Latent Space Model for Process Data

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Background-Abstract

New technologies enable interactive and adaptive items to be adopted in educational measurements. The recorded human-computer response process provides opportunities of extracting useful information of problem-solving.

However, the process data is typically complex, expensive, and noisy, which makes it a challenging to extract useful information.

Social network analysis with latent space model a possible solution for handling the process data and psychometrics modeling (e.g. partial scoring)

Research Question - Abstract

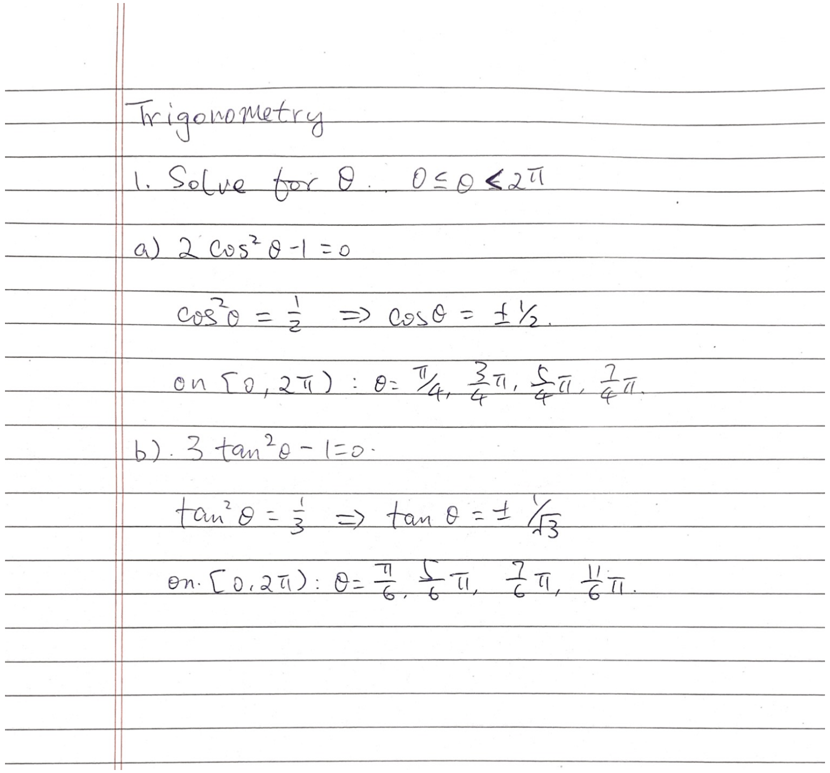
The purpose of this study is to discuss the use of latent space model for extracting the information from process data with an example of partial scoring ➔ Model

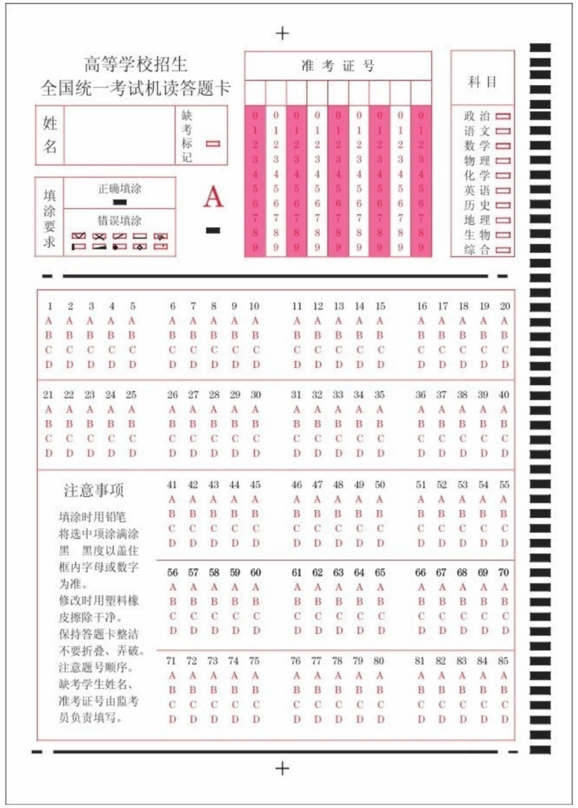
Meanwhile, we will introduce the simulation study to check the performance of the LSM in identifying the relative importance of actions and task-takers' latent proficiency under different situations . ➔ Simulation Study

