_t denotes the nominal wage rate, a_t is an exogenous productivity shock, and $F(\cdot)$ is an increasing and concave production function. The optimality condition determining the demand for labor is

$$a_t F'(h_t) = \frac{W_t}{P_t}, \tag{1}$$

which equates the marginal product of labor to the real wage.

The labor input h_t is assumed to be a composite of a continuum of labor varieties h_{jt} for $j \in [0, 1]$. The aggregation technology is of the form

$$h_t = \left[\int_0^1 h_{jt}^{1-\frac{1}{\eta}} dj \right]^{\frac{1}{1-\frac{1}{\eta}}}, \tag{2}$$

where $\eta > 0$ is the elasticity of substitution across labor varieties.[2] The firm chooses the quantity of each labor variety h_{jt} to minimize its total labor cost, $\int_0^1 W_{jt} h_{jt} dj$, subject to the aggregation technology (2), given its desired amount of the labor composite h_t and taking as given the wage of each variety of labor, denoted W_{jt}. This cost minimization problem yields the demand for labor of type j

$$h_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\eta} h_t, \tag{3}$$

where

$$W_t = \left[\int_0^1 W_{jt}^{1-\eta} dj \right]^{\frac{1}{1-\eta}} \tag{4}$$

is the cost-minimizing price of one unit of aggregate labor, that is, when h_{jt} is chosen optimally for all j, the aggregate wage rate W_t satisfies $W_t h_t = \int_0^1 W_{jt} h_{jt} dj$.

[2]Section 6 introduces heterogeneity in labor productivity across workers.

3.2 Households

The representative household has preferences over streams of consumption, denoted c_t, described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where $\beta \in (0,1)$ is a subjective discount factor, and $U(\cdot)$ is an increasing and concave period utility function.

The household supplies inelastically $\bar{h}$ units of labor of each variety $j \in [0,1]$.[3] The economy faces an exogenous natural rate of unemployment denoted u_t^n. The natural rate of unemployment reflects frictions in the labor market unrelated to nominal rigidity (Friedman, 1968). We interpret u_t^n as an aggregate supply shock. The effective supply of labor of each variety is then given by $\bar{h}(1-u_t^n)$. Sometimes the household will not be able to sell all the units of labor it supplies. In these circumstances, employment is demand determined and the household suffers involuntary unemployment above the natural rate. Formally, households supply labor of each variety j subject to the constraint

$$h_{jt} \leq \bar{h}(1-u_t^n). \tag{5}$$

Each period $t \geq 0$, households can trade a nominally risk free discount bond denoted B_t that pays the interest rate i_t when held between periods t and $t+1$. In addition, each period the household pays real lump-sum taxes in the amount τ_t and receives profits from the ownership of firms in the amount ϕ_t. Its sequential budget constraint is then given by

$$c_t + \frac{B_t/P_t}{1+i_t} + \tau_t = \int_0^1 \frac{W_{jt}}{P_t} h_{jt} dj + \frac{B_{t-1}/P_{t-1}}{1+\pi_t} + \phi_t,$$

where

$$\pi_t \equiv \frac{P_t}{P_{t-1}} - 1 \tag{6}$$

denotes the inflation rate. The household chooses contingent plans for bond holdings and consumption to maximize its lifetime utility subject to its sequential budget constraint and some no-Ponzi game borrowing limit. The optimality conditions associated with consumption and bond holdings give rise to the Euler equation

$$U'(c_t) = \beta(1+i_t) E_t \frac{U'(c_{t+1})}{1+\pi_{t+1}}. \tag{7}$$

[3]Section 5 endogenizes the supply of labor.

3.3 Heterogeneous Downward Nominal Wage Rigidity

Each period $t \geq 0$, the nominal wage of every variety $j \in [0,1]$ is assumed to be subject to a lower bound constraint of the form

$$W_{jt} \geq \gamma(j) W_{t-1}, \tag{8}$$

where $\gamma(j)$ is a positive and increasing function governing the degree of downward nominal wage rigidity of labor variety j. This formulation of downward nominal wage rigidity nests the homogeneous case studied in Schmitt-Grohé and Uribe (2016), which obtains when the function $\gamma(j)$ is independent of j. The wage lower bound is assumed to depend on the past average wage rate, W_{t-1}, instead of on the past variety-specific wage rate, W_{jt-1}, to facilitate aggregation. The function $\gamma(\cdot)$ need not be interpreted as representing a fixed ordering of labor varieties. For example, welders could be represented by $j = 0.45$ in period t and by $j = 0.73$ in period $t+1$. This could occur, for example, because employment in welding became more regulated.

The labor market closes with a slackness condition imposed at the level of each labor variety,

$$[\bar{h}(1-u_t^n) - h_{jt}][W_{jt} - \gamma(j) W_{t-1}] = 0. \tag{9}$$

According to this condition, when an occupation suffers unemployment above the natural rate, the wage rate must be stuck at its lower bound. The slackness condition also says that if in a given occupation the wage rate is above its lower bound, then the occupation must display full employment, defined as an unemployment rate equal to the natural rate.

3.4 The Government

The central bank sets the nominal interest rate according to a Taylor rule of the form

$$1 + i_t = \frac{1+\pi^*}{\beta} \left(\frac{1+\pi_t}{1+\pi^*} \right)^{\alpha_\pi} \left(\frac{y_t}{y} \right)^{\alpha_y} \mu_t, \tag{10}$$

where π^* denotes the central bank's inflation target, y_t denotes aggregate output, y denotes the steady-state value of y_t, α_π and α_y are parameters, and μ_t is an exogenous and stochastic monetary shock.

We assume that fiscal policy is passive in the sense that government solvency is satisfied independently of the path of the price level.

3.5 Equilibrium

In equilibrium, aggregate output is given by

$$y_t = a_t F(h_t). \tag{11}$$

Market clearing in the goods market requires that consumption equal output,

$$c_t = y_t. \tag{12}$$

We are now ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium)**.** *A competitive equilibrium is a set of processes c_t, y_t, h_t, h_{jt}, W_t, W_{jt}, P_t, π_t, and i_t satisfying* (1) *and* (3)-(12) *for all $j \in [0,1]$ and $t \geq 0$, given the initial wage W_{-1} and the exogenous disturbances a_t, μ_t, and u_t^n.*

Next, we show that the equilibrium conditions can be written in terms of a single labor variety.

3.6 Equilibrium in j^* Form

We consider an equilibrium in which for every $t \geq 0$ there exists a cutoff labor variety denoted $j_t^* \in (0,1)$ that operates at full employment, $h_{jt} = \bar{h}(1-u_t^n)$ for $j = j_t^*$, and for which the wage lower bound holds with equality, $W_{j_t^* t} = \gamma(j_t^*)W_{t-1}$. Evaluating the labor demand (3) at $j = j_t^*$ yields the condition

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{13}$$

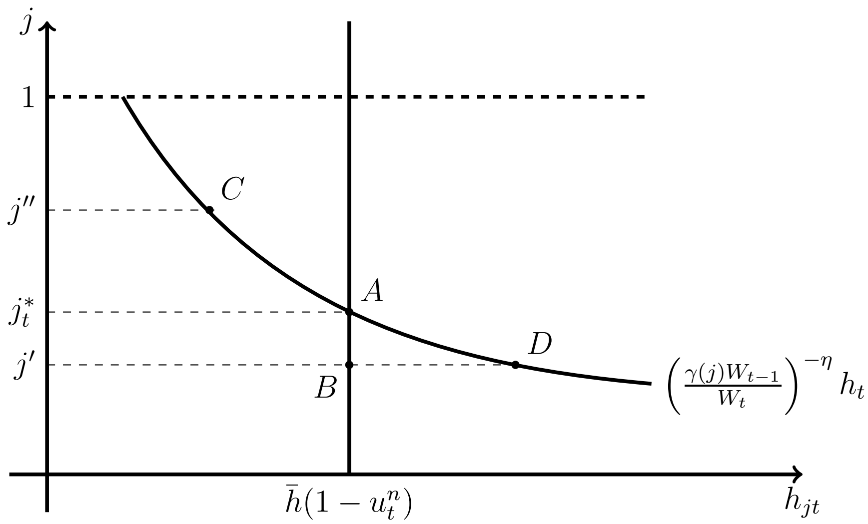
where

$$\pi_t^W \equiv \frac{W_t}{W_{t-1}} - 1 \tag{14}$$

denotes wage inflation in period t.

All varieties $j < j_t^*$ also pay the wage $\gamma(j_t^*)W_{t-1}$ and operate at full employment. To see this, let $W_t^* \equiv \gamma(j_t^*)W_{t-1}$ and suppose first, contrary to the claim, that $W_{jt} < W_t^*$ for some $j < j_t^*$. Then, by (3) we have that $h_{jt} = (W_{jt}/W_t)^{-\eta}h_t > (W_t^*/W_t)^{-\eta}h_t = \bar{h}(1-u_t^n)$, which violates the time constraint (5). Intuitively, since at W_t^* there is full employment, a wage lower than W_t^* would induce a demand for labor in excess of full employment, which is impossible. Suppose now that, contrary to the claim, $W_{jt} > W_t^*$ for some $j < j_t^*$. Then by the same logic $h_{jt} < \bar{h}(1-u_t^n)$. Further, $W_{jt} > W_t^* = \gamma(j_t^*)W_{t-1} > \gamma(j)W_{t-1}$. So we have

Figure 1: Determination of Wages and Employment Across Labor Varieties



Notes. The downward sloping line depicts the demand for labor when the wage constraint is binding in the space (h_{jt}, j), where $j \in [0, 1]$ indexes labor varieties and h_{jt} denotes the quantity of labor of variety j demanded by firms. The vertical line depicts the supply of labor net of natural unemployment as a function of the labor variety j. In the figure, the aggregate variables h_t and W_t/W_{t-1} are taken as given.

that in this case $\bar{h}(1-u_t^n) - h_{jt} > 0$ and $W_{jt} - \gamma(j)W_{t-1} > 0$, which violates the slackness condition (9).

It also follows that all labor varieties $j > j_t^*$ are stuck at their wage lower bound and suffer involuntary unemployment. To see this, use (3) and (8) to write, for any $j > j_t^*$, $h_{jt} = (W_{jt}/W_t)^{-\eta} h_t \leq (\gamma(j)W_{t-1}/W_t)^{-\eta} h_t < (\gamma(j_t^*)W_{t-1}/W_t)^{-\eta} h_t = \bar{h}(1-u_t^n)$. This shows that all labor varieties $j > j_t^*$ suffer involuntary unemployment. It then follows from the slackness condition (9) that $W_{jt} = \gamma(j)W_{t-1}$, that is, wages of all labor varieties $j > j_t^*$ are stuck at their lower bounds.

Summing up, in the equilibrium we are considering, we have that

$$\begin{cases} h_{jt} = \bar{h}(1-u_t^n) \text{ and } W_{jt} = \gamma(j_t^*)W_{t-1} & \text{for } j \leq j_t^* \\ h_{jt} < \bar{h}(1-u_t^n) \text{ and } W_{jt} = \gamma(j)W_{t-1} & \text{for } j > j_t^* \end{cases}. \tag{15}$$

The cutoff labor variety j_t^* is an important object in this model because it governs the extensive margin of unemployment, that is, how many occupations will operate below potential.

Figure 1 provides a graphical explanation of the determination of wages and employment across labor varieties, given the aggregate variables h_t and W_t/W_{t-1}. The downward sloping curve represents the demand for labor of each variety, h_{jt}, as a function of j when the variety-specific wage equals its lower bound, $W_{jt} = \gamma(j)W_{t-1}$. The vertical line represents the labor supply net of natural unemployment, $\bar{h}(1 - u_t^n)$, as a function of j. The intersection of the two lines at point A determines the cutoff variety j_t^*. This is because at point A there is full employment and the wage constraint exactly binds, which are the two conditions defining j_t^*. Points located to the right of the downward sloping line are infeasible because they imply that $W_{jt} < \gamma(j)W_{t-1}$, which violates the wage lower bound. Points located to the left of the downward sloping line imply that the wage lower bound is slack, $W_{jt} > \gamma(j)W_{t-1}$. Points located to the right of the vertical line are infeasible because they violate the resource constraint $h_{jt} \leq \bar{h}(1 - u_t^n)$. Points located to the left of the vertical line imply involuntary unemployment, $h_{jt} < \bar{h}(1 - u_t^n)$. Points to the left of both the downward sloping and the vertical lines are infeasible because they imply that the wage lower bound is slack, $W_{jt} > \gamma(j)W_{t-1}$, and that there is involuntary unemployment, $h_{jt} < \bar{h}(1 - u_t^n)$, which violates the slackness condition $(W_{jt} - \gamma(j)W_{t-1})(h_{jt} - \bar{h}(1 - u_t^n)) = 0$.

