

Retrospective comments from the authors, Joseph Abate and Ward Whitt

This year, 2024, is the 40th anniversary of a research collaboration begun at AT&T Bell Laboratories in 1984, which led to 39 co-authored publications by the authors from 1987 to 2011. Even though we were in different organizations, we shared many research interests. We were able to draw on our different backgrounds (theoretical physics from NYU Courant [Abate] and operations research from Cornell [Whitt]).

Our early papers, such as **Transient Behavior of the M/M/1 Queue: Starting at the Origin. *Queueing Systems: Theory and Applications*, vol. 2, No. 1, 1987, pp. 41-65**, focused on the transient behavior of basic queueing models. Drawing on Abate's longstanding interest in numerical inversion of Laplace transforms (H. Dubner and J. Abate, **Numerical Inversion of Laplace Transforms by Relating Them to the Finite Fourier Cosine transform. *JACM*, vol. 15, pp. 115-123**), these early papers exploited numerical inversion of Laplace transforms.

As part of the first author's efforts to educate the second author about this somewhat arcane subject, we conducted an extensive review, which led to the paper, **The Fourier-Series Method for Inverting Transforms of Probability Distributions. *Queueing Systems*, vol. 10, No. 1, 1992, pp. 5-88**. After completing that paper, we realized that there remained a need for a concise summary of effective algorithms. That led to the test-of-time awarded paper for the years 1994-1998, **Numerical Inversion of Laplace Transforms of Probability Distributions. *ORSA Journal on Computing*, vol. 7, 1995, pp. 36-43**. It could still serve as a concise introduction. It could be supplemented by the more recent tutorial survey, **An Introduction to Numerical Transform Inversion and its Application to Probability Models**, in *Computational Probability*, W. Grassman (ed.), Kluwer, Boston, 1999, pp. 257-323 (also with Gagan L. Choudhury).

Our paper receiving the previous test-of-time award for the years 2002-2006, **A Unified Framework for Numerically Inverting Laplace Transforms.. *INFORMS Journal on Computing*, vol. 18, No. 4, 2006, pp. 408-421**, was our last (so far) paper on numerical transform inversion. We think that there is still opportunity for these computational methods to be further exploited.