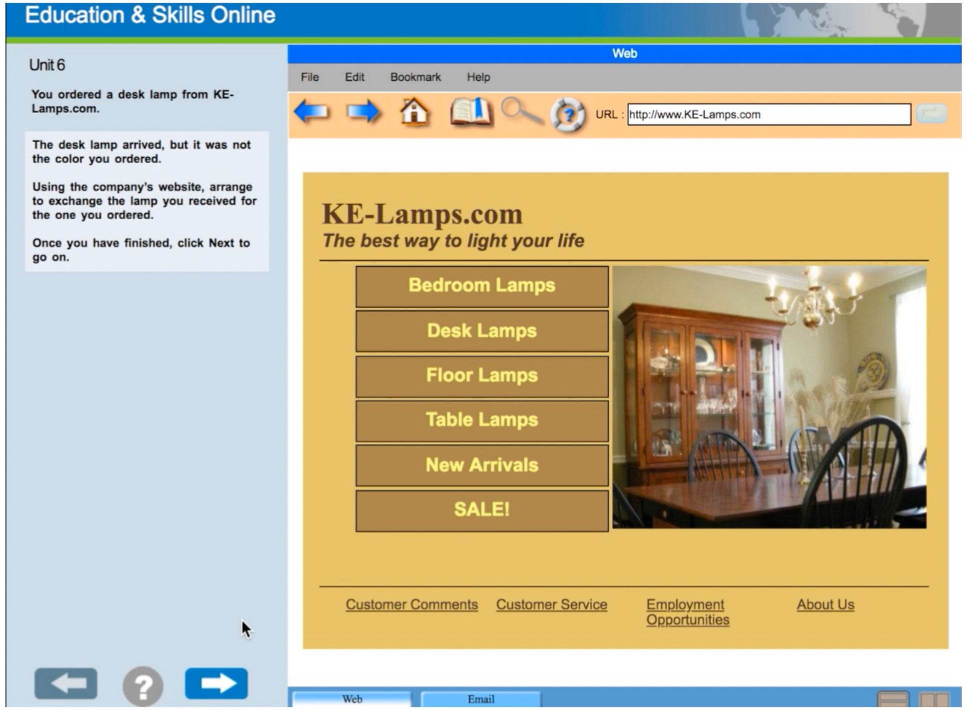
Finally, the proposed model will be applied in PISA 2012 ➔ Real Case Study

A little bit literature review

Paper-pencil test, standard test, computer-based interactive test

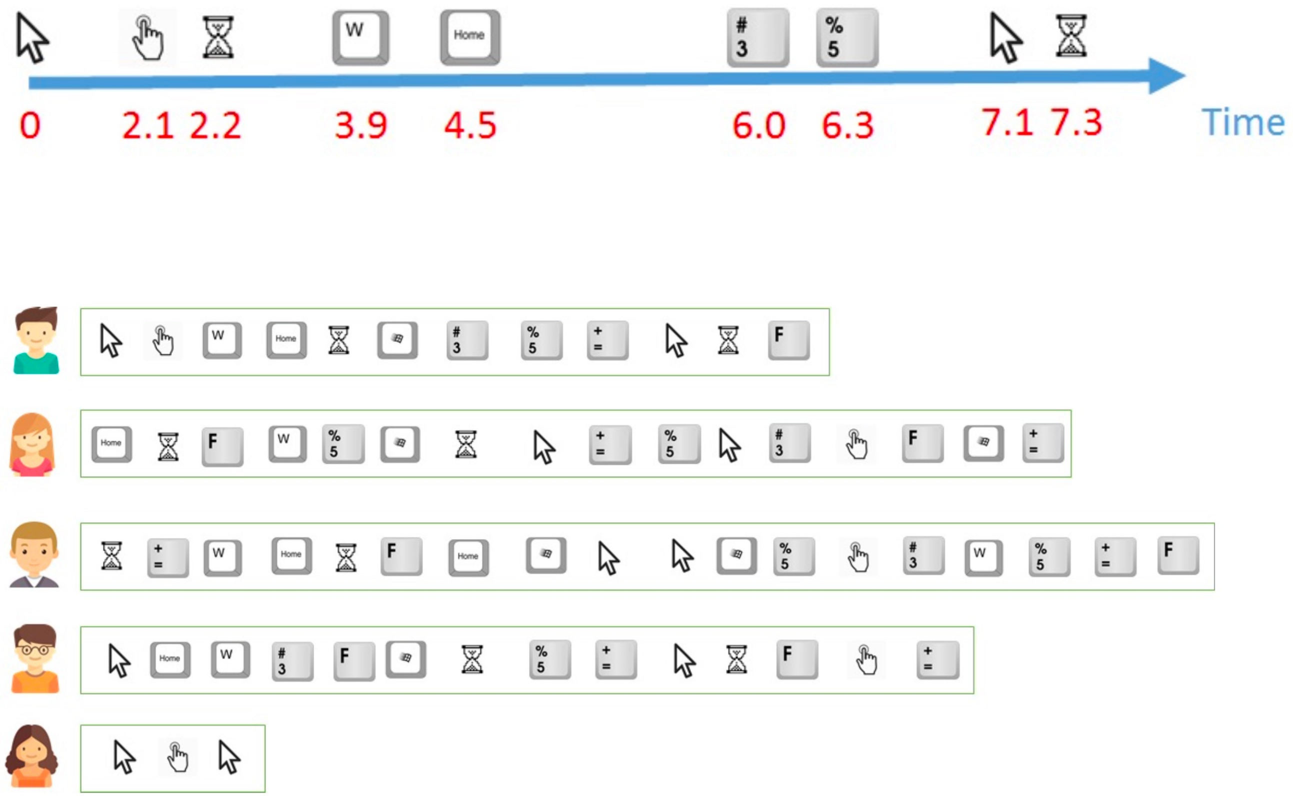






A little bit literature review

Process Data in computer-based interactive test



StIDStd	cnt	school_id	Time	Event_number	Event	Event_value
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09372	ARE	0000372	25.5	2	click	ItemContent
09372	ARE	0000372	26.7	3	click	paragraph02
09372	ARE	0000372	33.8	4	END_ITEM	NULL
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09394	ARE	0000372	168.2	9	click	hit_MarketPark
09394	ARE	0000372	173.7	10	click	map
09394	ARE	0000372	175.7	11	ACER_EVENT	'10000000100010011000000
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09394	ARE	0000372	181.2	16	click	hit_DiamondSilver
09394	ARE	0000372	183.8	17	ACER_EVENT	'01000000000000000001000