Since points located to the right of either curve or to the left of both are infeasible, it follows that in equilibrium pairs (j, h_{jt}) must lie on the vertical line if $j < j_t^*$ and on the downward sloping curve if $j > j_t^*$. For example, for variety $j' < j_t^*$ in the figure, the equilibrium is at point B, where there is full employment and the wage is unconstrained, i.e., workers receive a wage strictly above the lower bound $\gamma(j')W_{t-1}$. How much above? If the wage rate of variety j' were at its lower bound, $\gamma(j')W_{t-1}$, then there would be excess demand given by the distance between points B and D. Since wages are upwardly flexible, the wage rate has to increase until the excess demand disappears. It is clear from the figure that this occurs when variety j' earns the same wage as variety j_t^*, namely, $\gamma(j_t^*)W_{t-1}$. By contrast, for variety $j'' > j_t^*$, the equilibrium is at point C, where there is involuntary unemployment and the wage lower bound is binding, i.e., the wage is stuck at its lower bound.

The fact that the equilibrium is on the vertical line for $j < j_t^*$ and on the downward sloping curve for $j > j_t^*$ means that for varieties $j < j_t^*$ employment is supply determined and that for varieties $j > j_t^*$ employment is demand determined. This marks a difference with the new-Keynesian model with Calvo-type nominal wage rigidity in which employment is demand determined for all labor varieties. In the figure, the triangular area located above the downward sloping line, to the left of the vertical line, and below 1 represents the aggregate amount of unemployment above the natural rate.

Next, we analyze the determination of j_t^* in general equilibrium. To this end, write the wage aggregation equation (4) as

$$\begin{aligned} W_t^{1-\eta} &= \int_0^1 W_{jt}^{1-\eta} dj \\ &= \int_0^{j_t^*} [\gamma(j_t^*)W_{t-1}]^{1-\eta} dj + \int_{j_t^*}^1 [\gamma(j)W_{t-1}]^{1-\eta}\, dj \\ &= W_{t-1}^{1-\eta}\left[j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj \right]. \end{aligned}$$

The second equality follows from the results summarized in (15). Using the definition of wage inflation given in (14) and rearranging gives

$$(1+\pi_t^W)^{1-\eta} = j_t^*\gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj. \tag{16}$$

According to this expression, wage inflation is increasing in the cutoff labor variety j_t^*. To understand why, suppose that the cutoff variety increases from $j_t^{*'}$ to $j_t^{*''} > j_t^{*'}$. Then, all varieties from 0 to $j_t^{*'}$ are unconstrained before and after the increase in j_t^*. As a result, their wages increase from $\gamma(j_t^{*'})W_{t-1}$ to $\gamma(j_t^{*''})W_{t-1}$. Varieties j between $j_t^{*'}$ and $j_t^{*''}$ were constrained before the change and become unconstrained after. For these workers, the wage rate increases from $\gamma(j)W_{t-1} < \gamma(j_t^{*''})W_{t-1}$ to $\gamma(j_t^{*''})W_{t-1}$. Finally labor varieties $j > j_t^{*''}$ are constrained before and after the change in j_t^*, so their wages remain unchanged. Since for every variety j the nominal wage either increases or stays the same, it follows that the aggregate wage, W_t, and hence wage inflation, π_t^W, increase.

We are now ready to define the competitive equilibrium in j_t^* form.

Definition 2 (Competitive Equilibrium in j^* Form)**.** *A competitive equilibrium is a set of processes j_t^*, y_t, h_t, $w_t \equiv W_t/P_t$, i_t, π_t, and π_t^W, satisfying*

$$y_t = a_t F(h_t), \tag{17}$$

$$U'(y_t) = \beta(1+i_t)E_t\frac{U'(y_{t+1})}{1+\pi_{t+1}}, \tag{18}$$

$$a_t F'(h_t) = w_t, \tag{19}$$

$$1+i_t = \frac{1+\pi^*}{\beta}\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\alpha_\pi}\left(\frac{y_t}{y}\right)^{\alpha_y}\mu_t, \tag{20}$$

$$1+\pi_t^W = \frac{w_t}{w_{t-1}}(1+\pi_t), \tag{21}$$

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t, \tag{22}$$

and

$$(1+\pi_t^W)^{1-\eta} = j_t^* \gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj, \tag{23}$$

given the initial condition w_{-1} and the stochastic processes a_t, μ_t, and u_t^n.

Equilibrium conditions (17)–(21) are standard components of optimizing monetary models, with or without nominal rigidity. The Keynesian features of the model appear in the last two equilibrium conditions. Equation (22) says that there is one labor variety, j_t^*, for which there is full employment and the wage constraint just binds. Equation (23) says that wage inflation is a weighted average of the wage increase across varieties relative to the average wage prevailing the previous period. For equation (23) to hold with equality at all times it must be the case that in equilibrium wage inflation be neither too high nor too low so as to rule out the corner solutions $j_t^* = 0$ and $j_t^* = 1$.[4]

In this model, monetary disturbances have real effects. To see this, it suffices to consider, as an example, a situation in which the economy is initially in steady state and in period 0 experiences an unexpected purely transitory fall in the monetary disturbance μ_t. Suppose that after the shock there is perfect foresight. Suppose, contrary to the claim, that the fall in μ_t does not affect the real allocation (y_t or h_t for any $t \geq 0$). Then, by the Euler equation (18) and the Taylor rule (20), we have that the inflation rate π_t must change either at $t = 0$ or at $t = 1$ or both. Also, by the labor demand (19), the real wage w_t must stay constant, otherwise h_t would move. Then, by (21), wage inflation, π_t^W, must change either at $t = 0$ or at $t = 1$ or both. In turn, by (22), j_t^* must change either at $t = 0$ or at $t = 1$ or both, but in such a way as to keep constant the ratio $\gamma(j_t^*)/(1+\pi_t^W)$, otherwise h_t would be affected. But, according to (23), the ratio $\gamma(j_t^*)/(1+\pi_t^W)$ can stay constant only if $\gamma'(j) = 0$, which is a contradiction.

[4]Formally, for equilibria displaying small fluctuations around the steady state, an interior solution is guaranteed if $\left[\int_0^1 \gamma(j)^{1-\eta} dj\right]^{1/(1-\eta)} < 1 + \pi^* < \gamma(1)$.

4 The Wage Phillips Curve

The aggregate unemployment rate, denoted u_t, is given by the integral of the unemployment rates across all labor varieties. Formally,

$$\begin{aligned} u_t &\equiv \int_0^1 \left(\frac{\bar{h} - h_{jt}}{\bar{h}} \right) dj \\ &= u_t^n j_t^* + \int_{j_t^*}^1 \left(\frac{\bar{h} - h_{jt}}{\bar{h}} \right) dj \\ &= u_t^n j_t^* + (1 - j_t^*) - \frac{h_t}{\bar{h}} \int_{j_t^*}^1 \left(\frac{W_{jt}}{W_t} \right)^{-\eta} dj \\ &= u_t^n j_t^* + (1 - j_t^*) - \left(\frac{W_{t-1}}{W_t} \right)^{-\eta} \frac{h_t}{\bar{h}} \int_{j_t^*}^1 \gamma(j)^{-\eta} dj. \end{aligned}$$

The second and fourth equalities follow from (15) and the third from (3). Using the definition of wage inflation given in (14) and equilibrium condition (22) to eliminate $(W_{t-1}/W_t)^{-\eta}\, h_t$, we can write

$$u_t = u_t^n + (1 - u_t^n) \left[(1 - j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)} \right)^{-\eta} dj \right]. \tag{24}$$

The right hand side of equation (24) is decreasing in j_t^*. It follows that as j_t^* increases, the unemployment rate falls. This is intuitive because all activities below the cutoff threshold j_t^* operate at full employment, so the higher the cutoff threshold is, the smaller the set of activities displaying involuntary unemployment above the natural rate will be.

Given the natural rate of unemployment, u_t^n, equations (23) and (24) parametrically represent a contemporaneous relationship involving only unemployment and wage inflation (u_t and π_t^W). Further, u_t and π_t^W are negatively related. To see this, recall that equation (23) implies that π_t^W is increasing in j_t^* and that equation (24) implies that u_t is decreasing in j_t^*. Thus, the model's implied relationship between unemployment and wage inflation represents a downward sloping wage Phillips curve.

We note that the model implies a contemporaneous wage Phillips curve. In particular, it does not feature future expected inflation. In this sense, the present model departs from the new-Keynesian framework in which the wage Phillips curve is forward looking (Erceg, Henderson, and Levin, 2000; Galí, 2011). In both models, households and firms are rational, optimizing, and forward looking. The reason why the new-Keynesian model produces a forward-looking Phillips curve is its assumption that workers have monopoly power. By contrast, in the heterogeneous downward nominal wage rigidity model proposed here, households and firms are assumed to be price takers in the labor market. In this way, the present model provides microfoundations to Phillips's original formulation of a contemporaneous wage Phillips curve (Phillips, 1958). The following proposition summarizes this result.

Proposition 1 (The Wage Phillips Curve)**.** *The model with heterogeneous downward nominal wage rigidity implies a contemporaneous negative relationship between wage inflation,* π_t^W*, and the unemployment rate,* u_t*. This relationship is parametrically defined by equations* (23) *and* (24) *and depends on the exogenous supply shock* u_t^n*.*

We now turn to the characterization of the short- and long-run wage Phillips curves, with a special interest in the curvature of the former.

4.1 The Short-Run Wage Phillips Curve

The short-run wage Phillips curve is the locus of points (u_t, π_t^W) satisfying equations (23) and (24) for a given value of the natural rate of unemployment u_t^n.

To illustrate the properties of the short-run wage Phillips curve implied by the model, we consider a linear functional form for $\gamma(j)$ and calibrate the parameters of the model.[5] Specifically, assume that

$$\gamma(j) = (1 + \pi^*)^\delta(\Gamma_0 + \Gamma_1 j). \tag{25}$$

Here, the parameter $\delta \in [0, 1]$ captures the degree of wage indexation to long-run inflation, and the parameters $\Gamma_0, \Gamma_1 > 0$ govern the degree of downward nominal wage rigidity. The time unit is a quarter. The calibration period is 1986 to 2007. This period is of interest for two reasons. First it excludes the missing inflation episode in the recovery from the financial crisis of 2008 and the missing unemployment episode in the post Covid-19 disinflation, which the model aims to explain. Second, there exists an estimate of a linear wage Phillips curve using U.S. data over this period, which we use to pin down the slope of the Phillips curve at the steady state values of wage inflation and unemployment. We fix the natural rate of unemployment u_t^n at its steady-state value, denoted u^n. We set u^n equal to 0.04 (or 4 percent) to match the minimum unemployment rate observed over the calibration period 1986 to 2007. This value is in line with the 2024 Long-Range Consensus Forecast of the Blue Chip Survey (2024) of 4.1 percent for the period 2031-2035 and with the average of the U.S. full employment unemployment rate of 4.1 percent estimated by Michaillat and Saez (2024) over the period 1930 to 2024.

We set the elasticity of substitution across labor varieties to 11 ($\eta = 11$). This number is an average of the values used in Erceg, Henderson, and Levin (2000), Christiano, Eichenbaum, and Evans (2005), and Galí (2015). We assume full indexation of wages ($\delta = 1$),

[5] In section 6 below, we introduce an alternative formulation in which $\gamma(j)$ depends on the realization of an idiosyncratic labor productivity shock.

Table 1: Parameter Values

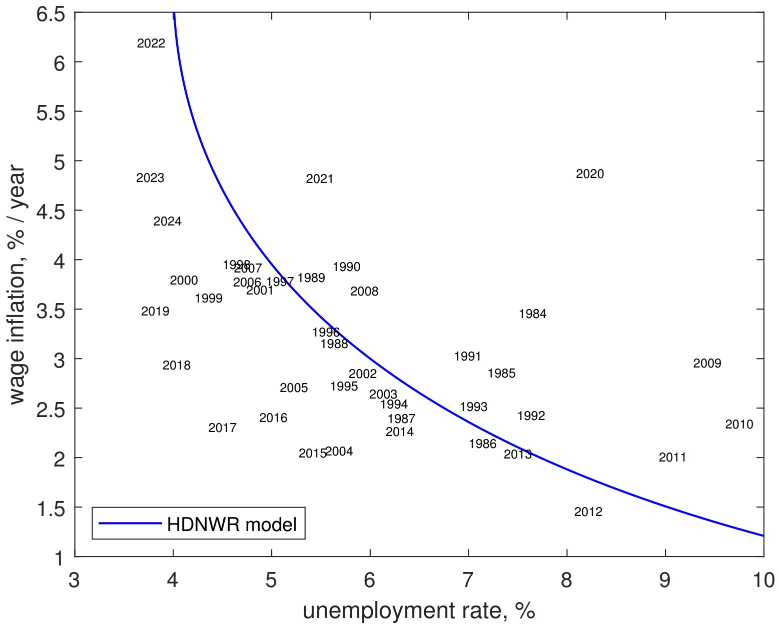
Parameter	Value	Description
Γ_0	0.978	Parameter of the $\gamma(j)$ function
Γ_1	0.031	Parameter of the $\gamma(j)$ function
δ	1	Wage indexation parameter of the $\gamma(j)$ function
π^*	$1.03^{1/4} - 1$	Steady state inflation rate
u^n	0.04	Natural rate of unemployment
η	11	Elasticity of substitution across labor varieties
β	0.99	Subjective discount factor
σ	1	Inverse of intertemporal elasticity of substitution
θ	5	Inverse of Frisch elasticity of labor supply
α	0.75	Labor elasticity of output
α_π	1.5	Inflation coefficient of Taylor rule
α_y	0.125	Output coefficient of Taylor rule
ρ_μ	0.5	Persistence of monetary shock
ρ_a	0.9	Persistence of technology shock

Note. The time unit is a quarter.

as in much of the related literature (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007). We set π^* to 3 percent per year, which corresponds to the median value of wage inflation over the calibration period. We calibrate Γ_0 and Γ_1 to meet two restrictions. First, we impose that the Phillips curve contains the point π_t^W equal to 3 percent per year and u_t equal to 6 percent. These values correspond to the medians of wage inflation and unemployment observed over the calibration period. Second, we require that at that point, the slope of the wage Phillips curve is -0.74, that is, a one-percentage point increase in the unemployment rate is associated with a 74 basis points decline in the annual wage inflation rate. This value for the slope of the wage Phillips curve is taken from Galí and Gambetti (2019), who estimate a linear wage Phillips curve using U.S. data from 1986 to 2007. The resulting values are $\Gamma_0 = 0.978$ and $\Gamma_1 = 0.031$. The top panel of Table 1 summarizes the parameter values used in the computation of the Phillips curve. (The bottom panel of this table is discussed in section 7.)

