# *Experiment 1: Velocity, acceleration and measurement of g*

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*Office Hour: Monday, 5:30PM-6:30PM @ Pupin 1216*

INTRO TO EXPERIMENTAL PHYS-LAB
1493/1494/2699

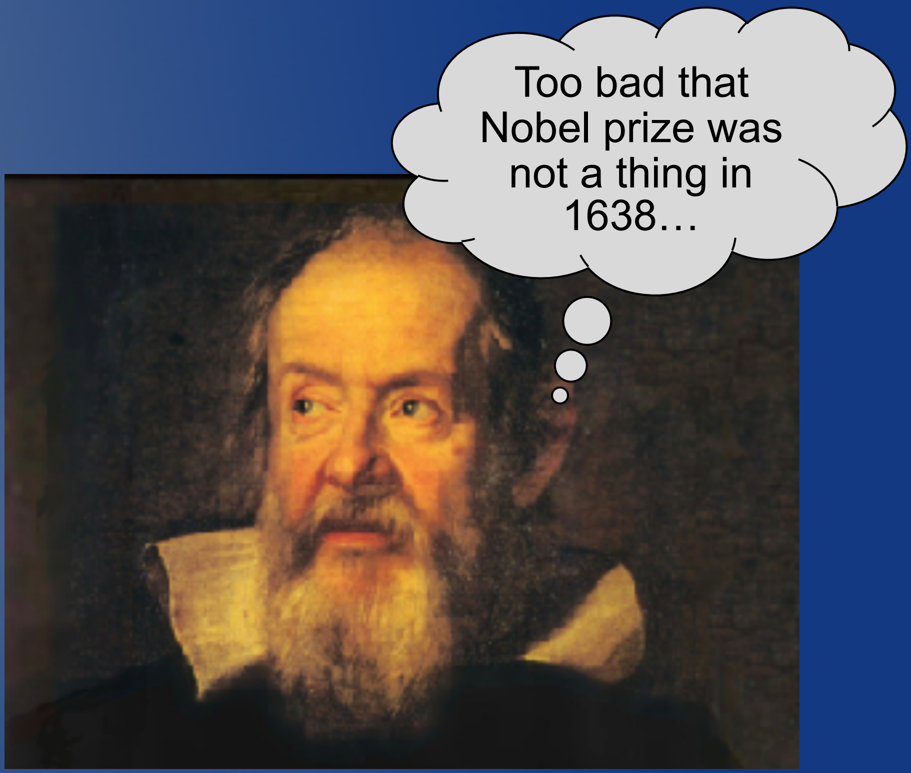
# Overview

- ***Introduction***
  - Brief historical introduction (Galilei and Newton)
  - The physics behind the experiment (equations of kinematics, force equation, elastic collisions)
- ***The experiment***
  - Description of the apparatus
  - <u>Part 1:</u> the coefficient of restitution
  - <u>Part 2:</u> acceleration of gravity, *g*
  - Tips

# History: Galieo Galilei

- Experiments aimed at studying the motion of bodies undergoing a uniform acceleration are literally the first scientific experiments of human history!

- **Galileo** (circa 1638):
  - Built his own smooth, frictionless inclined plane
  - He realized that bodies possess inertia
  - He showed for the first time that bodies undergoing constant acceleration move with displacement proportional to a squared time




- In doing this he set the criteria for the **scientific method**:
  - Observation → Prediction → Experiment

# History: Sir Isaac Newton

- After Galileo, other very important experimental physicists studied the kinematics of bodies on Earth and in the sky (Johannes Kepler, Tycho Brahe, …)
- They paved the road to the first great theoretical physicist: **Sir Isaac Newton**

*"If I have seen further, it is by standing on the shoulders of giants"*

*Isaac Newton*

- Newton formulated a theory that has been regarded as the definitive, exact one for centuries
- To change this paradigm we will have to wait the revolutionary work of Albert Einstein in 1905

# History: Sir Isaac Newton

- **Three Newtonian laws of motion:**
  - First Law: A body will stay in **constant** motion unless it is acted upon by a **force**.
  - Second Law: the acceleration due to a force is proportional to the force itself:

$$\boxed{\vec{F}_{\mathrm{tot}} = m\vec{a}_{\mathrm{tot}}}$$

  - Third Law: For every force there will be a **reaction force** equal in magnitude, but opposite in direction.

- The first law is due to the inertial nature of mass
- If a body is at rest it will stay so. If it is in motion with constant velocity it will stay so.
- Every ***change*** in motion must be due to a force!