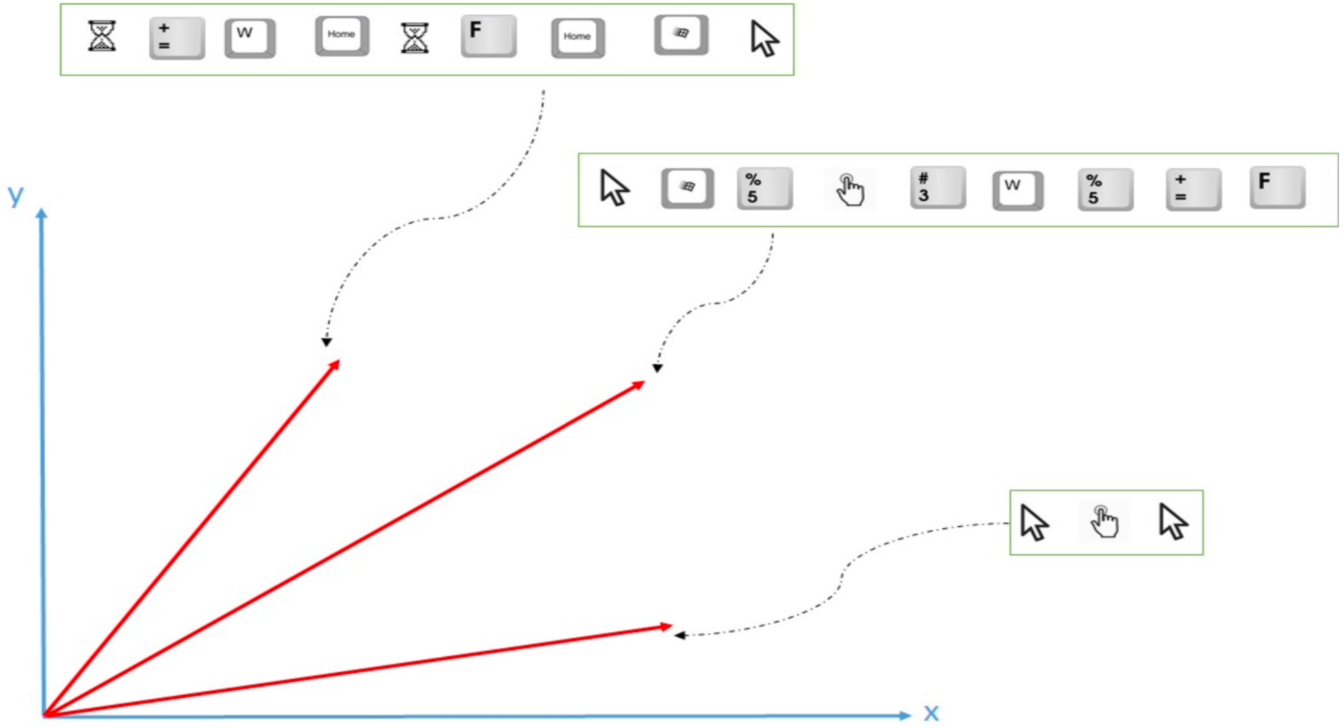
A little bit literature review

How to extract the information from Process Data?

Task-taker/Response				
1	1	1	0	0
2	1	1	0	1
3	1	1	1	0
4	1	1	0	0
5	0	1	0	0

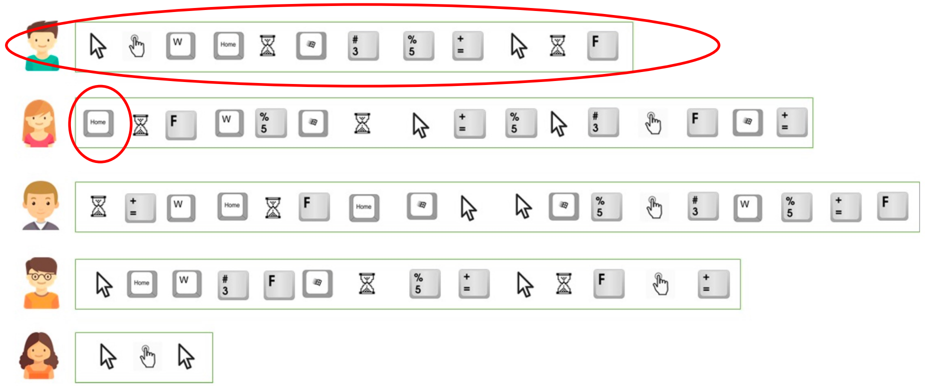
n-gram

Direction 1: aggregating or summarizing actions in sequence to generate the features of the task-taker (popular)