Figure 2 shows with a solid line the short-run wage Phillips curve predicted by the calibrated heterogeneous downward nominal wage rigidity model in the space (u_t, π_t^W). By construction, when the unemployment rate is 6 percent, the annual wage inflation rate is 3 percent. Also by construction, at that point, the slope of the Phillips curve is equal to -0.74. However, the curvature of the relationship between unemployment and wage inflation is endogenously determined. The predicted Phillips curve is nonlinear, relatively steep at high levels of inflation and relatively flat at low levels of inflation implying that the costs

Figure 2: The Short-Run Wage Phillips Curve



Notes. The figure shows with a solid line the short-run wage Phillips curve implied by the calibrated heterogeneous downward nominal wage rigidity (HDNWR) model for $u_t^n = u^n = 0.04$. The figure also shows the (u_t, π_t^W) pairs observed in annual U.S. data over the period 1984 to 2024. The observation labeled 2024 refers to unemployment and wage inflation in the first three months of 2024.

in terms of unemployment of reducing high inflation and that the costs in terms of inflation of reducing high unemployment are both relatively small. For example, lowering inflation from 6 to 5 percent would increase the unemployment rate by only 0.3 percentage points, whereas lowering inflation from 2 to 1 percent would increase the unemployment rate by 3 percentage points.

The nonlinearity of the predicted wage Phillips curve provides a unified explanation for the apparent flattening of the Phillips curve (or missing inflation) in the recovery from the 2008 great contraction and for the resilience of the labor market during the tightening cycle that curbed the post Covid-19 inflation. This feature of the data is apparent in Figure 2, which, along with the predicted wage Phillips curve, displays annual observations of U.S. unemployment and wage inflation for the period 1984 to 2024.[6] The nonlinearity of the Phillips curve, which was not targeted in the calibration—recall that the calibration targets

[6] Annual wage inflation is computed as the average of year-over-year monthly wage inflation. The measure of monthly nominal wages is Average Hourly Earnings of Production and Nonsupervisory Employees, FRED series AHETPI. The unemployment rate is the arithmetic mean of monthly unemployment rates, FRED series UNRATE. The observation labeled 2024 in the figure refers to unemployment and wage inflation in the first three months of 2024.

only one point along the Phillips curve and the slope at that point using pre-great-contraction data—captures relatively well the overall shape of the observed cloud of unemployment and wage inflation pairs. In particular, the post Covid-19 observations (2022 to 2024), characterized by high inflation and low unemployment, fall reasonably close to the steep portion of the Phillips curve implied by the calibrated model. The same is true for the large fall in unemployment with little uptake in inflation over the period 2012 to 2014, when the U.S. economy emerged from the financial crisis. At the same time, as we will see in section 4.3 below, the model also predicts that during the pandemic (2020 and 2021) the economy was buffeted by large negative supply shocks (increases in u_t^n), which shifted the position of the wage Phillips curve.

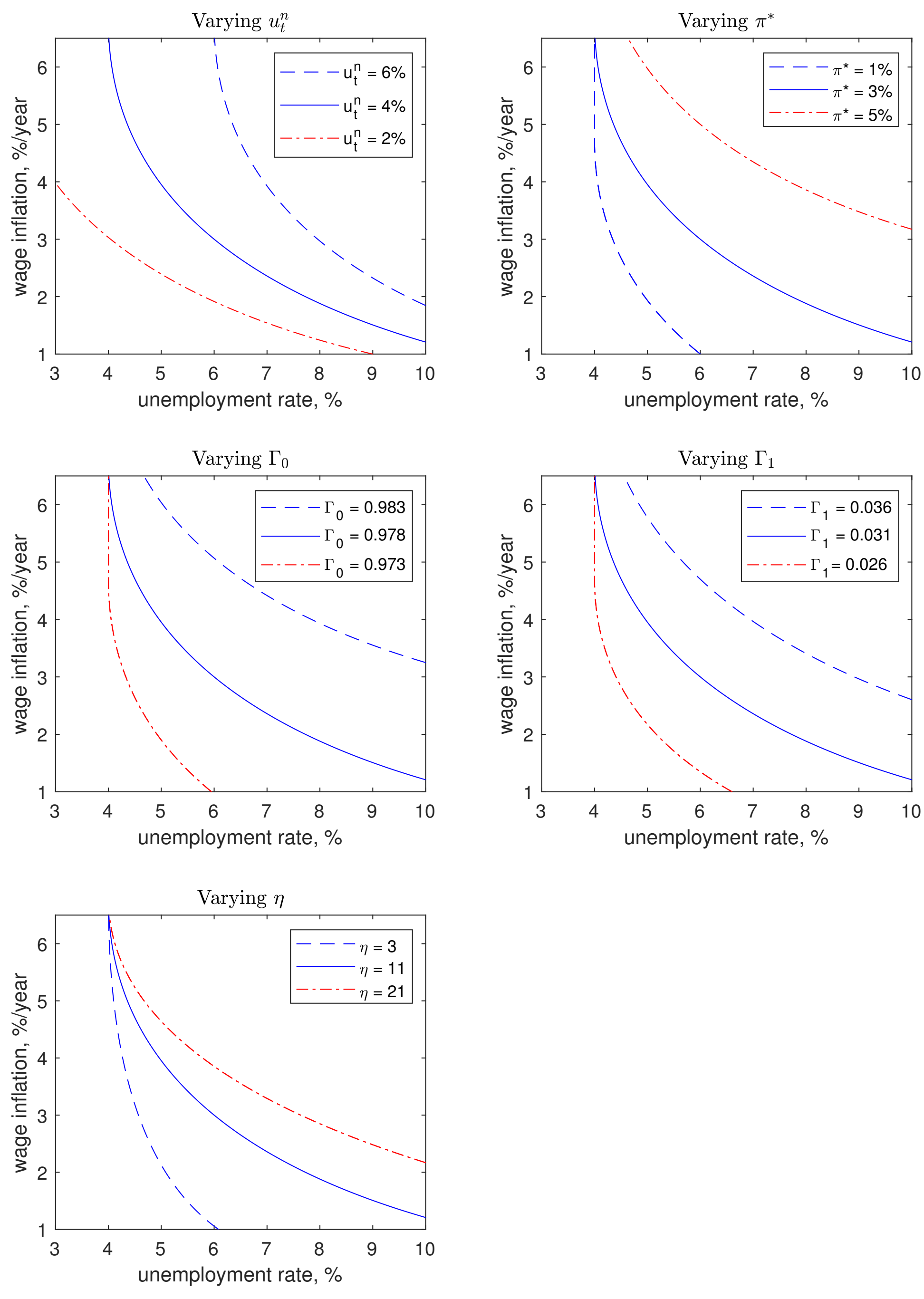
4.2 Shifters of the Short-Run Wage Phillips Curve

Figure 3 displays how changes in the natural rate of unemployment and in key structural parameters of the model shift the short-run wage Phillips curve. A negative aggregate supply shock in the form of an increase in the natural rate of unemployment, u_t^n, shifts the Phillips curve up and to the right. Intuitively, an exogenous increase in the number of unemployed workers requires more grease in the labor market (i.e., an increase in wage inflation) to maintain a given rate of overall unemployment. Similarly, an increase in the inflation target (an increase in π^*) or an increase in the degree of downward nominal wage rigidity (an increase in Γ_0 or Γ_1) shifts the short-run Phillips curve up and to the right. The intuition behind these effects is as follows. Wage inflation acts as a lubricant of the labor market because the higher wage inflation is, the larger the number of activities that are not constrained by the wage lower bound will be. An increase in π^*, Γ_0, or Γ_1 raises the wage lower bound. Thus, the economy needs more lubricant to maintain the same level of unemployment. An increase in the elasticity of substitution across labor varieties, η, flattens the wage Phillips curve. Intuitively, the larger is η, the more sensitive will be the demand for labor to changes in the relative wage rate. Thus, an increase in inflation, by reducing the relative wage of constrained labor varieties, causes a larger increase in employment the larger η is.

4.3 Demand and Supply Shocks in the Pandemic Era

According to conventional wisdom, the Covid-19 pandemic caused large negative supply shocks due to lockdowns and other restrictions. These types of shocks are captured by u_t^n in the model, which, as explained earlier, is a shifter of the Phillips curve. At the same time, both monetary and fiscal policy reacted aggressively during and after the pandemic,

Figure 3: Shifters of the Short-Run Wage Phillips Curve



Notes. Solid lines correspond to the baseline calibration. The variable u_t^n is the shock to the natural rate of unemployment. The parameter π^* is the inflation target. The parameters Γ_0 and Γ_1 pertain to the wage lower bound function $\gamma(j) = (1+\pi^*)^\delta(\Gamma_0 + \Gamma_1 j)$ (equation 25). The degree of indexation, δ, is set at 1. The parameter η represents the elasticity of substitution across labor varieties.

Table 2: Aggregate Supply Shocks During the Pandemic

Year	Actual Wage Inflation π_t^W	Actual Unemployment u_t	Predicted Supply Shocks $u_t^n - u^n$
2020	4.88	8.09	3.70
2021	4.83	5.35	0.92
2022	6.20	3.63	-0.40
2023	4.84	3.63	-0.81

Note. Wage inflation is expressed in percent per year and the unemployment rate and the supply shock in percent.

suggesting that demand forces were also at work. An open question is what was the timing and impact of demand and supply shocks in the pandemic era.

The Phillips curve plotted in Figure 2 is calibrated with data prior to the global financial crisis and hence prior to the pandemic. Thus, there is nothing in the Phillips curve of Figure 2 that targets the pandemic years. In that figure, the supply shock u_t^n is set at its steady-state value ($u_t^n = u^n = 4$ percent). That is, the plot displays the relationship between wage inflation and unemployment predicted by the model in the absence of supply shocks. This is how a Phillips curve is typically depicted in the related literature. But this does not mean that the model predicts no supply shocks shifting the Phillips curve during the pandemic—or in other periods for that matter.

Assuming that the pandemic did not affect the long-run level of inflation, π^*,[7] and assuming no measurement errors in the observed values of unemployment and wage inflation (u_t and π_t^W), the supply shocks hitting the economy can be backed out by evaluating equilibrium conditions (23) and (24) at the actually observed levels of u_t and π_t^W. This yields a system of two equations in two unknowns, u_t^n (the object of interest) and the equilibrium cutoff variety j_t^* for each period t.

Table 2 displays the result of this exercise. The second and third columns display the actually observed values of wage inflation and unemployment, π_t^W and u_t, and the last column displays the supply shocks, $u_t^n - u^n$, that according to the model hit the economy in the pandemic years. The model predicts that the largest negative supply shock occurred in 2020. This is consistent with the fact that 2020 was characterized by severe lockdowns

[7]This is arguably the case of greatest empirical relevance, see Brent and Smith (2023), Lebow and Peneva (2024), and Schmitt-Grohé and Uribe (2024).

in the United States. The second largest negative supply shock occurs the following year, 2021, when lockdowns were still in place but had been relaxed in many parts of the country. According to the model, by 2022 the negative supply shocks had ceased, which is in line with the lifting of restrictions and the recovery of aggregate activity observed at the time. This analysis suggests that the predicted curvature of the Phillips curve is not at odds with the prediction that the economy was buffeted by significant supply shocks during the worst of the pandemic.

The fact that according to the model by 2022 supply conditions had largely returned to normal implies that the model interprets the 2022 inflation surge as driven by demand shocks. This finding is in line with the empirical analyses of Bergholt et al. (2024) and Giannone and Primiceri (2024) identifying demand shocks as the key drivers of the post-Covid inflation surge.