# Motion under constant acceleration

- If a body is subject to a constant acceleration in 2 dimensions it is easy to find the velocity as a function of time:

$$\frac{dv_x}{dt} = a_x = \text{const.} \qquad \frac{dv_y}{dt} = a_y = \text{const.} \quad \Longrightarrow \quad \int dv_x = a_x \int dt \qquad \int dv_y = a_y \int dt \quad \Longrightarrow \quad v_x(t) = v_{0,x} + a_x t \qquad v_y(t) = v_{0,y} + a_y t$$

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- Integrating again we find the position as a function of time:

$$\int dx = \int v_x(t) dt = \int (v_{0,x} + a_x t) dt \qquad \int dy = \int v_y(t) dt = \int (v_{0,y} + a_y t) dt \implies \boxed{\begin{aligned} x(t) &= x_0 + v_{0,x} t + \frac{1}{2} a_x t^2 \\ y(t) &= y_0 + v_{0,y} t + \frac{1}{2} a_y t^2 \end{aligned}}$$

- This is exactly what Galileo observed experimentally

# Vector equations

- ***<u>Very important:</u>***

Newton's law is a vectorial equation

⬇

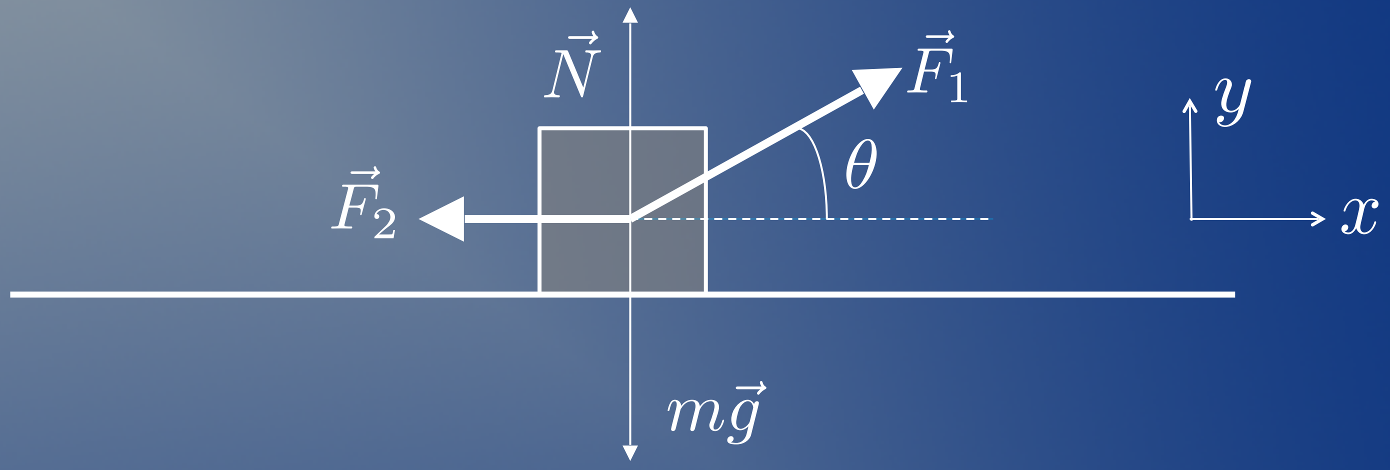
It contains one equation for each direction in space!

- This means that in general one has to <u>apply the following procedure</u>:
  1. Choose a suitable, convenient system of Cartesian axes
  2. Consider ***<u>all</u>*** the forces in play and compute their vector sum, $\vec{F}_{\mathrm{tot}}$
  3. Each component of $\vec{F}_{\mathrm{tot}}$ will correspond to one Newton's equation for the body:

$$F_{\mathrm{tot},x} = ma_x$$
$$F_{\mathrm{tot},y} = ma_y$$
$$F_{\mathrm{tot},z} = ma_z$$

# Vector equations: an example

- Let's take a quick look at a simple example: a body on a table with two external forces applied as follow




- Newton's law along the x- and y-axes:

$$F_{\mathrm{net},x} = F_1 \cos\theta - F2 = ma_x$$

$$F_{\mathrm{net},y} = N + F_1 \sin\theta - mg = ma_y$$

- If we know every force we can then solve for the accelerations. Viceversa, if we know the accelerations we can find some of the forces