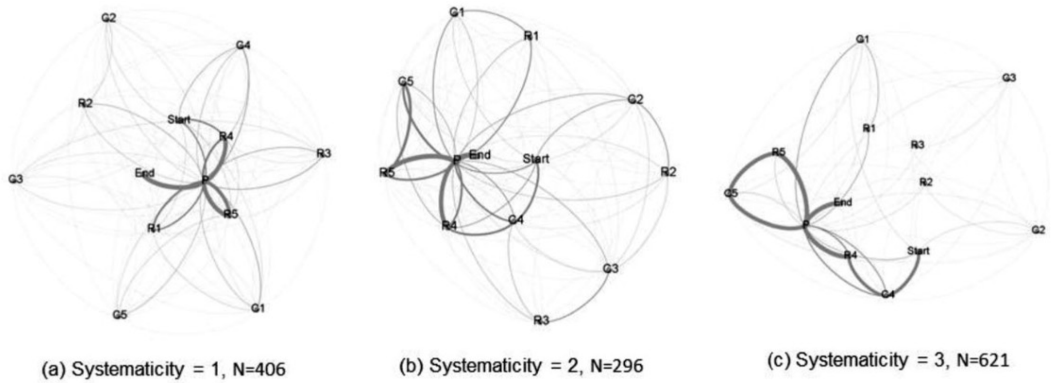


A little bit literature review

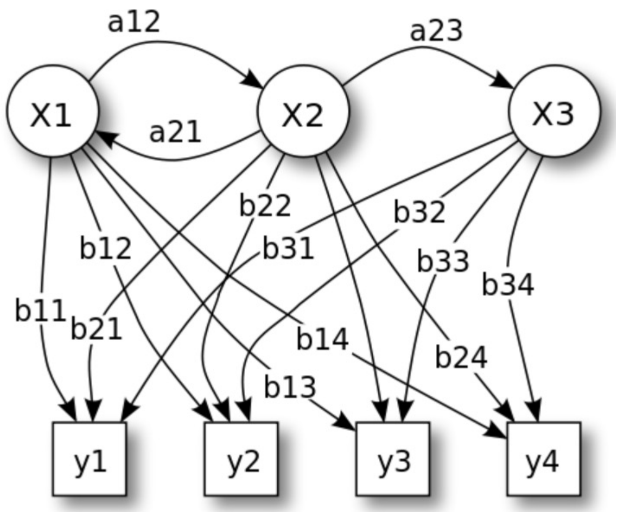
How to extract the information from Process Data?



Direction 2: aggerating or summarizing over task-takers (or response sequence) to generate the features of action



social network analysis



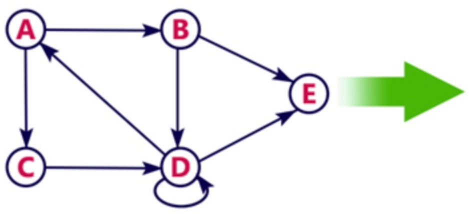
Hidden Markov model

A little bit literature review

How to extract the information from Process Data?

unstandardized categorical sequential data with possible covariates

unstandardized:
1. the length of each response sequence is unstandardized;
2. total number of possible actions are known and fixed;



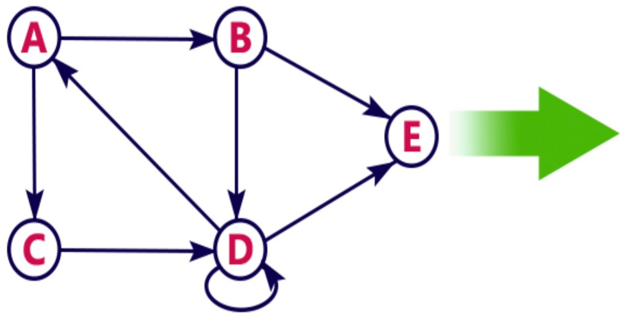
	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	1	1
C	0	0	0	1	0
D	1	0	0	1	1
E	0	0	0	0	0

Categorical & Sequential: adjacent matrix

Covariate: Response time and type of action

Model

Latent Space Model for Process Data



	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	1	1
C	0	0	0	1	0
D	1	0	0	1	1
E	0	0	0	0	0

…

We start with transferring the response sequence into the $n \times n$ adjacent matrix A, with each element A_{ij} representing how many times task-takers choose the jth action after the ith action.

All possible (or necessary) actions in the process sequence could be viewed as the actors in the social network. Edges represent the frequency of transition/connection among actions.

Model

Latent Space Model for Process Data

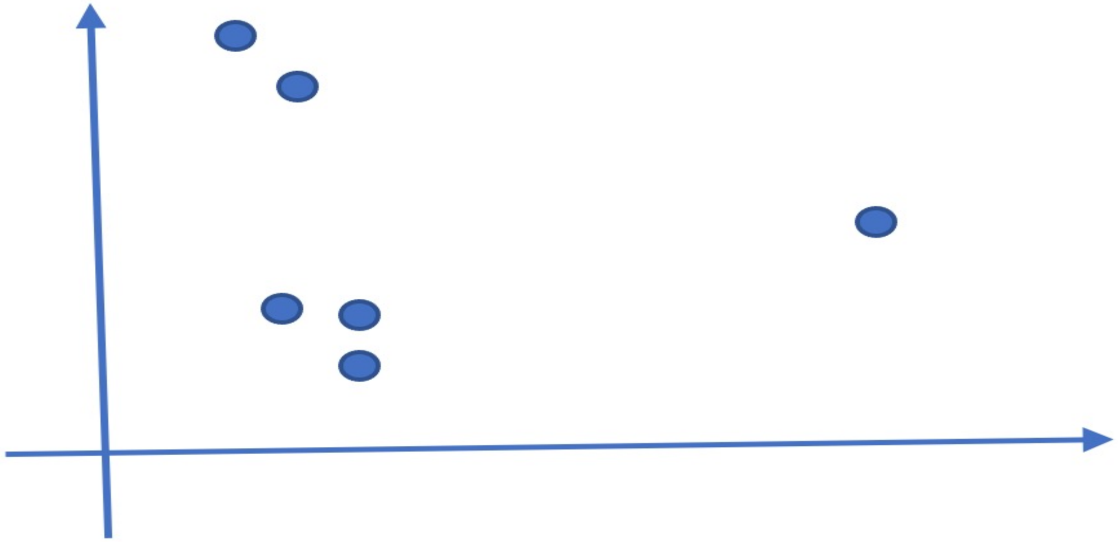
The latent space model (LSM) is a technique to model the social network base on positing the existence of a latent space of characteristics of the actors (Hoff, Raftery, & Handcock, 2002). Fundamentally, LSM is an extension of the exponential random graph model (ERGM; Robins, et al., 2007) with latent positions as primary covariates along with other additional covariates.