4.4 The Long-Run Wage Phillips Curve

The long-run wage Phillips curve is the locus of points $(u_t, \pi_t^W) = (u, \pi^W)$ satisfying equations (23) and (24) for $u_t^n = u^n$, where variables without a time subscript denote steady-state values. The difference between the short- and long-run Phillips curves is that in the long run wage inflation and price inflation are both equal to the inflation target. Specifically, because output is constant in the steady state, the Euler equation (18) implies the long-run Fisher relationship

$$i = \frac{1+\pi}{\beta} - 1.$$

This expression and the Taylor rule (20) imply that in the steady state inflation must be at its target level,

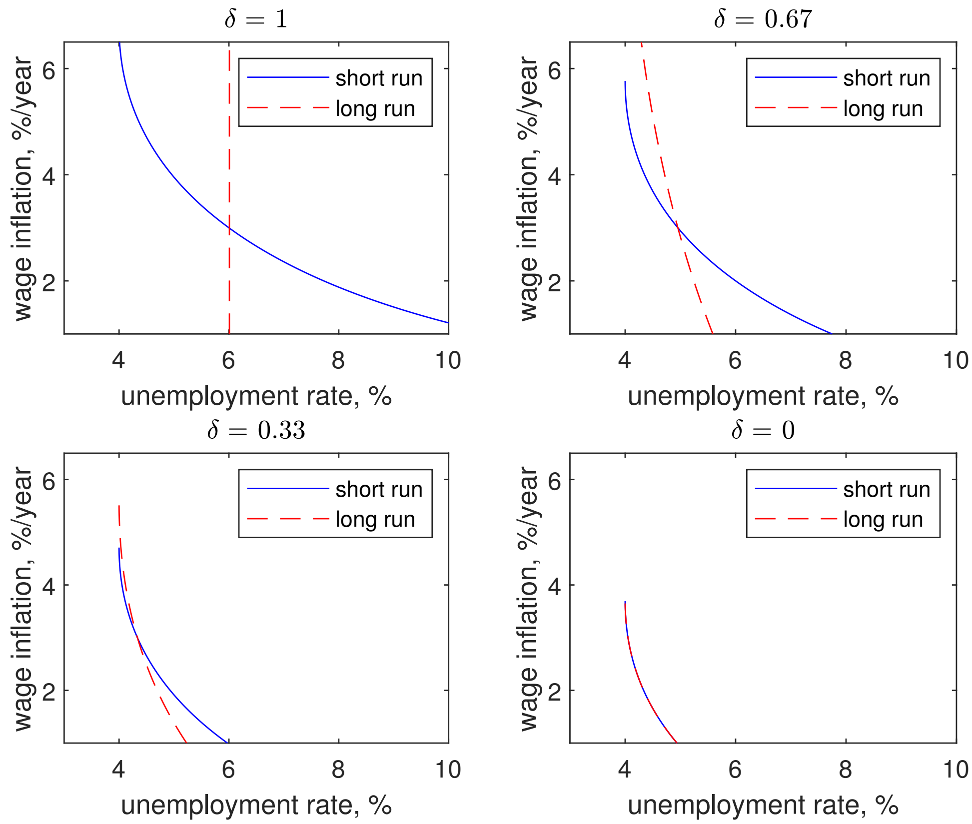
$$\pi = \pi^*.$$

Since in the steady state the real wage is constant, equilibrium condition (21) implies that wage inflation equals product-price inflation,

$$\pi^W = \pi^*.$$

Equilibrium conditions (23) and (24) evaluated at $u_t = u$, $\pi_t^W = \pi^*$, and $u_t^n = u^n$ constitute a relationship between inflation and unemployment in the steady state, which we call the long-run wage Phillips curve. It follows immediately that in the absence of wage indexation ($\delta = 0$), that is, when the function $\gamma(\cdot)$ is independent of π^*, the short- and long-run Phillips curves coincide. But this ceases to be the case when wages are indexed to steady-state inflation. To see this, consider again the linear functional form for $\gamma(\cdot)$ given in equation (25).

Figure 4: The Long-Run Wage Phillips Curve



Notes. The parameter δ pertains to the wage lower bound function $\gamma(j) = (1+\pi^*)^\delta(\Gamma_0 + \Gamma_1 j)$ (equation 25). The figure shows that the long-run wage Phillips curve is in general downward sloping and steeper than its short-run counterpart. The long-run wage Phillips curve is vertical when $\delta = 1$ (baseline calibration) and identical to the short-run wage Phillips curve when $\delta = 0$.

In this case, equilibrium conditions (23) and (24) evaluated at the steady state become

$$(1+\pi^W)^{(1-\eta)(1-\delta)} = j^* \widetilde{\gamma}(j^*)^{1-\eta} + \int_{j^*}^1 \widetilde{\gamma}(j)^{1-\eta} dj, \tag{26}$$

$$u = u^n + (1-u^n)\left[(1-j^*) - \int_{j^*}^1 \left(\frac{\widetilde{\gamma}(j)}{\widetilde{\gamma}(j^*)}\right)^{-\eta} dj\right], \tag{27}$$

where $\widetilde{\gamma}(j) \equiv \Gamma_0 + \Gamma_1 j$.

It is clear from (26) and (27) that under full wage indexation ($\delta = 1$, the baseline calibration) the long-run wage Phillips curve is perfectly vertical in the space (u, π^W). This is intuitive. Under full indexation, an increase in inflation fails to inject grease in the labor market in the long run, as indexation soaks it up one for one. By contrast, under imperfect indexation ($\delta < 1$), only a fraction δ of an increase in inflation is absorbed by indexation and the rest is grease to the labor market.

To see more precisely what happens for intermediate degrees of wage indexation, Figure 4 displays the long-run wage Phillips curve for four different degrees of wage indexation, $\delta = 1, 2/3, 1/3,$ and 0. For comparison, it also displays the corresponding short-run Phillips curves. The figure illustrates that absent full indexation the long-run wage Phillips curve is downward sloping and that as the degree of wage indexation goes down the slope of the long-run wage Phillips curve falls. In fact, the long-run wage Phillips curve rotates around the point $(u, \pi^W) = (0.06, 0)$ counterclockwise as δ declines. To see why this is so, recall that the calibration targets an unemployment rate of 6 percent and assumes full indexation. When $\delta = 1$, the left-hand side of (26) is equal to 1, regardless of the value of π^W. This uniquely pins down the steady-state value of j^* and by equation (27) also the steady state value of u. When $\delta < 1$ and inflation is zero ($\pi^W = 0$), then the left-hand side of equation (26) is also equal to 1, regardless of the value of δ. Thus, the long-run wage Phillips curve must contain the point $(u, \pi^W) = (0.06, 0)$ for any value of δ. When $\delta = 1$, the unemployment rate associated with this rotation point can be interpreted as the non-accelerating inflation rate of unemployment (NAIRU), because it is the rate of unemployment that can be sustained in the long run at the target rate of inflation.

Comparing the long-run and the short-run Phillips curves, the figure shows that for positive degrees of wage indexation $\delta \in (0, 1]$, the long-run Phillips curve is steeper than its short-run counterpart. The intuition why the wage Phillips curve is steeper in the long run is as follows. In the short run, movements in the inflation rate are not accompanied by movements in the long-run rate of inflation, so they grease the labor market one for one. By contrast, to the extent that δ is greater than zero, only a fraction $(1 - \delta)$ of an increase in inflation greases the labor market in the long run.

5 The HDNWR Model with Endogenous Labor Supply

Suppose now that the representative household derives disutility from supplying labor. Specifically, replace the lifetime utility function considered thus far with the function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_0^1 V(h^s_{jt}) dj \right],$$

where h^s_{jt} denotes the amount of labor of type j supplied in period t, and $V(\cdot)$ is a convex labor disutility function. To facilitate aggregation, we use the functional form

$$V(h) = \frac{h^{1+\theta}}{1+\theta}, \tag{28}$$

which is often used in the related literature (e.g., Galí, 2015). As before, there can be rationing in the labor market: for each labor type j, at the going wage W_{jt} households may not be able to sell all the units of labor they offer. The household sets its desired supply of labor of variety j to equate the marginal rate of substitution between labor and consumption to the variety-specific real wage. Formally, the supply of labor of type j is given by

$$\frac{V'(h^s_{jt})}{U'(c_t)} = w_{jt}, \tag{29}$$

where $w_{jt} \equiv W_{jt}/P_t$. We continue to assume that there is an exogenous amount of involuntary unemployment unrelated to wage stickiness, embodied in the variable u^n_t denoting the natural rate of unemployment. The restriction that employment is voluntary now takes the form

$$h_{jt} \leq h^s_{jt}(1-u^n_t).$$

This expression says that the household is not willing to have more members employed than the ones it voluntarily supplies to the market net of the ones that are naturally unemployed.

The household's budget constraint and the optimality conditions associated with consumption and bond holdings are unchanged. The firm's demand for labor of variety $j \in [0,1]$, given by equation (3), is also unchanged.

As before, we consider an equilibrium in which each period $t \geq 0$ there is a cutoff labor variety, j^*_t, that operates at full employment,

$$h_{j^*_t t} = h^s_{j^*_t t}(1-u^n_t), \tag{30}$$

and for which the wage constraint holds with equality,

$$W_{j^*_t t} = \gamma(j^*_t)W_{t-1}. \tag{31}$$

Combining these two conditions with the labor demand (3) and the labor supply (29) yields

$$\frac{V'\left(\frac{h_t}{1-u^n_t}\left(\frac{\gamma(j^*_t)}{1+\pi^W_t}\right)^{-\eta}\right)}{U'(c_t)} = \frac{\gamma(j^*_t)w_{t-1}}{1+\pi_t}. \tag{32}$$

It can be shown that, as in the case of an inelastic labor supply, all labor varieties $j < j_t^*$ operate at full employment and are paid the same wage as variety j_t^*. Also, all varieties $j > j_t^*$ are constrained by the wage lower bound and suffer unemployment above the natural rate.

The definition of a competitive equilibrium with an endogenous labor supply is then identical to that given in Definition 2, except that equation (22) is replaced by equation (32).

With an endogenous labor supply, the unemployment rate is the ratio of unemployed labor to the total labor supply. Formally,

$$u_t = \frac{\int_0^1 (h_{jt}^s - h_{jt})dj}{\int_0^1 h_{jt}^s dj}.$$

Using the functional form (28) for the disutility of labor and equations (3), (29), (30), and (31), we can rewrite the unemployment rate as

$$u_t = u_t^n + (1 - u_t^n)\frac{\int_{j_t^*}^1 \left[\left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{\frac{1}{\theta}} - \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta}\right] dj}{j_t^* + \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{\frac{1}{\theta}} dj}. \tag{33}$$

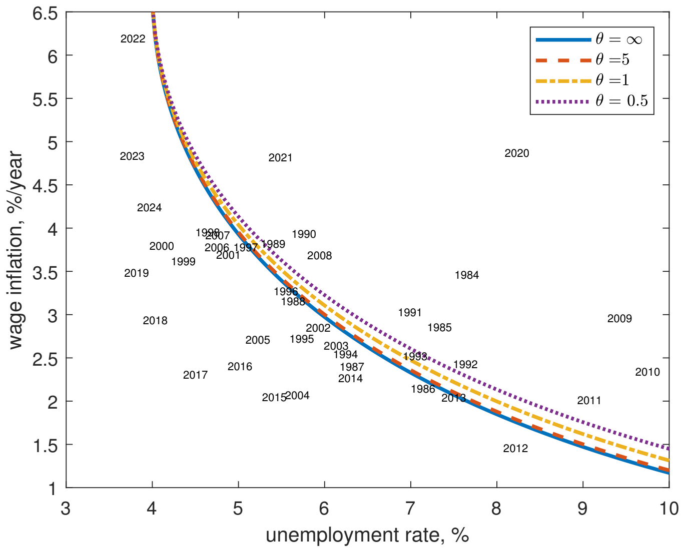
Note that as the elasticity of labor supply approaches zero ($\theta \to \infty$), equations (32) and (33) converge to equations (22) and (24) with $\bar{h}$ normalized to 1, and the model becomes the one with inelastic labor supply studied in sections 3 and 4.

The following definition summarizes the equilibrium with endogenous labor supply.

Definition 3 (Competitive Equilibrium with Endogenous Labor Supply)**.** *A competitive equilibrium in the economy with endogenous labor supply is a set of processes j_t^*, y_t, h_t, u_t, w_t, i_t, π_t, and π_t^W, satisfying* (17)-(21), (23), (32), *and* (33), *given the initial condition w_{-1} and the stochastic processes a_t, μ_t, and u_t^n.*

As in the case of an inelastic labor supply, the model features a wage Phillips curve implicitly given by equations (23) and (33) linking current unemployment, u_t, and current wage inflation, π_t^W. The Phillips curve now features a new parameter, θ, representing the inverse of the wage elasticity of labor supply. To depict the implied short-run wage Phillips curve we assign a value of 5 to this parameter following Galí (2015). This value implies a labor supply elasticity of 0.2. The parameters Γ_0 and Γ_1 of the variety-specific wage lower bound function $\gamma(j) = (1 + \pi^*)(\Gamma_0 + \Gamma_1 j)$ were recalibrated using the same targets for the steady-state unemployment-inflation pair and for the slope of the Phillips curve at that point as in the economy with an inelastic labor supply. The resulting values are $\Gamma_0 = 0.9781$

Figure 5: The Short-Run Wage Phillips Curve in the Model with Endogenous Labor Supply



Note. See notes to Figure 2.

and $\Gamma_1 = 0.0310$, which are the same as those associated with the HDNWR model with inelastic labor supply up to the third significant digit. Figure 5 displays with a solid line the predicted short-run wage Phillips curve. It is virtually identical to the one predicted by the economy with an inelastic labor supply. This is not surprising given that the calibrated labor supply elasticity is relatively small. The figure also shows that as the labor supply elasticity increases, the wage Phillips curve becomes flatter. However, quantitatively the differences are minor.