# (Quasi) elastic collision

- In a perfectly elastic collision both energy and momentum are conserved
- If you have a single body of mass *m* and velocity *v* bouncing ***elastically*** off a wall it will reverse its direction. Hence, from conservation of momentum:

$$p_i = p_f \Rightarrow mv_i = -mv_f \Rightarrow v_i = v_f$$

Mass of the body clearly stays the same

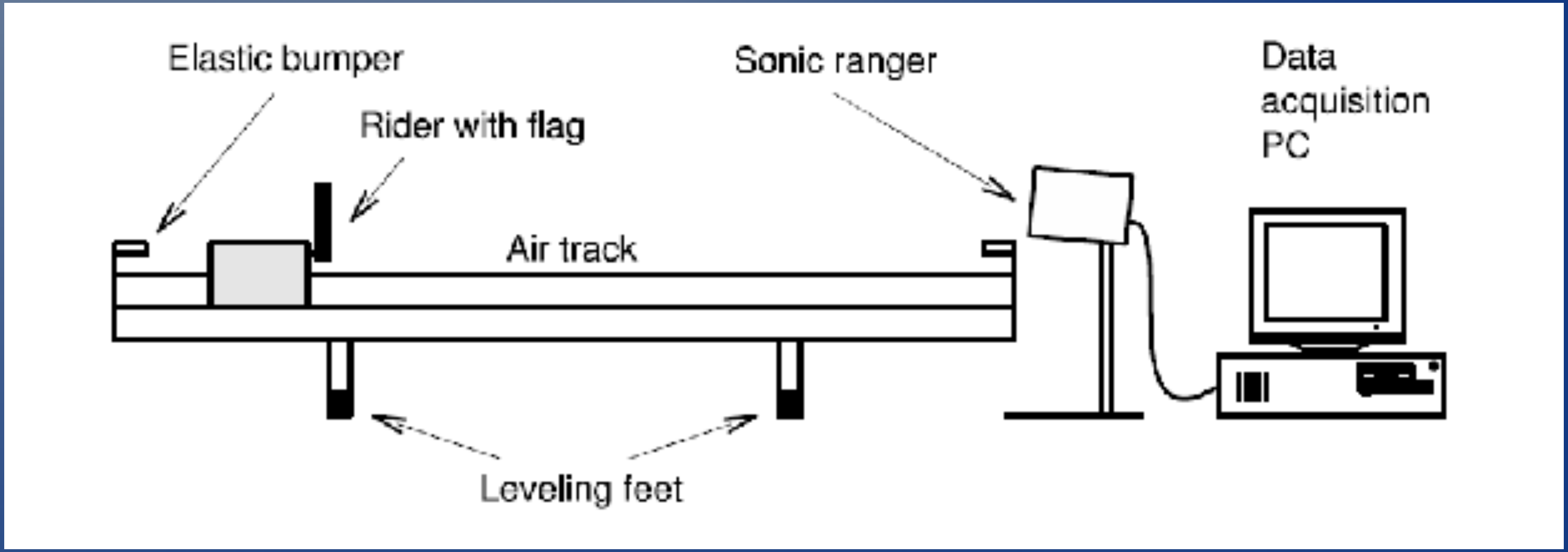
- If the collision is perfectly elastic the velocity only changes direction but the magnitude remains the same
- One can then have a measure of the elasticity of a collision by computing the **coefficient of restitution**:

$$\boxed{e = \left|\frac{v_f}{v_i}\right|} = \begin{cases} 1 & \text{elastic} \\ < 1 & \text{non elastic} \end{cases}$$

# The Experiment

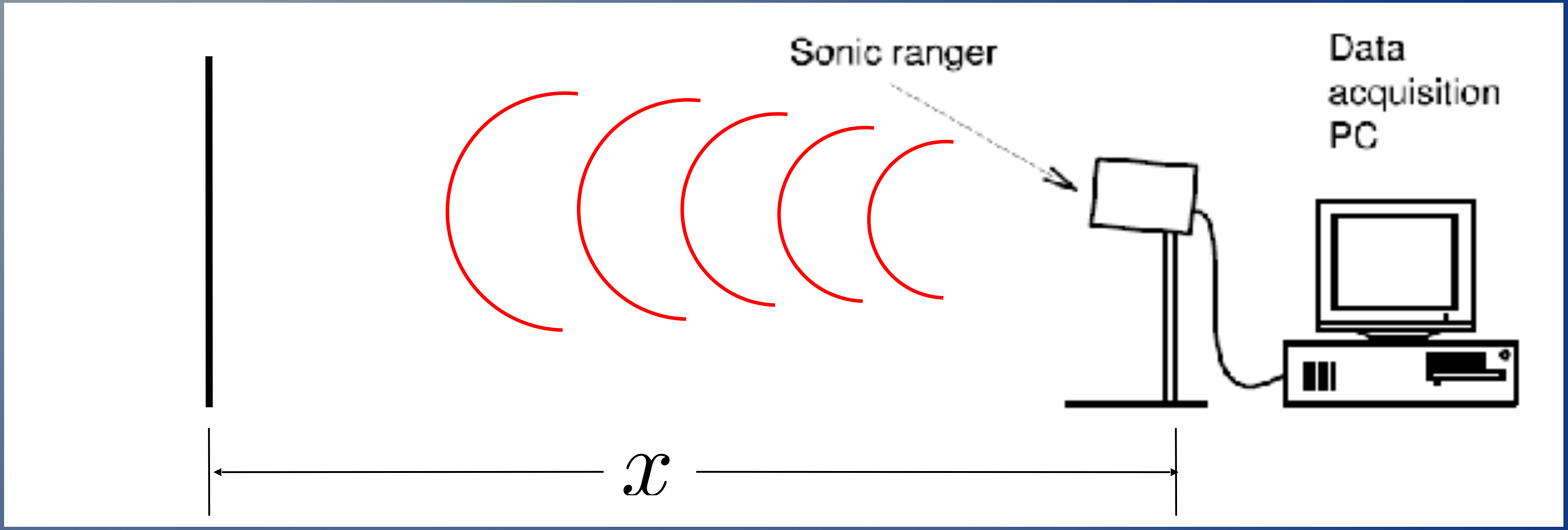
# Apparatus: the “frictionless” air-track