$$P(A|\beta, x, Z) = \prod_{(i,j)\in A} P(A_{ij} = a_{ij}|\beta, x, Z),$$

$$P(A_{ij} = a_{ij}|\beta, x_{ij}, Z) = f\left(a_{ij}\middle|E(A_{ij}|\beta, x_{ij}, |Z_i - Z_j|)\right),$$

$$E(A_{ij}|\beta, x_{ij}, |Z_i - Z_j|) = g^{-1}\left(\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|)\right),$$

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ijk}\beta_k - |Z_i - Z_j|,$$



Model

Latent Space Model for Process Data

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ijk}\beta_k - |Z_i - Z_j|,$$

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ik}\beta_{ik} + \sum_{l=1}^{Q} y_{jl}\beta_{jl} - |Z_i - Z_j|,$$

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{K} f(x_{ik}, y_{jk})\beta_k - |Z_i - Z_j|$$

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \alpha_i + \gamma_j + |Z_i - Z_j|$$

Covariates included in LSM could be edge covariates (as shown in Equation 4) and actor covariates. Meanwhile, LSM can also incorporate random effects (e.g., the receiver effect or sender effect when the social network is directional).

Model

Latent Space Model for Process Data

$$P(A|\beta, x, Z) = \prod_{(i,j)\in A} P(A_{ij} = a_{ij}|\beta, x, Z),$$

$$P(A_{ij} = a_{ij}|\beta, x_{ij}, Z) = f\left(a_{ij}\middle|E(A_{ij}|\beta, x_{ij}, |Z_i - Z_j|)\right),$$

$$E(A_{ij}|\beta, x_{ij}, |Z_i - Z_j|) = g^{-1}\left(\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|)\right),$$

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ijk}\beta_k - |Z_i - Z_j|,$$

The latent space model could be estimated by Markov Chain Monte Carlo (MCMC) algorithm. In a Bayesian context, we can specify the hyperpriors on the LSM as following:

$$\beta_k \sim N\,(\xi_k, \psi_k^2), \quad k = 1,2, \dots, p$$

$$Z_i \sim MVN_d(\mu, \sigma^2 I_d), \quad i = 1,2, \dots, n$$

Model

How to model response time?

	A	B	C	D	Sender
A		1	2	3	2
B	4		5	6	5
C	7	8		9	8
D	10	11	12		11
Receiver	7	6.67	6.33	6	

adjacent matrix for response time

- Approach 1: average time intervals between two consecutive actions could be included in the LSM as edge covariate

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ijk}\beta_k - |Z_i - Z_j|,$$

- Approach 2: response time could also be viewed as the actor covariate
  - Average Receiver Time
  - Average Sender Time

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ik}\beta_{ik} + \sum_{l=1}^{Q} y_{jl}\beta_{jl} - |Z_i - Z_j|,$$

- Approach 3: incorporate response time as the weight of latent position.

$$\eta_{ij}(\beta, x_{ij}, |Z_i - Z_j|) = \sum_{k=1}^{P} x_{ijk}\beta_k - T_{ij}|Z_i - Z_j|,$$

Model

Latent Space Model for Process Data

Procedure 1. (Feature extraction for process data using LSM)

1. Build the adjacent matrix of actions based on the whole response sequences;
2. Add edge or actor covariate or assign the weight to the latent position distance into LSM with additional information
2. Estimating the d-dimensional latent position for each action using LSM.

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10123	ARE	0000402	795.7	2	click	map
10123	ARE	0000402	805.3	3	dblclick	noID
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09394	ARE	0000372	175.7	11	ACER_EVENT	'10000000100010011000000
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09394	ARE	0000372	177.5	14	click	hit_nowhereEinstein
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09394	ARE	0000372	181.2	16	click	hit_DiamondSilver
09394	ARE	0000372	183.8	17	ACER_EVENT	'01000000000000000001000

→

	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	1	1
C	0	0	0	1	0
D	1	0	0	1	1
E	0	0	0	0	0