6 A Model with Heterogeneous Labor Productivity

Thus far we have taken the cross-sectional distribution of the wage lower bound, $\gamma(j)$, as given. In this section, we provide an example of how this formulation could be micro-founded. A recent literature has emphasized the role of heterogeneity in labor productivity for macroeconomic adjustment (see, for example, Kaplan, Moll, and Violante, 2018; Bayer, Born, and Luetticke, 2024, and the references cited therein). Motivated by this literature, here we incorporate idiosyncratic productivity shocks into the model and show that this source of heterogeneity can give rise to a convex relationship between wage inflation and unemployment. Because workers suffering negative idiosyncratic productivity shocks require a fall in their real wage to remain fully employed, they are the most likely to be affected by downward nominal wage rigidity. The empirical literature has additionally documented that firms have an easier time cutting wages when worker productivity is low (Fehr and Goette, 2005; Bewley, 1999; and Campbell and Kamlani, 1997). This fact amounts to cross-sectional heterogeneity in the degree of downward nominal wage rigidity. To isolate this effect, we assume that given the level of labor productivity, all workers have the same wage lower bound, but that the wage lower bound is lower for less productive workers. All other aspects of the model are as in the baseline formulation.

The labor input, h_t, is assumed to be a composite of efficiency units of labor varieties with the aggregation technology $h_t = \left[\int_0^1 (z_{jt}h_t(z_{jt}))^{1-\frac{1}{\eta}} dj\right]^{\frac{1}{1-\frac{1}{\eta}}}$, where $z_{jt} > 0$ is a stochastic variety-specific level of labor productivity with density function $f(\cdot)$ and $h_t(z_{jt})$ is the level of employment of workers with productivity z_{jt}. It is convenient to conduct the analysis in the domain of labor productivities as opposed to in the domain of labor varieties. To this end, we perform a change of variable as follows. Assume without loss of generality that z_{jt} is increasing in j. It follows that $j = \mathcal{F}(z_{jt})$, where $\mathcal{F}(\cdot)$ is the cumulative density function associated with the density function $f(\cdot)$. Now introduce the change of variable $z = z_{jt}$. Then we have that $dj = f(z)dz$. Note also that $z \to 0$ as $j \to 0$ and that $z \to \infty$ as $j \to 1$. The aggregation technology can then be written as

$$h_t = \left[\int_0^\infty (zh_t(z))^{1-\frac{1}{\eta}} f(z)dz\right]^{\frac{1}{1-\frac{1}{\eta}}}.$$

The firm chooses employment of each variety to maximize profits, $P_t a_t F(h_t) - \int_0^\infty W_t(z)h_t(z)f(z)dz$, taking P_t and $W_t(z)$ as given, where $W_t(z)$ is the nominal wage of labor variety z. The firm's cost-minimizing demand for labor with productivity z is

$$h_t(z) = z^{\eta-1}\left(\frac{W_t(z)}{W_t}\right)^{-\eta} h_t, \tag{34}$$

where W_t is given by

$$W_t = \left[\int_z \left(\frac{W_t(z)}{z}\right)^{1-\eta} f(z)dz\right]^{\frac{1}{1-\eta}}. \tag{35}$$

When $h_t(z)$ is chosen optimally, the aggregate wage rate W_t satisfies $W_t h_t = \int_z W_t(z)h_t(z)f(z)dz$. The demand for aggregate units of labor, h_t, satisfies the optimality condition $P_t a_t F'(h_t) = W_t$.

The household supplies inelastically $\bar{h}$ units of labor of each variety z, subject to the constraint

$$h_t(z) \le \bar{h}(1 - u_t^n).$$

The nominal wage of every variety z is assumed to be subject to a lower bound constraint of the form

$$W_t(z) \geq z^{\xi}\gamma W_{t-1}. \tag{36}$$

The parameter $\gamma > 0$ measures the common degree of downward nominal wage rigidity across productivity levels. The parameter $\xi \geq 0$ measures the degree of productivity specific wage rigidity. It aims to capture the empirical regularity that workers experiencing lower productivity are more willing to tolerate wage cuts (Fehr and Goette, 2005). The higher ξ is, the more sensitive the degree of downward wage flexibility to changes in productivity will be. We make the following assumption about the parameter ξ:

Assumption 1. *The parameter ξ satisfies $\eta(1-\xi) > 1$.*

This assumption states that wage flexibility cannot increase too quickly as productivity falls. The labor market closes with a slackness condition imposed at the level of each labor variety z:

$$[\bar{h}(1-u_t^n) - h_t(z)][W_t(z) - z^{\xi}\gamma W_{t-1}] = 0. \tag{37}$$

We consider an equilibrium in which for every $t \geq 0$ there exists a cutoff labor productivity, denoted z_t^*, such that workers with this level of productivity are fully employed,

$$h_t(z_t^*) = \bar{h}(1-u_t^n) \tag{38}$$

and for which the wage lower bound holds with equality,

$$W_t(z_t^*) = z_t^{*\xi}\gamma W_{t-1}. \tag{39}$$

This cutoff is of interest because, as in the baseline model, it determines the entire cross-sectional distribution of wages and employment. It can be shown that for any $z > z_t^*$, there is full employment, $h_t(z) = \bar{h}(1-u_t^n)$, and the wage rate is unconstrained, $W_t(z) > z^{\xi}\gamma W_{t-1}$ (see Claim 1 in the appendix). It can also be shown that for any $z < z_t^*$, there is involuntary unemployment, $h_t(z) < \bar{h}(1-u_t^n)$, and the wage rate is stuck at its lower bound, $W_t(z) = z^{\xi}\gamma W_{t-1}$ (see Claim 2 in the appendix). In words, relatively productive workers (those with $z > z_t^*$) are unaffected by downward nominal wage rigidity and are fully employed, whereas relatively unproductive workers (those with $z < z_t^*$) are constrained by downward nominal wage rigidity, which prevents their wages from falling to a level that is consistent with full employment.

Next we analyze the relationship between the cutoff labor productivity level z_t^* and wage inflation. To this end, following steps similar to those given in the derivation of equation (16) of the baseline model, we write the wage inflation rate as

$$(1+\pi_t^W)^{1-\eta} = \int_{z<z_t^*} \left(\frac{\gamma}{z^{1-\xi}}\right)^{1-\eta} f(z)dz + \int_{z>z_t^*} \left(\frac{\gamma}{{z_t^*}^{1-\xi}} \left(\frac{z}{z_t^*}\right)^{-\frac{1}{\eta}}\right)^{1-\eta} f(z)dz. \tag{40}$$

This equation implies that wage inflation and the cutoff productivity level are negatively related (a formal proof is in Claim 3 in the appendix). This result is intuitive: higher inflation reduces the real value of the wage lower bound. In turn this allows the real wage of constrained workers to fall. As a consequence workers with productivity levels below but close to z_t^*, which were previously unemployed, now become fully employed. This means that z_t^* falls with wage inflation.

The aggregate unemployment rate is defined as

$$u_t = \int_z \frac{\bar{h} - h_t(z)}{\bar{h}} f(z)dz.$$

As shown above, highly productive workers, namely those with $z > z_t^*$, are fully employed, $h_t(z) = \bar{h}(1-u_t^n)$. Thus the unemployment rate for highly productive workers is u_t^n. Relatively unproductive workers, those with $z < z_t^*$, suffer involuntary unemployment. Evaluating the labor demand schedule, equation (34), at $z < z_t^*$ and at $z = z_t^*$ and taking ratios, we have

$$\frac{h_t(z)}{h_t(z_t^*)} = \left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1},$$

which says that the lower productivity is, the lower the employment level will be (recall that by Assumption 1, $\eta(1-\xi) > 1$). This is so because, due to downward nominal wage rigidity, the marginal product of labor falls more quickly than the nominal wage. Unemployment can then be written as

$$u_t = u_t^n + (1-u_t^n) \int_{z<z_t^*} \left[1 - \left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1}\right] f(z)dz. \tag{41}$$

According to this expression, the unemployment rate is increasing in the productivity threshold z_t^* (a formal proof is in Claim 4 in the appendix). This is intuitive, because when z_t^* increases, some workers who were previously fully employed become unemployed.

Equations (40) and (41) parametrically represent a contemporaneous negative relationship between wage inflation and unemployment. Thus the model with heterogeneous labor productivity of workers and downward nominal wage rigidity implies that in equilibrium the economy moves along a downward sloping wage Phillips curve.

We assume that $\ln(z)$ follows a normal distribution with mean 0 and standard deviation σ_z. Then, equations (40) and (41) can be written as

$$(1+\pi_t^W)^{1-\eta} = \gamma^{1-\eta} e^{\frac{(\eta-1)^2(1-\xi)^2\sigma_z^2}{2}} \Phi\left(\frac{\ln z_t^* - (\eta-1)(1-\xi)\sigma_z^2}{\sigma_z}\right) + \gamma^{1-\eta} z_t^{*(\eta-1)(1-\xi)+\frac{1}{\eta}-1} e^{\frac{\left(1-\frac{1}{\eta}\right)^2\sigma_z^2}{2}} \left[1 - \Phi\left(\frac{\ln z_t^* - \left(1-\frac{1}{\eta}\right)\sigma_z^2}{\sigma_z}\right)\right] \tag{42}$$

and

$$u_t = u_t^n + (1-u_t^n)\left[\Phi\left(\frac{\ln z_t^*}{\sigma_z}\right) - z_t^{*1-(1-\xi)\eta} e^{\frac{(\eta(1-\xi)-1)^2\sigma_z^2}{2}} \Phi\left(\frac{\ln z_t^* - (\eta(1-\xi)-1)\sigma_z^2}{\sigma_z}\right)\right], \tag{43}$$

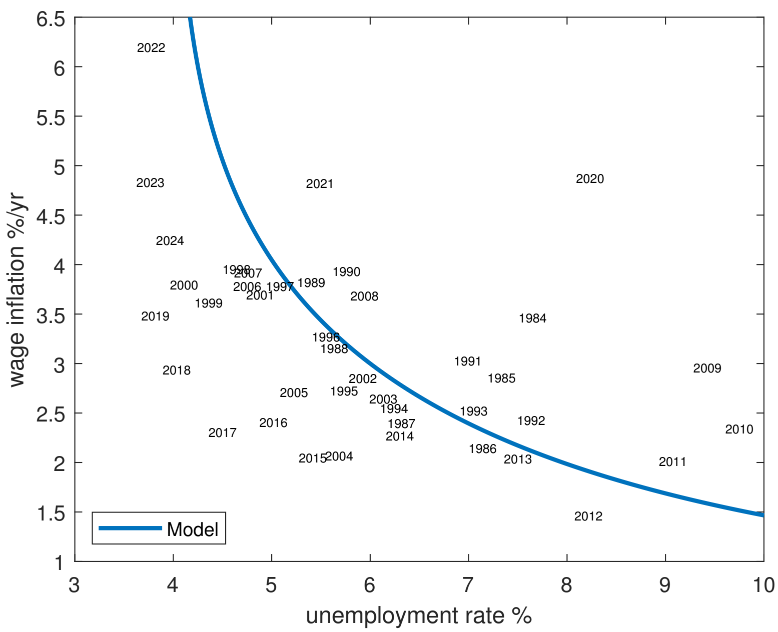
where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution.

Following McKay and Wolf (2023) we set the standard deviation of $\ln(z)$ equal to 0.2825.[8] As in the baseline model, to guarantee that the long-run wage Phillips curve is vertical, we assume that there is full indexation to long-run inflation, that is, we assume that $\gamma = (1+\pi^*)\tilde{\gamma}$, where $\tilde{\gamma}$ is a parameter. In the steady state, wage inflation is equal to the inflation target π^*. The parameters π^* and η, and the steady state value of u_t^n take the values given in Table 1. We set the steady-state unemployment rate equal to 6 percent as in the calibration of the baseline model. Following the same calibration strategy as in the baseline model, we set $\tilde{\gamma}$ and ξ to ensure that the pair $(u_t, \pi_t^W) = (u, \pi^*)$ lies on the wage Phillips curve and that at that point the slope of the wage Phillips curve is equal to -0.74/4. The implied values are $\tilde{\gamma} = 1$ and $\xi = 0.8835$.