- **Air-track**: metal bar with pinholes for air to flow
- Thanks to the flow of air the rider runs on a nearly frictionless cushion of air
- Timing is performed by Sonic Ranger



# Apparatus: position measurement

- Sends out pulses of sound waves with a frequency *f = 20 Hz*.



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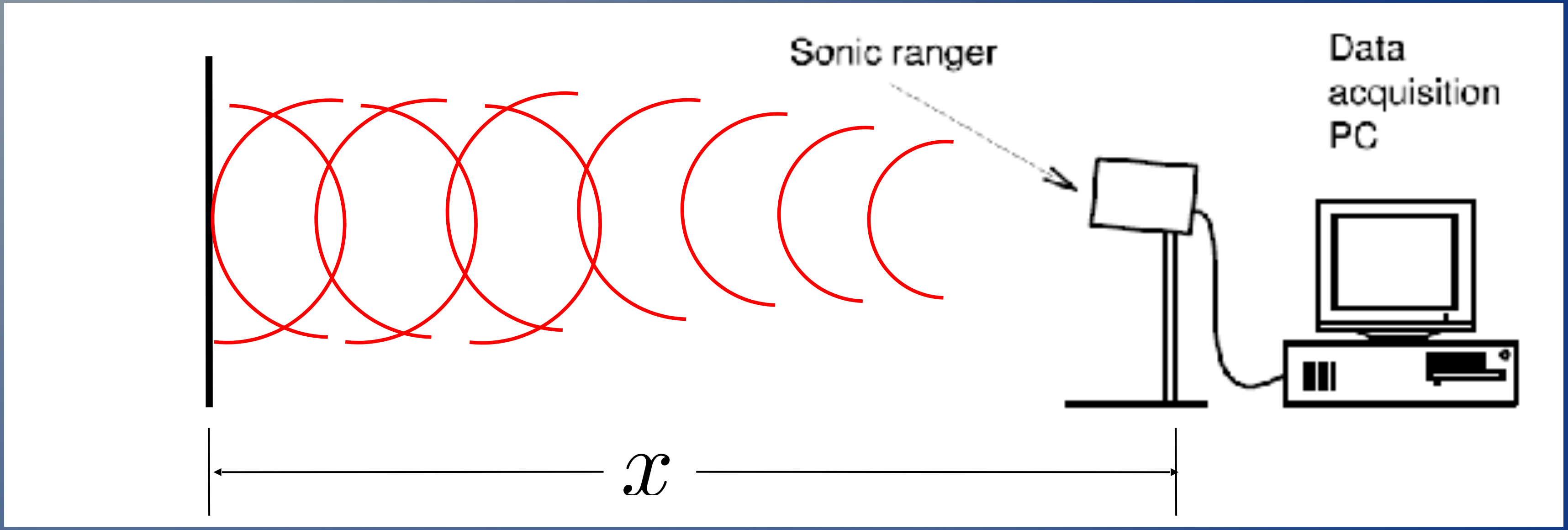


- Detects reflected sound waves and records “round trip” time ($\tau$) for each pulse.
- It then measures the distance to the reflecting object using the knowledge of $\tau$ and of the speed of sound ($v_s$):

$$x = \frac{v_s \tau}{2}$$

# Apparatus: velocity measurement

- Traditional radars measure the velocity of a body by detecting the shift in frequency between sent and received waves (Doppler effect)
- Our radar as a less sophisticated method. It simply computes the average velocity over very small time intervals:

$$v(t) = \frac{\Delta x}{\Delta t} = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

# Apparatus: velocity measurement

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The output will look like this:

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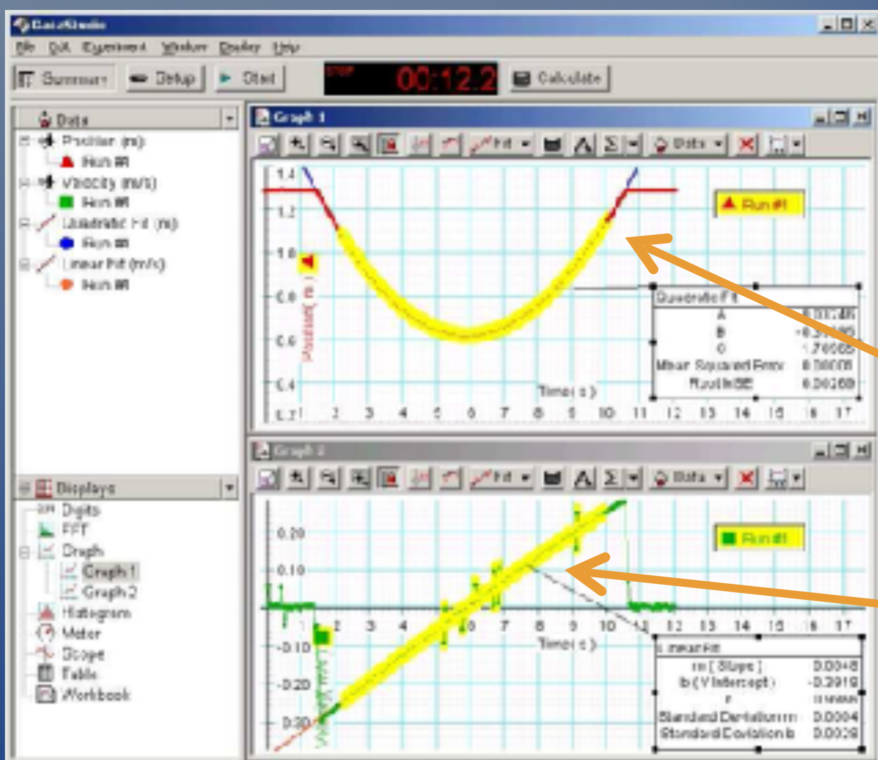



Position vs. time - - - *x(t) vs. t*

Velocity vs. time - - - *v(t) vs. t*

# Main goals

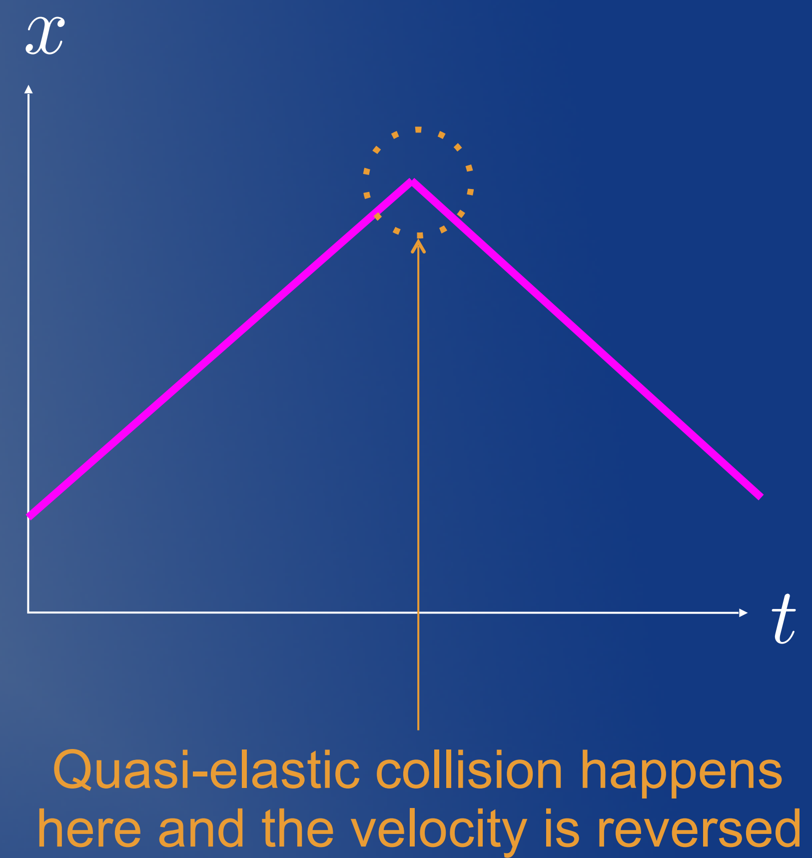
- Measure *x(t)* vs. t and *v(t)* vs. t under different physical circumstances
- ***Part 1:***
  - Achieve *motion with constant velocity* with level air-track
  - Collide the rider with one end of the air-track
  - Measure coefficient of restitution
- ***Part 2:***
  - Achieve *motion with constant acceleration* with inclined air-track
  - Extract the acceleration of gravity, *g*, from the *x(t)* and *v(t)* curves
  - Estimate effect of friction