	A	B	C	D
A		1	2	3
B	4		5	6
C	7	8		9
D	10	11	12	

→ **LSM** →

Action	Dimension 1	Dimension 2	Dimension 3
1	0.6	-0.4	3
2	-3.1	1.2	0.7
3	1	1	0

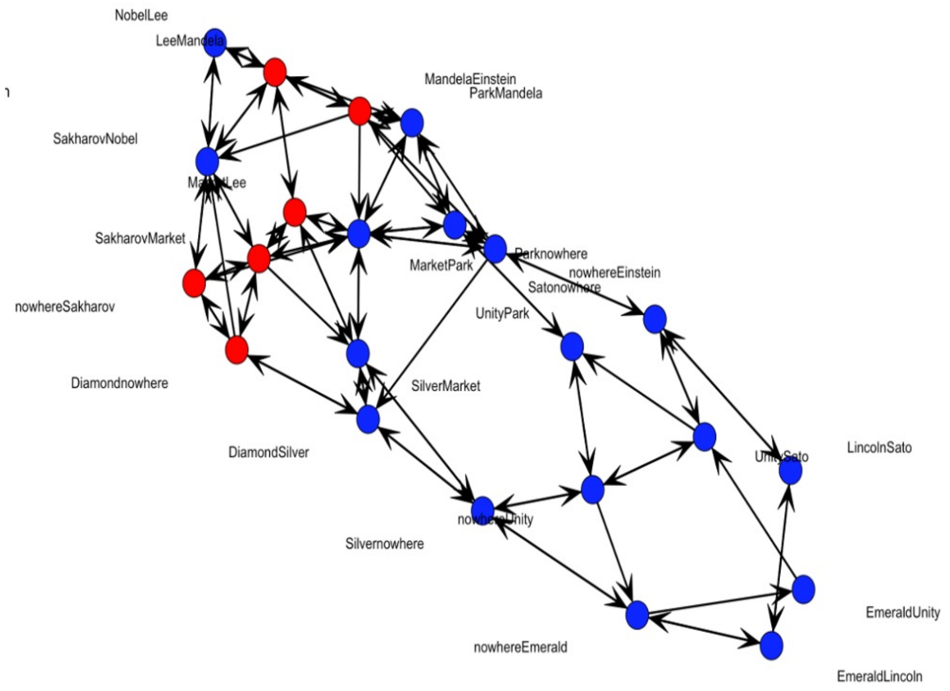
```
Formula:   G ~ euclidean(d = D) + sendercov("sender_RT")
Attribute: weight
Model:     Poisson
MCMC sample of size 8000, draws are 100 iterations apart, after burnin of 5e+05 iterations.
Covariate coefficients posterior means:
                     Estimate    2.5%   97.5% 2*min(Pr(>0),Pr(<0))
(Intercept)           5.32522  5.24132  5.4063             < 2.2e-16 ***
sendercov.sender_RT -0.24262 -0.31957 -0.1637             < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Overall BIC:        19180.74
Likelihood BIC:     18894.05
Latent space/clustering BIC:     286.692

Covariate coefficients MKL:
                        Estimate
(Intercept)           5.12315858
sendercov.sender_RT -0.04397887
```

Application

Partial Scoring with Latent Space Model



For a problem-solving item with process data, there usually exists a sequence representing the minimum number of actions needed for giving a correct answer. We denote this sequence as 'minimum key sequence'.

We expect to see the task-takers with a high ability to choose as few unnecessary actions as possible. Too many unnecessary actions indicate struggling or randomly guessing. Consequently, by calculating the distance/similarity between minimum key sequence with any response process, we can determine how far away the response sequence from the best solution.

$$d_{ij} = \frac{1}{KL}\sum_{i=1}^{K}\sum_{j=1}^{L} d(X_i, Y_j)$$

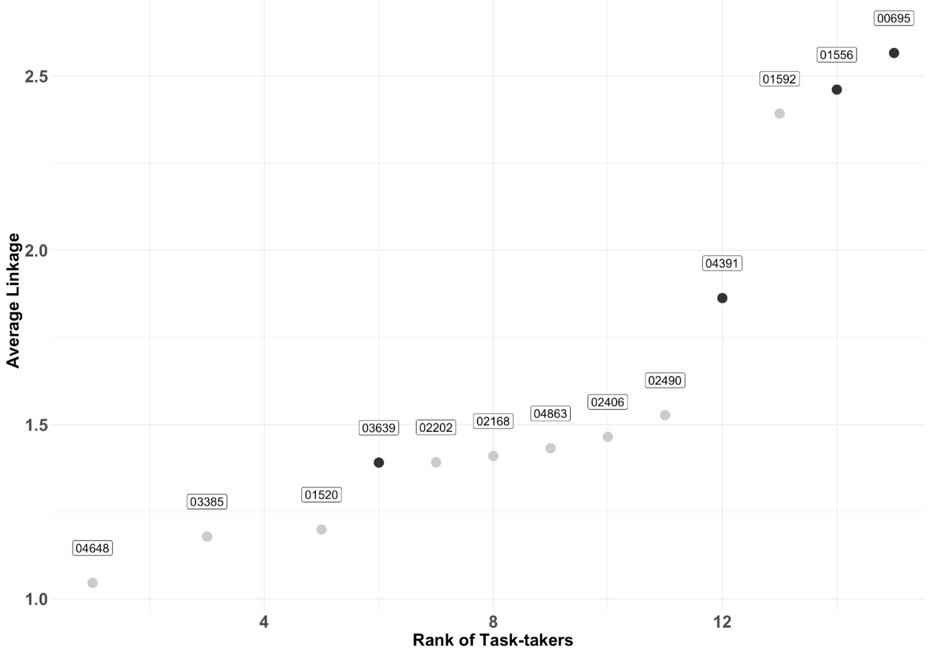
Average Linkage

Application

Partial Scoring with Latent Space Model

Procedure 2. (Partial Scoring Based on Latent Space Model)

1. Generate the latent positions of actions using latent space model;
2. Calculate the average linkage (or any other linkage criteria) between the response process and the minimum key sequence;
2. Interpreter or score the item correctly: high average linkage means low ability.