Figure 6 displays the wage Phillips curve implied by the model. As in the baseline model the Phillips curve is convex, implying a relatively small cost in terms of inflation of reducing high unemployment as well as a relatively small cost in terms of unemployment of reducing high levels of inflation. The intuition behind this result is as follows. Take, for example, a situation in which the unemployment rate is high. In this case, a large number of workers is constrained by the wage lower bound. These workers have relatively low productivity, and their wages are too high for firms to have an incentive to fully employ them. A given increase in wage inflation reduces the relative wage of all constrained workers, that is, $W_t(z)/W_t$ goes down for this type of worker, fostering firms' demand for their services. Because the mass of unemployed workers is large, the reduction in unemployment is also relatively large. If the

[8]Specifically, McKay and Wolf (2023, Appendix B1) postulate a continuum of regular workers indexed by $j \in [0,1]$ with productivity $z_{j,t}$ following the AR(1) process $\ln z_{j,t} = 0.9^{1/4} \ln z_{j,t-1} + \sigma_\epsilon \epsilon_{j,t}$, with $\epsilon_{j,t}$ following a standard normal distribution. They estimate σ_ϵ to be 0.064. The implied unconditional standard deviation of $\ln z_{j,t}$, which is the relevant object for the present analysis, is 0.2825.

Figure 6: The Short-Run Wage Phillips Curve of the Heterogeneous Labor Productivity Model



Note. See notes to Figure 2.

initial situation is one in which unemployment is relatively low, the effect of an increase in wage inflation is qualitatively the same, that is, the relative wage of workers whose wages are constrained by the wage lower bound goes down, stimulating demand for this type of labor. However, the pool of workers constrained by the wage lower bound is small, so the fall in unemployment will also be relatively small.

7 Regular Dynamics

We have established that the heterogeneous downward nominal wage rigidity (HDNWR) model predicts that the unemployment costs of reducing inflation are much lower at high inflation rates than at low inflation rates. This result concerns the global properties of the model. A natural question is whether for regular fluctuations in a neighborhood around the intended inflation target, the HDNWR model produces equilibrium dynamics that are consistent with conventional intuition. To address this question we compare its predicted dynamics to those induced by the most widely used framework for nominal wage rigidities, namely, the new-Keynesian model with Calvo wage staggering. We find that for calibrations and shock processes typically considered in the related literature, the predicted dynamics are fairly similar. This result is of interest because unlike the HDNWR model, the new-

Keynesian model with Calvo staggering features a forward-looking wage Phillips curve. The claim in this section is not that the similarity in the dynamics predicted by the two models is necessarily valid for all possible calibrations and shock specifications. Rather the claim is that it is valid for conventional ones.

To derive this result the section characterizes the equilibrium dynamics of the HDNWR model and compares them to those implied by a new-Keynesian (NK) model with Calvo-type wage stickiness. The HDNWR model considered here is the one with endogenous labor supply developed in section 5.

It is evident from Definitions 2 and 3 that in spite of the fact that the HDNWR model with inelastic or elastic labor supply features occasionally binding constraints at the level of individual varieties of labor, its complete set of equilibrium conditions does not. This means that the model is amenable to a characterization of the equilibrium dynamics using perturbation methods. We summarize this result in the following proposition:

Proposition 2 (HDNWR and Perturbation)**.** *The equilibrium dynamics of the HDNWR model with inelastic or elastic labor supply described in Definitions 2 and 3, respectively, can be approximated using perturbation techniques.*

Thus, to obtain the implied impulse responses of the model to exogenous shocks we can follow the customary approach of linearizing the equilibrium conditions around the nonstochastic steady state. The quantitative analysis that follows adopts this approach.

The calibration of the model is summarized in Table 1. The parameters appearing in the top panel of the table were already discussed in section 4. We assume a period consumption subutility function of the form $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$ and a production function of the form $F(h) = h^{\alpha}$. Following Galí (2015), we set $\sigma = 1$, $\alpha = 0.75$, $\beta = 0.99$, $\theta = 5$, $\alpha_\pi = 1.5$, and $\alpha_y = 0.5/4$.

7.1 Response to a Monetary Shock

The monetary shock μ_t in the Taylor rule (10) is assumed to follow an autoregressive process of order one

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \epsilon_t^\mu, \tag{44}$$

where ϵ_t^μ is a mean zero i.i.d. innovation, and $\rho_\mu \in [0, 1)$ is a parameter. Following Galí (2015), we set $\rho_\mu = 0.5$.

Figure 7 displays with solid lines the impulse response to a one percent annualized increase in μ_t. In equilibrium this monetary contraction results in a 0.11 percentage point increase in the policy interest rate (from its steady-state value of 7.23 percent to 7.34 percent). The increase in the interest rate is smaller than the increase in μ_t because of the contemporaneous adjustment of the endogenous variables that enter the Taylor rule, π_t and y_t. The HDNWR model predicts that the tightening in monetary conditions is deflationary. An efficient adjustment of the labor market would require a fall in nominal wages large enough to perfectly offset the fall in prices. However, due to the presence of downward nominal wage rigidity, the decline in nominal wages is insufficient. That is, a larger number of job varieties become constrained by the lower bound on nominal wages. This frictional adjustment is reflected in a decline in the labor variety cutoff j_t^*. In turn, the fact that the real wage is inefficiently high for more labor varieties causes an increase in involuntary unemployment and hence a decline in output and consumption.

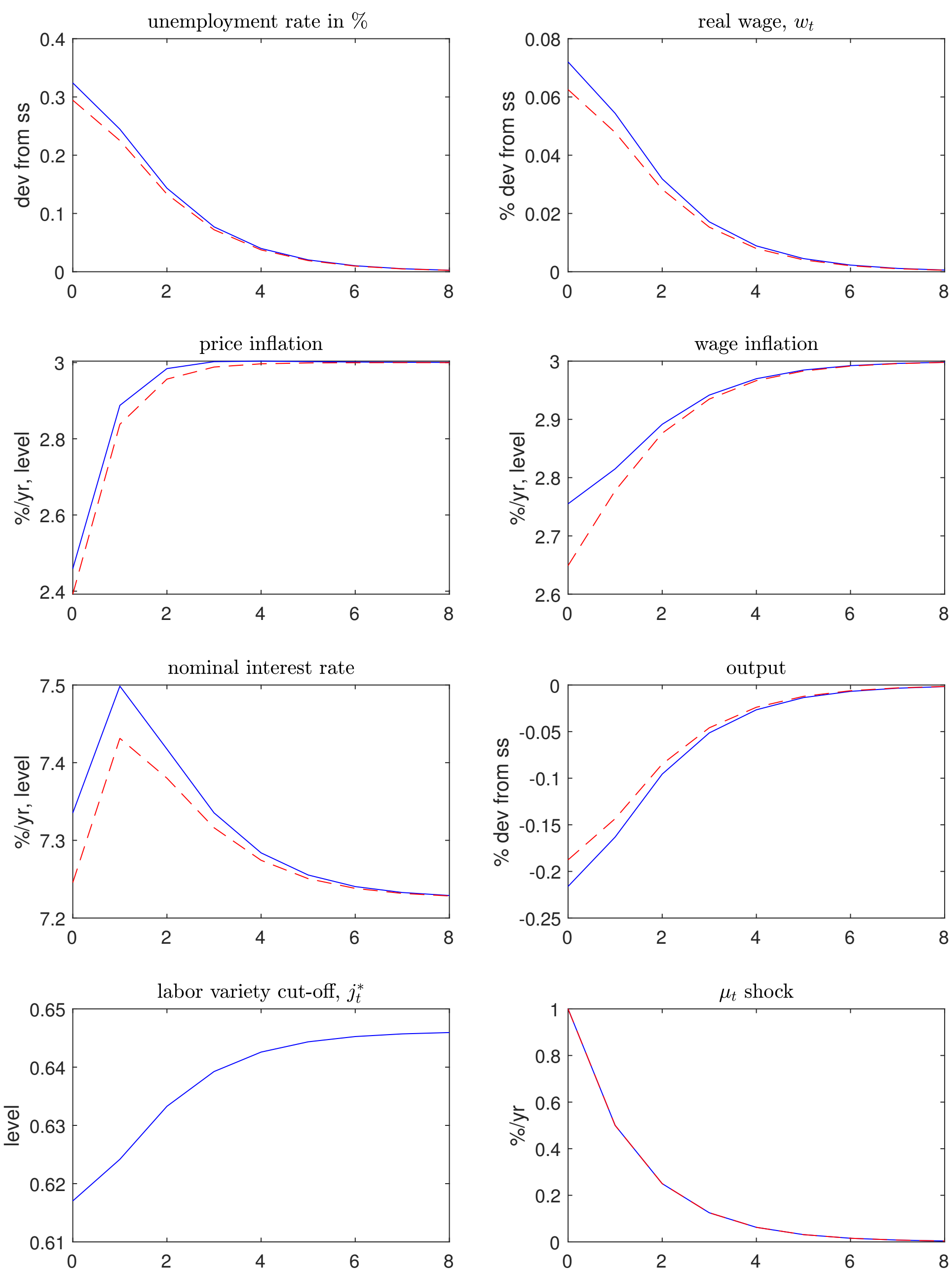
For comparison, we consider a canonical NK model with wage staggering. This model departs from the HDNWR model only in its wage setting module. Specifically, we assume that wages are set in a Calvo fashion as in Erceg, Henderson, and Levin (2000) and define the unemployment rate as in Galí (2011). A detailed derivation of the NK model we use here can be found in a technical appendix (Schmitt-Grohé and Uribe, 2022). All parameters of the NK model that are common to the HDNWR model are assigned the same values, namely, those given in Table 1. The common parameters are π^*, η, β, σ, θ, α, α_π, α_y, and ρ_μ. As in the HDNWR model, in the NK model we assume full indexation of wages to steady-state inflation.

It remains to explain how we calibrate the degree of nominal wage rigidity in the NK model. We cannot directly adopt the strategy used to calibrate the HDNWR model, namely, to match the slope of the implied wage Phillips curve to the one estimated by Galí and Gambetti (2019). The reason is that the empirical Phillips curve estimated by Galí and Gambetti is not forward looking and therefore does not have a natural theoretical counterpart in the NK model. Instead, we assume that the fraction of types of labor that cannot reoptimize wages in any given period in the NK model is equal to the steady-state fraction of types of labor that are stuck at the wage lower bound in the HDNWR model. Formally, letting θ_w denote the fraction of wages that are not set optimally in any given period in the NK model, we impose

$$\theta_w = 1 - j^*,$$

where j^* is the deterministic steady-state value of j_t^*, the fraction of labor varieties that are not stuck at their wage lower bounds in the HDNWR model. The resulting value of θ_w is 0.35. This value is low relative to those typically used to calibrate NK models. For example, Galí (2015) sets θ_w to 0.75. To address this issue, we also consider a calibration in which $\theta_w = 1 - j^* = 0.75$. In this case, we recalibrate the parameters Γ_0 and Γ_1 of the function $\gamma(j)$ in the HDNWR model. Specifically, we continue to impose that the steady-state unemployment-inflation pair matches its observed median value but drop the

Figure 7: Impulse Responses to a Monetary Tightening



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum and its serial correlation is 0.5. The horizontal axes measure quarters after the shock.

requirement that the model matches the slope of the Galí-Gambetti wage Phillips curve and instead target a value of 0.75 for $1-j^*$. The resulting values of Γ_0 and Γ_1 are 0.9908 and 0.0175.

Figure 7 displays with dashed lines the response of the NK model to a 1 percent per annum increase in the monetary shock μ_t when $\theta_w = 1 - j^* = 0.35$. The figure shows that for most variables the responses predicted by the NK and HDNWR models are quite close. Figure 8 compares the impulse responses of the two models when $\theta_w = 1 - j^* = 0.75$. Understandably, because now wages are more rigid, both models predict a more subdued response of wage inflation and a larger response of unemployment. The important point for the purpose of the present discussion, however, is that both models deliver quite similar dynamics.