# Leveling Air-track

- It is essential for the whole experiment to level the air-track carefully before performing your measurements
- Use the following <u>procedure</u>:
  1. Turn on air and place the rider on the air-track
  2. Adjust the feet of the track until the air-track doesn't move on its own anymore
  3. Check for different positions of the rider
- ***<u>IMPORTANT NOTES:</u>***
  - Some track might be slightly bent on different spots and therefore the rider might never be at rest. Check that the motion is random and slow
  - Do not use shims to raise the track! You will need them later. You can use paper instead

# Part 1: quasi-elastic collision

- Put the <u>rubber band</u> on the rider
- Push the rider away from the sonic ranger and let it <u>bounce off the other side</u>
- Your *x(t) vs. t* plot will look like the one on the right
- Use Data Studio to **obtain the slopes** (with **errors**!) of the lines before and after the collision




- Perform the collision **10 times** and repeat the above steps

# Part 1: analysis

- The slope of each *x(t) vs. t* is the (constant) velocity of the rider
- For each of the 10 measurement you can then compute the coefficient of restitution:

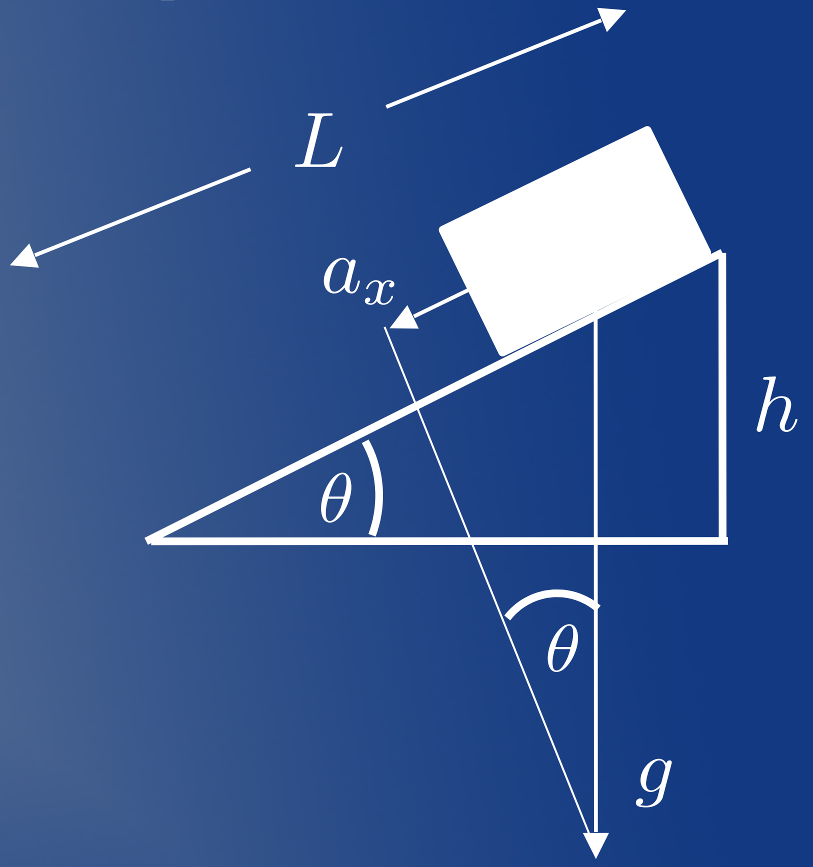
$$e = \left|\frac{v_f}{v_i}\right| = \left|\frac{\text{slope after}}{\text{slope before}}\right|$$

- <u>Remember to propagate errors!</u>
- After that you will have 10 values of $e \pm \sigma_e$
- Use them to compute ~~both unweighted and~~ weighted average and error for the coefficient of restitution
- This will be your final result

# Part 2: set up

- In the set up of this experiment (typical inclined plane) the perpendicular component of *g* is canceled by the normal reaction
- The parallel component is instead given by:

$$a_x = g \sin\theta = g\frac{h}{L}$$



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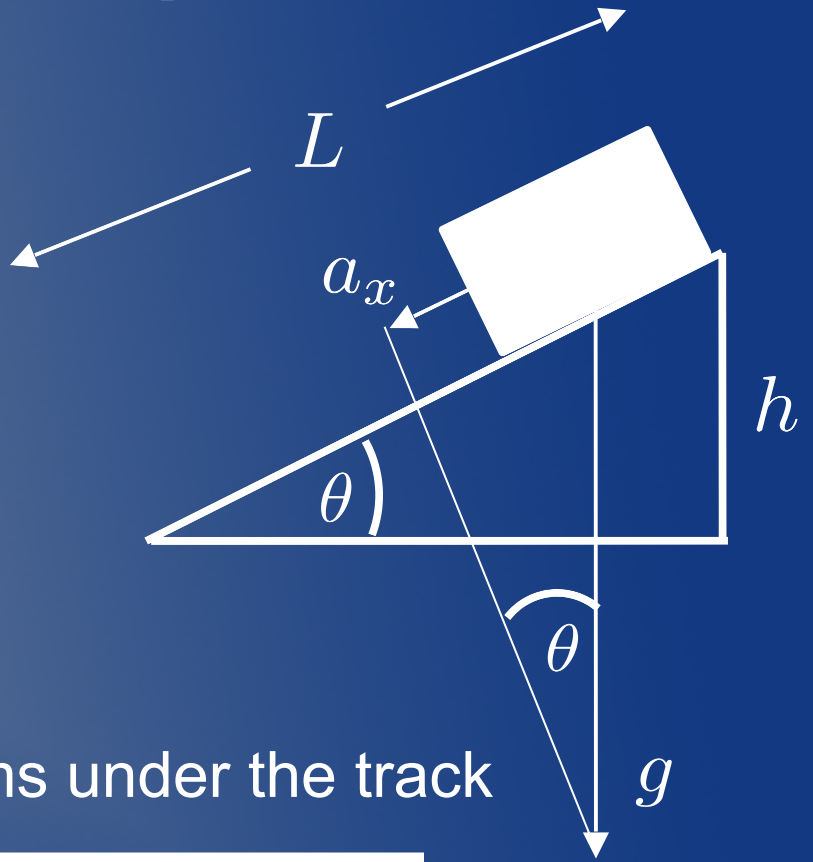


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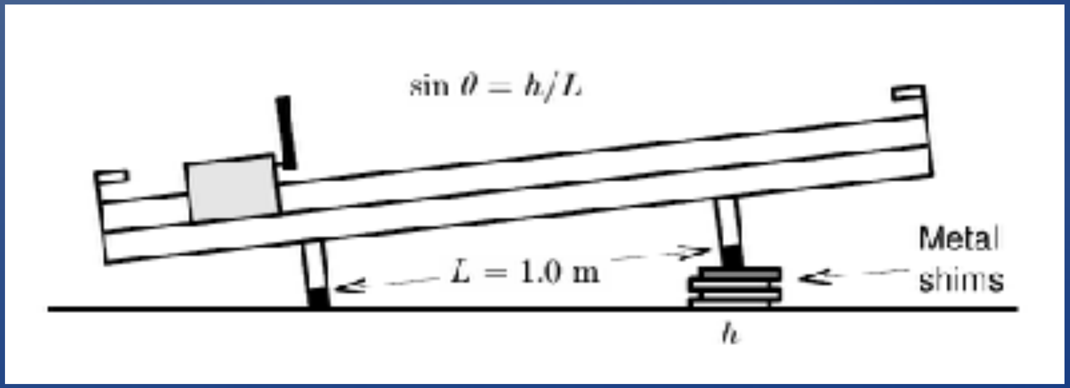
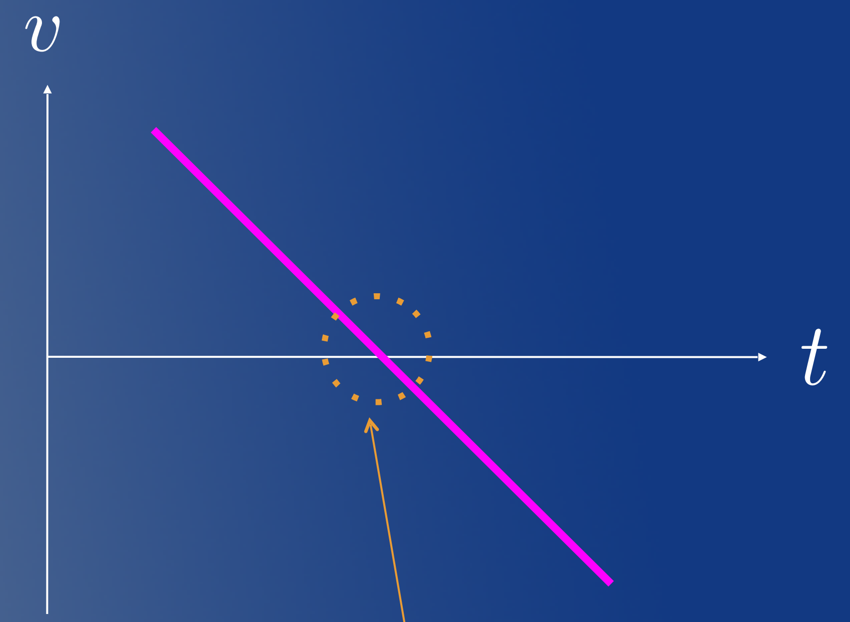
- You can change *h* by placing shims under the track