Simulation

In this simulation study, twenty-six actions ($N = 26$) are involved in the item (e.g., $\boldsymbol{A} = \{A, B, \ldots, Z\}$). Meanwhile, we assume that all action sequences have to start with A and end with Z. Including the shared starting and ending action allows us to generate action sequences with different lengths stochastically. The Markov model for generating action sequences is determined by the remaining $N - 1$ actions. According to $\boldsymbol{P}$, we can generate an action sequence by starting from A and ending until Z appears. Meanwhile, we resample test-takers' response until at least five actions are included in the response sequence.

Based on the scenario of partial scoring, we assume that the key minimum sequence required 5 actions (i.e., A-H-K-U-Z). Then ideal matrix $\widetilde{\boldsymbol{P}}$ has 1 at the corresponding position of action transition and 0 at all the other positions. Thus, the task-takers with the highest proficiency would have the same probability transition matrix as the ideal matrix $\widetilde{\boldsymbol{P}}$. Consequently, we add more random noise to the ideal matrix $\widetilde{\boldsymbol{P}}$ when the latent proficiency level of the task-taker is low.

Simulation

In this simulation study, task-takers latent abilities (θ) are randomly generated from the standard normal distribution. We add the random noise to each task-taker's probability transition matrix $\boldsymbol{P}$, based on their latent ability. Random noise threshold is determined by ability using the following formula:

$$T = (1 - \frac{1}{1 + e^{-\theta}})/10$$

For each task-taker's action transition matrix $\boldsymbol{P}$, we generate random noise u_{ij} independently from the uniform distribution on the interval $[0, T]$. Then, these random noises are added to the ideal matrix $\widetilde{\boldsymbol{P}}$, except for the positions of action transitions in key minimum sequence. Since random noise threshold T range from 0 to 1, the transition position of incorrect action transitions will have a lower probability than the correct action transitions. Finally, we need to normalized probability using the following formula:

$$P_{ij} = \frac{\exp{(\mu_{ij})}}{\sum_{l=1}^{N} \exp{(\mu_{il})}}$$

Simulation

$\widetilde{P}$	1	2
1	0	1
2	1	0

High Ability:10
T=0

μ_1	1	2
1	0	0
2	0	0

p_1	1	2
1	0	1
2	1	0

{1,2,1,2}

add to $\widetilde{P}$

Normalization

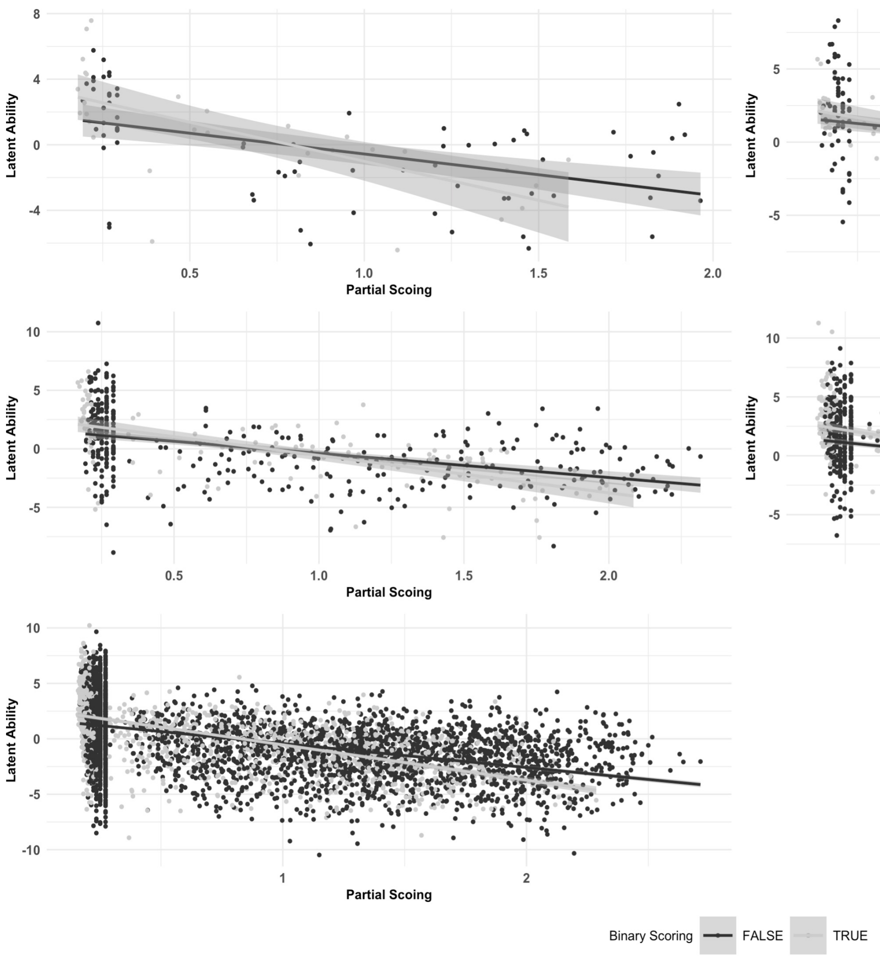
Low Ability:-10
T=0.1

μ_2	1	2
1	0.06	0.04
2	0.04	0.06

p_2	1	2
1	0.055	0.945
2	0.945	0.055

{1,2,2,1,2}

Simulation



As we can see from Figure 1, we explore three simulations with the number of test-takers equal to 100, 500, 1000, 2000, and 5000.