7.2 Response to a Technology Shock

Figure 9 displays the response of the HDNWR model to a 1-percent positive productivity shock, that is, a 1 percent increase in the exogenous variable a_t buffeting the production function (11). The shock is assumed to follow a first-order autoregressive process of the form

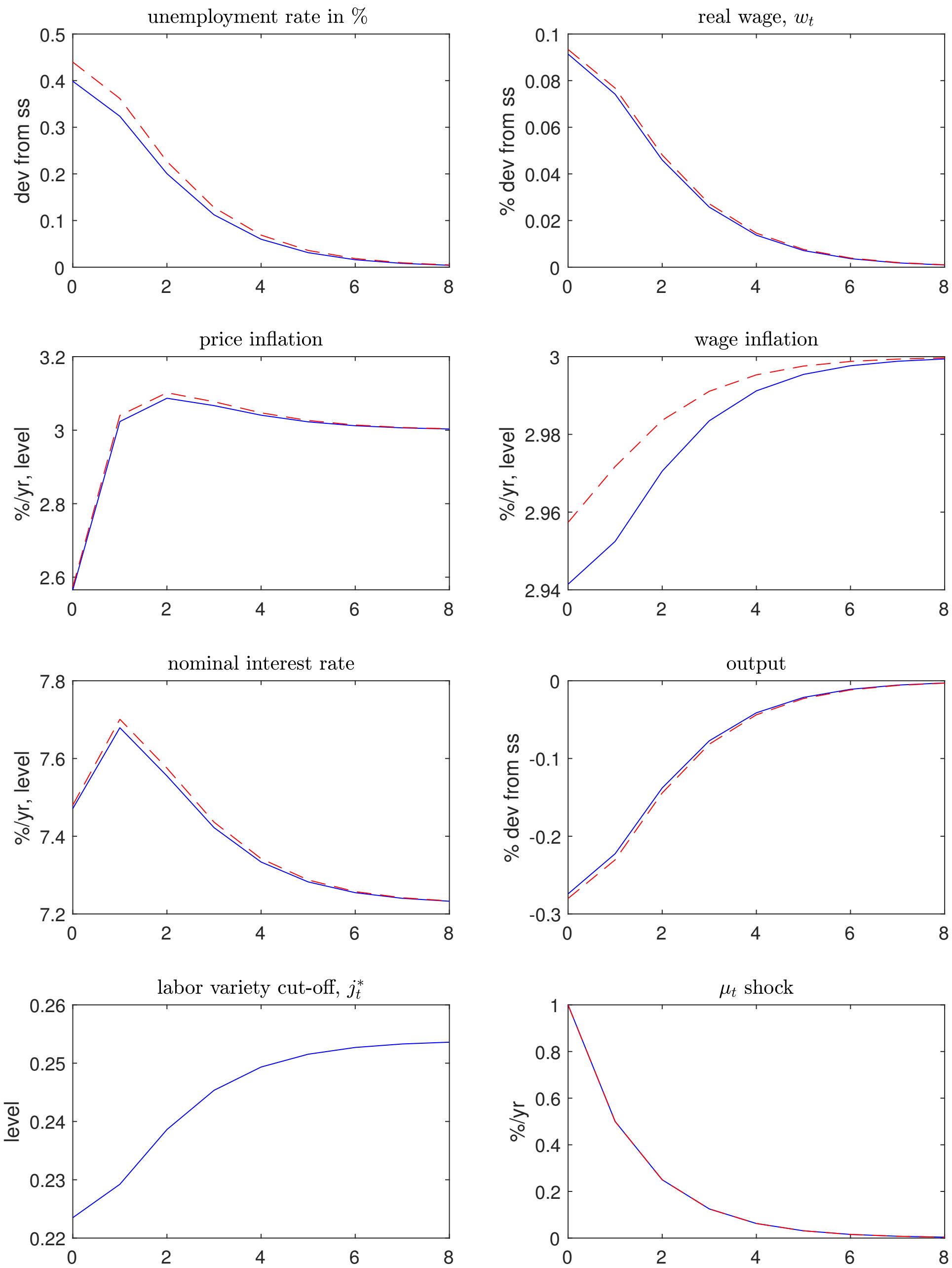
$$\ln a_t = \rho_a \ln a_{t-1} + \epsilon_t^a,$$

where ϵ_t^a is a mean-zero i.i.d. disturbance and ρ_a is a parameter. Following Galí (2015), we set ρ_a equal to 0.9.

The increase in output following the positive technology shock puts downward pressure on product-price inflation. The increase in labor productivity following the technological improvement pushes nominal wages up. This relaxes the wage constraint for some wage varieties (j_t^* goes up on impact), inducing a fall in unemployment in the initial period. As the technology shock begins to return to its stationary position, real wages fall. However, due to the presence of wage rigidity, they fall at a slower pace than the one consistent with full employment. As a result, unemployment rises and remains above steady state throughout the transition.

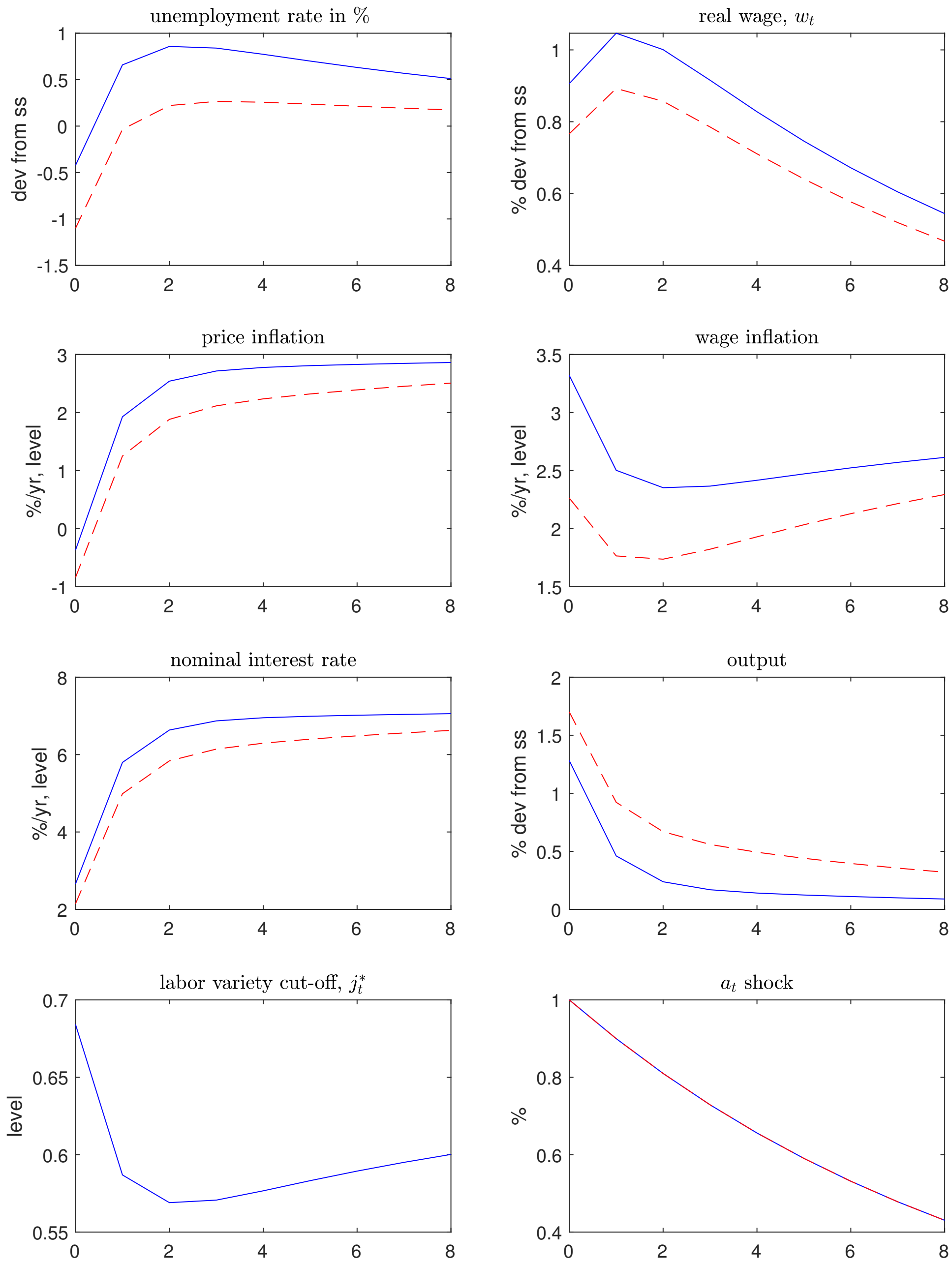
The response of the NK model to the positive productivity shock, shown with dashed lines in Figure 9, is similar. The main difference is that under the present calibration in the NK model unemployment experiences a larger decline on impact and a smaller subsequent increase. This difference in the response of unemployment is due to the relatively low value picked for the degree of wage rigidity ($\theta_w = 0.35$). When we set θ_w to the more conventional value of 0.75 and recalibrate the HDNWR model to target $1 - j^* = 0.75$, then the response of unemployment to the technological improvement is almost the same in both models. This

Figure 8: Impulse Responses to a Monetary Tightening with a Higher Degree of Wage Rigidity ($\theta_w = 1 - j^* = 0.75$)



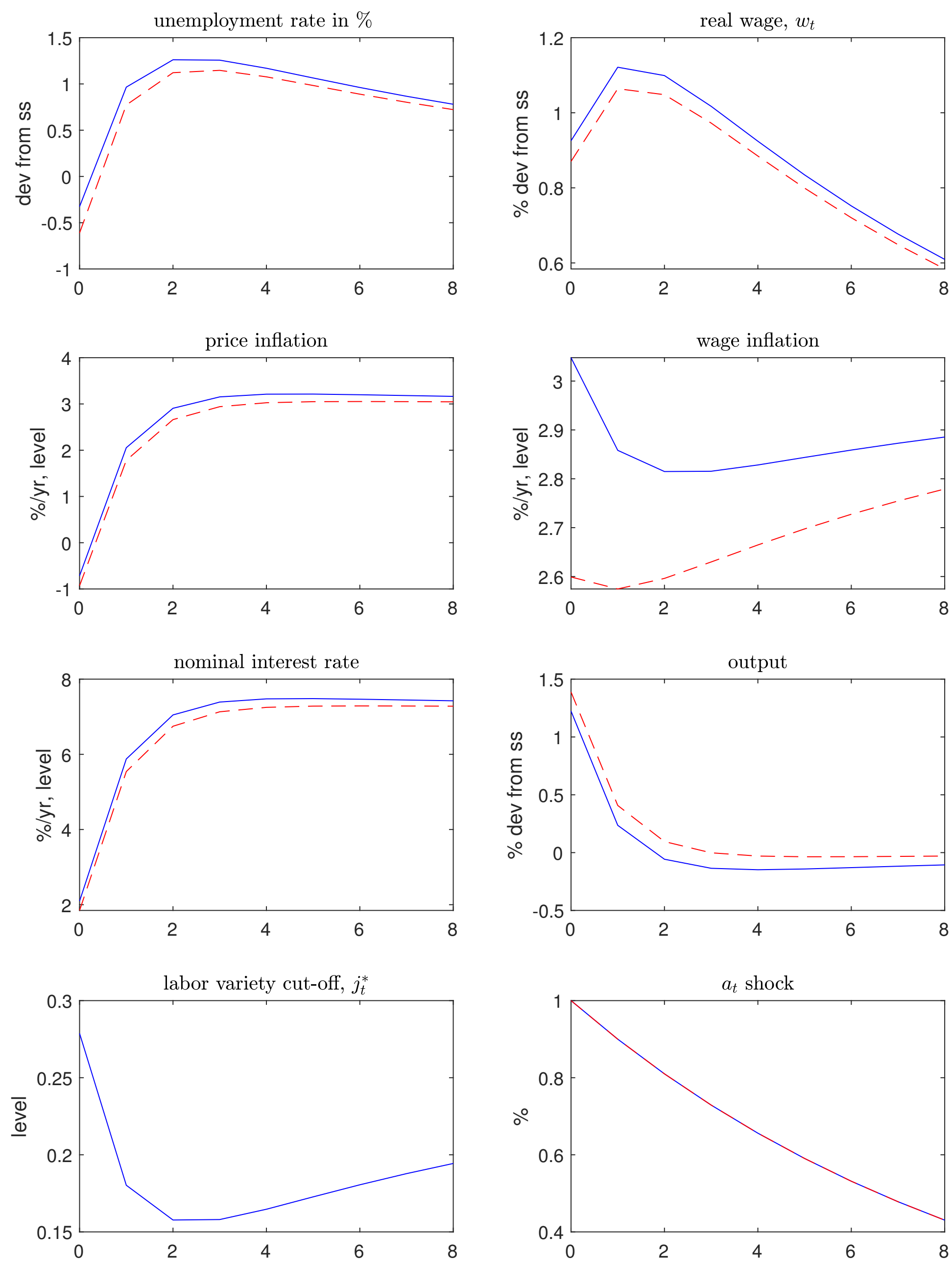
Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum and its serial correlation is 0.5. The horizontal axes measure quarters after the shock.

Figure 9: Impulse Responses to a Productivity Shock



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the shock is 1 percent and its serial correlation is 0.9.

Figure 10: Impulse Responses to a Productivity Shock with a Higher Degree of Wage Rigidity ($\theta_w = 1 - j^* = 0.75$)



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the shock is 1 percent and its serial correlation is 0.9.

result is shown in Figure 10. The two models produce virtually the same responses to the positive productivity shock not only for unemployment but also for most of the other endogenous variables displayed in the figure.

Overall, the results of the present section suggest that for regular fluctuations in a neighborhood around the steady state, at least for the conventional calibrations considered here, the forward-looking nature of the wage Phillips curve in the NK framework, which up to first order is the key difference between the HDNWR model and the NK model, does not appear to play a crucial role for the predicted response to monetary and productivity disturbances.