# Part 2: measurements

- For a fixed value of the height do:
  1. Release the rider on the plane
  2. Start taking data right before the rider hits the elastic bumper
  3. Take data until the next collision with the bumper
  4. Obtain the slope (with error!) of the *v(t) vs. t* line
  5. Repeat 10 times




Here the rider stops rising, reverse its velocity and starts falling back

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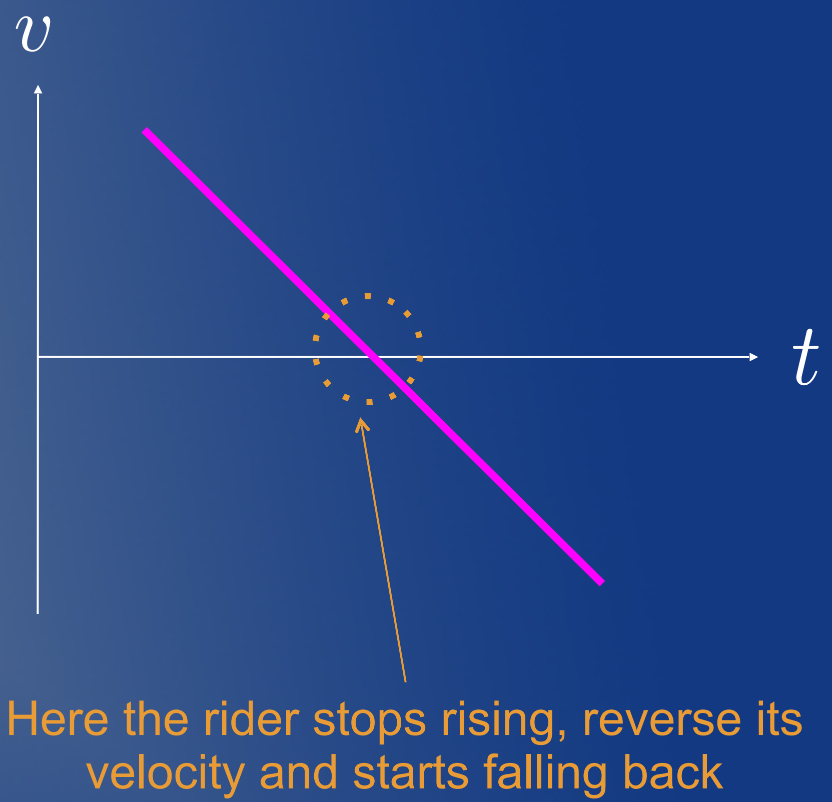


- For a few trial ( ~ 3 ) check the consistency of the value of $a_x$ obtained from *v(t)* with that obtained from a <u>quadratic fit of *x(t)*</u>
- Repeat everything for a total of 4 heights

# Part 2: analysis

- For a fixed value of the height *h* you will have 10 values of the slope with errors
- Starting from those values compute the weighted $\bar{a}_x \pm \sigma_{a_x}$
- Now you have 4 pairs of measured *($a_x$, h)*
- Recall that:

$$a_x = \left(\frac{g}{L}\right) h$$

- Make a plot of $a_x$ *vs. h* and show that the shape is linear
- Using a **weighted fit** estimate slope and intercept:
  - From the slope compute $\bar{g} \pm \sigma_g$. Compare with the expected value *g = 9.807 m/s²*
  - Compute the intercept and check if it is statistically compatible with zero. If not, discuss why.

# Part 2: estimating friction

- From the *v(t)* data estimate the initial velocity (immediately after the riders hits the bumper) $\longrightarrow$ $v_1$
- From the x(t) data estimate the total distance traveled between collisions (distance from top to bottom of the parabola) $\longrightarrow$ $l_2$
- If energy is conserved: $\frac{1}{2}mv_1^2 = mgl_2 \sin\theta$
- For a few sample trials <u>compute</u> (w/ error) $\boxed{\Delta = \frac{v_1^2}{2a_x l_2} - 1}$
- Statistical deviations of this quantity from zero indicate that energy has been lost because of friction

# Tips

- ***IMPORTANT:*** never place or move the rider on the air-track without the air flow on! That would ruin the track

- The experiment is fairly easy and hence not so many tips are needed. However:

1. Make sure very carefully that the track is level before every new set of measurements. This is particularly important for part 2 since a small unaccounted angle would affect severely your acceleration $a_x$
2. Do not trust the height of the shims reported on them. Once you selected a certain number of them measure their total height directly using a caliper