To obtain some intuition for this result, consider the linear versions of the wage Phillips curves in the HDNWR and NK models, which can be written, respectively, as

$$\hat{\pi}_t^W = \kappa_1 \hat{u}_t$$

and

$$\hat{\pi}_t^W = \beta E_t \hat{\pi}_{t+1}^W + \kappa_2 \hat{u}_t,$$

where a ˆ superscript denotes deviations from the nonstochastic steady state. Iterating the NK Phillips curve forward, yields

$$\hat{\pi}_t^W = \sum_{t=0}^{\infty} \beta^j \kappa_2 E_t \hat{u}_{t+j}.$$

Assume for simplicity that in equilibrium unemployment follows an AR(1) law of motion of the form $E_t \hat{u}_{t+j} = \lambda(\rho, \chi)^j \hat{u}_t$, where the persistence parameter $\lambda(\rho, \chi)$ is an endogenous object that depends not only on the persistence of the exogenous shock in question, $\rho = \rho_\mu, \rho_a$, but also on the vector χ containing other structural parameters of the NK model that influence the endogenous persistence in unemployment.[9] Then, we have that

$$\hat{\pi}_t^W = \frac{\kappa_2}{1 - \lambda(\rho, \chi)\beta} \hat{u}_t.$$

The two models will deliver more similar dynamics the more similar are κ_1 and $\kappa_2/(1 - \lambda(\rho, \chi)\beta)$. More importantly, the similarity of the two models will not be much affected by changes in the persistence of the exogenous shock, ρ, if $\lambda(\rho, \chi)$ is relatively insensitive to ρ. As it turns out, for the two shocks considered, λ is not too sensitive to changes in ρ for values of $\rho = \rho_\mu, \rho_a$ between 0 and the respective calibrated values. For this range, the endogenous persistence in unemployment built in the NK model dominates the persistence induced by

[9] In the NK model with wage stickiness and a Taylor rule endogenous persistence arises from past real wages being a state variable.

the exogenous shocks. As the exogenous shocks became highly serially correlated, their persistence might dominate the persistence of unemployment. In this range, the dynamics of the HDNWR and NK models can be quantitatively dissimilar.

8 Conclusion

This paper contributes to understanding nonlinearities in the trade off between inflation and unemployment. To this end, it proposes a model with heterogeneous downward nominal wage rigidity originating either in cross-sectional heterogeneity in nominal fairness standards or in cross-sectional heterogeneity in labor productivity. These innovations result in a convex short-run wage Phillips curve. At high inflation levels, the predicted Phillips curve is relatively steep, indicating that the costs of reducing high inflation can be low in terms of employment. Conversely, at high levels of unemployment, the curve is relatively flat, suggesting that the cost of reducing high unemployment can be low in terms of inflation.

The wage Phillips curve predicted by the model captures relatively well the observed pattern of wage inflation and unemployment in the United States since the mid 1980s. In particular, it predicts that the inflation-unemployment pairs corresponding to the post-Covid-19 inflation spike lie in the steep portion of the curve. The predicted nonlinearity in the Phillips curve provides an explanation for the observed robustness of the labor market during the significant monetary tightening triggered by the post-pandemic inflation episode in the United States and elsewhere. The model also predicts that the U.S. economy was buffeted by large negative aggregate supply shocks during the pandemic but that the post-pandemic inflation spike was primarily due to demand shocks. Further, the model predicts that the relatively mute response of inflation to the decline in unemployment observed during the recovery of the U.S. economy from the 2008 financial crisis lies on a relatively flat portion of the short-run wage Phillips curve, providing a rationale for the missing inflation puzzle associated with that episode.

The model predicts a long-run unemployment rate larger than the natural or full-employment unemployment rate, $u > u^n$. Recently, it has been argued that this gap, which is also observed in the data, is a measure of the failure of the monetary authority to achieve its maximum employment mandate. However, the theory presented here suggests that the central bank cannot close this gap without generating an ever accelerating rate of inflation. Put differently, the steady-state rate of unemployment, u, coincides with the notion of NAIRU. According to the model, narrowing the difference between the NAIRU and the full-employment rate of unemployment, $u - u^n$, requires structural reforms aimed at mitigating heterogeneity in the degree of downward nominal wage rigidity across labor varieties.

Methodologically, the paper contributes to macroeconomic modeling by providing a theoretical framework with downward nominal wage rigidity that is amenable to analysis using perturbation techniques. This property arises because, although the model features occasionally binding constraints at the micro level, no such constraints appear at the aggregate level. This is not the case in models with homogeneous downward nominal wage rigidity, which, we suspect, has impeded their adoption for the formulation, computation, and estimation of medium scale models for policy evaluation. In this regard, the proposed framework aims to lower the entry barrier for models with this empirically compelling type of nominal rigidity.

Appendix

This appendix presents proofs to various claims made in the body of the paper.

Claim 1 (Full employment for all labor varieties $z > z_t^*$)**.** *If Assumption 1 holds, then for all $z > z_t^*$ there is full employment, $h_t(z) = \bar{h}(1 - u_t^n)$ and the wage rate is unconstrained, $W_t(z) > z^\xi \gamma \tilde{W}_{t-1}$.*

Proof. Suppose that contrary to the claim, $h_t(z) < \bar{h}(1 - u_t^n)$ for some $z > z_t^*$. Then by equilibrium condition (37), the wage lower bound must bind, $W_t(z) = z^\xi \gamma \tilde{W}_{t-1}$, or $\tilde{W}_t(z) = z^{\xi-1}\gamma \tilde{W}_{t-1}$, Using this expression to eliminate $\tilde{W}_t(z)$ from the labor demand function (3) yields

$$zh_t(z) = \left(\frac{z^{\xi-1}\gamma \tilde{W}_{t-1}}{\tilde{W}_t} \right)^{-\eta} h_t.$$

By definition, for the cutoff productivity z_t^*, the wage lower bound also binds, so the above expression also holds when evaluated at z_t^*,

$$z_t^* h_(z_t^*) = \left(\frac{z_t^{*\xi-1}\gamma \tilde{W}_{t-1}}{\tilde{W}_t} \right)^{-\eta} h_t.$$

Taking the ratio of these two expressions, we have that

$$\frac{h_t(z)}{h(z_t^*)} = \left(\frac{z}{z_t^*} \right)^{\eta(1-\xi)-1}.$$

By Assumption at the cutoff productivity z_t^* there is full employment, we can write this expression as

$$h_t(z) = \left(\frac{z}{z_t^*} \right)^{\eta(1-\xi)-1} \bar{h}(1 - u_t^n).$$

Since, by assumption $z > z_t^*$ and $\eta(1-\xi) - 1 > 0$, we have that

$$h_t(z) > \bar{h}(1 - u^n),$$

which is a contradiction. We have thus shown that for $z > z_t^*$, $h_t(z) = \bar{h}(1 - u_t^n)$. It remains to show that $W_t(z) > z^\xi \gamma \tilde{W}_{t-1}$. Evaluate the labor demand function (34) at z and at z_t^* and use the fact that $h_t(z) = h(z_t^*) = \bar{h}(1 - u_t^n)$ and that $W(z_t^*) = z_t^{*\xi}\gamma \tilde{W}_{t-1}$. This yields

$$z\bar{h}(1 - u_t^n) = \left(\frac{W_t(z)}{z\tilde{W}_t} \right)^{-\eta} h_t$$

and

$$z_t^* \bar{h}(1-u_t^n) = \left(\frac{z_t^{*\xi-1} \gamma \tilde{W}_{t-1}}{\tilde{W}_t} \right)^{-\eta} h_t.$$

Dividing the first expression by the second, and solving for $W_t(z)$ we obtain

$$W_t(z) = \left(\frac{z}{z_t^*} \right)^{\frac{\eta(1-\xi)-1}{\eta}} z^{\xi} \gamma \tilde{W}_{t-1} > z^{\xi} \gamma \tilde{W}_{t-1},$$

where the inequality follows from Assumption 1 and the fact that $z > z_t^*$. Thus we have shown that (36) holds with strict inequality for $z > z_t^*$, which is what we had wanted to show. □

Claim 2 (Unemployment for all labor varieties $z < z_t^*$)**.** *If Assumption 1 holds, then for all $z < z_t^*$, there is involuntary unemployment, $h_t(z) < \bar{h}(1-u_t^n)$ and $W_t(z) = z^{\xi} \gamma \tilde{W}_{t-1}$.*

Proof. Suppose, contrary to the claim, that for $z < z_t^*$, $h_t(z) = \bar{h}(1-u_t^n)$. Then evaluate equilibrium condition (34) at z to obtain

$$z\bar{h}(1-u_t^n) = \left(\frac{\tilde{W}_t(z)}{\tilde{W}_t} \right)^{-\eta} h_t = \left(\frac{W_t(z)}{z\tilde{W}_t} \right)^{-\eta} h_t.$$

Evaluate (34) at z_t^* and use the fact that by definition $h(z_t^*) = \bar{h}(1-u_t^n)$. This yields

$$z_t^* \bar{h}(1-u_t^n) = \left(\frac{W_t(z_t^*)}{z_t^* \tilde{W}_t} \right)^{-\eta} h_t$$

Dividing the above equation by this expression we obtain

$$\frac{z}{z_t^*} = \left(\frac{z_t^* W_t(z)}{z W_t(z_t^*)} \right)^{-\eta}.$$

Solve for $W_t(z)$ and note that the maintained assumptions, namely, that $\eta > 0$ and $\eta(1-\xi) > 1$, imply that $0 < 1 - \frac{1}{\eta} < 1$. This yields

$$W_t(z) = \left(\frac{z}{z_t^*} \right)^{1-\frac{1}{\eta}} W_t(z_t^*) = \left(\frac{z}{z_t^*} \right)^{1-\frac{1}{\eta}} z_t^{*\xi} \gamma \tilde{W}_{t-1} = \left(\frac{z}{z_t^*} \right)^{\frac{\eta(1-\xi)-1}{\eta}} z^{\xi} \gamma \tilde{W}_{t-1} < z^{\xi} \gamma \tilde{W}_{t-1},$$

which violates (36). Thus for $z < z_t^*$, $h_t(z) = \bar{h}(1-u_t^n)$ is impossible. The only equilibrium outcome therefore is $h_t(z) < \bar{h}(1-u_t^n)$. It then follows immediately from equilibrium condition (37) that $W_t(z) = z^{\xi} \gamma \tilde{W}_{t-1}$. □

Claim 3 (Relation between π_t^W and z_t^*)**.** *If Assumption 1 holds, then in equilibrium* π_t^W *and* z_t^* *are negatively related.*

Proof. Let RHS denote the right-hand side of equilibrium condition (40), that is,

$$RHS \equiv \int_{z<z^*} \left(\frac{\gamma}{z^{1-\xi}}\right)^{1-\eta} f(z)dz + \int_{z>z^*} \left(\frac{\gamma}{{z_t^*}^{1-\xi}} \left(\frac{z}{z_t^*}\right)^{-\frac{1}{\eta}}\right)^{1-\eta} f(z)dz$$

and let LHS denote the left-hand side of equilibrium condition (40), that is,

$$LHS = (1+\pi_t^W)^{1-\eta}.$$

Totally differentiating (40) yields

$$\frac{dLHS}{d\pi_t^W} d\pi_t^W = \frac{dRHS}{dz_t^*} dz_t^*.$$

Rearrange to obtain

$$\frac{d\pi_t^W}{dz_t^*} = \frac{\frac{dRHS}{dz_t^*}}{\frac{dLHS}{d\pi_t^W}}.$$

The derivative of RHS with respect to z_t^* is given by

$$\frac{dRHS}{dz_t^*} = (\eta-1)\,\frac{\frac{\eta(1-\xi)-1}{\eta}}{z_t^*} \int_{z>z^*} \left(\frac{\gamma}{{z_t^*}^{1-\xi}} \left(\frac{z}{z_t^*}\right)^{-\frac{1}{\eta}}\right)^{1-\eta} f(z)dz > 0,$$

where the inequality follows from Assumption 1 and the fact that the assumption that $\eta > 0$ together with Assumption 1 imply that $\eta > 1$. The derivative of LHS with respect to π_t^W is given by

$$\frac{dLHS}{d\pi_t^W} = (1-\eta)(1+\pi_t^W)^{-\eta} < 0,$$

where the inequality follows from the fact that the assumption that $\eta > 0$ together with Assumption 1 imply that $\eta > 1$. Thus we have that

$$\frac{d\pi_t^W}{dz_t^*} = \frac{\frac{dRHS}{dz_t^*}}{\frac{dLHS}{d\pi_t^W}} < 0.$$

□

Claim 4 (Relation between u_t and z_t^*)**.** *If Assumption 1 holds, then in equilibrium* u_t *and* z_t^* *are positively related.*

Proof. Differentiating equilibrium condition (41) with respect to z_t^* yields

$$\frac{d\,u_t}{d\,z_t^*} = (1 - u_t^n)\frac{\eta(1-\xi) - 1}{z_t^*}\int_{z<z_t^*}\left(\frac{z}{z_t^*}\right)^{\eta(1-\xi)-1} f(z)dz > 0,$$

where the inequality follows from Assumption 1. □

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