The item difficulties for the 5 conditions are: .3, .265, .232, .281, and .261 based on inclusive rubric (whether all actions in the minimum key sequences are included without requirement of action order).

Simulation

Table 1: Results of Simulation Study

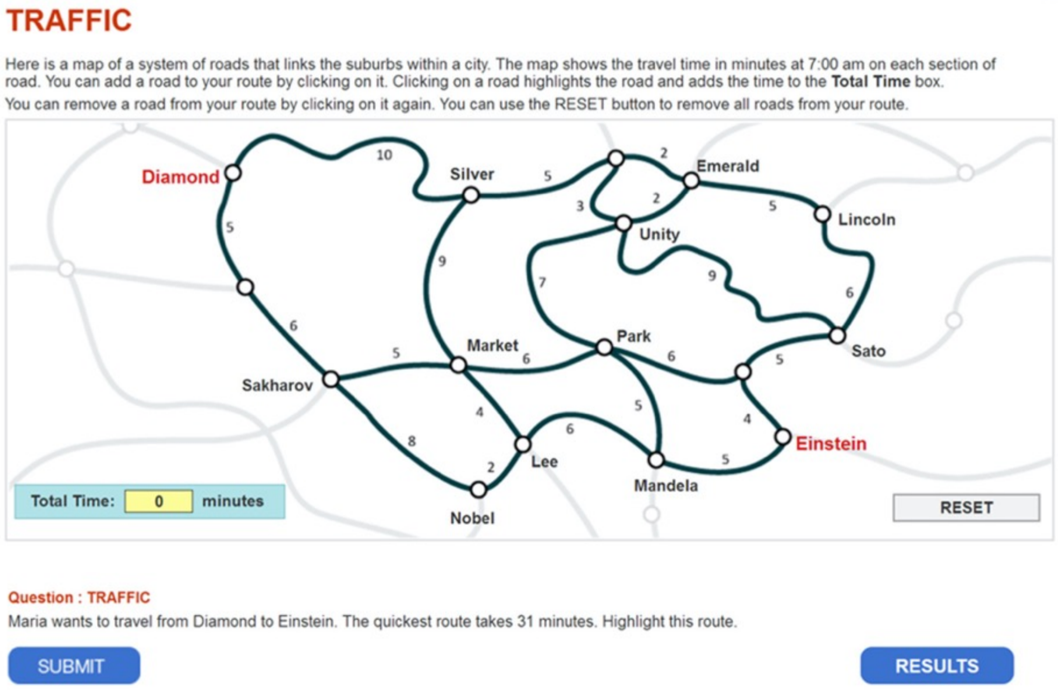
# of test-taker	Correlation[a]	Regression[b]			
		Intercept	Partial Score	Length of Sequence	Binary Score
100	-.549	2.216*** (.557)	-3.32*** (.581)	.039 (.039)	-.304 (.780)
200	-.493	1.901*** (.344)	-2.914*** (.398)	.032 (.028)	-.632 (.524)
500	-.504	1.876*** (.197)	2.242*** (.209)	-.005 (.014)	.062 (.336)
1000	-.544	1.995*** (.151)	-2.847*** (.169)	.000 (.011)	.419 (.228)
5000	-.527	1.919*** (.066)	2.360*** (.065)	-.001 (.005)	.092 (.103)

Note: a. linear correlation between partial scoring as average linkage and latent ability; b. linear regression with latent ability as dependent variable, and partial scoring, length of sequence, and binary score (based on the inclusive rubric) as independent variables.

Table 1 indicates the linear correlation between the partial score and latent ability. A smaller partial score represents smaller differences from the minimum key sequence and higher ability. Thus, the partial scores and latent abilities are negatively correlated with value around −.5. Meanwhile, Table 1 also shows the linear regression with latent ability as dependent variable and partial scoring, length of sequence, and binary as independent variables. Partial score is the only statistically significant independent variable in predicting the latent ability. For binary score, the estimated coefficient is unsignificant and even negative for the cases with number of test-taker as 100 and 200. This illustrates the limitation of using the binary score alone for analyzing the process data.

Real Case Study

Sample



CBA Program for International Student Assessment (PISA) 2012 computer-based items

Table 2: Description of the Process Actions

Label	Meaning	Included in the correct routes	Average Response Time
P1	Diamond-Nowhere	1	5.37
P2	Diamond-Silver	0	3.64
P3	Emerald-Lincoln	0	1.26
P4	Emerald-Unity	0	1.26
P5	Lee-Mandela	1	1.26
P6	Lincoln-Sato	0	1.48
P7	Mandela-Einstein	1	1.68
P8	Market-Lee	1	1.63
P9	Market-Park	0	1.39
P10	Nobel-Lee	0	1.41
P11	Nowhere-Einstein	0	1.83
P12	Nowhere-Emerald	0	1.31
P13	Nowhere-Sakharov	1	1.61
P14	Nowhere-Unity	0	1.41
P15	Park-Mandela	0	1.62
P16	Park-Nowhere	0	1.46
P17	Sakharov-Market	1	1.51
P18	Sakharov-Nobel	0	1.63
P19	Sato-nowhere	0	1.43
P20	Sliver-Market	0	1.78
P21	Sliver-nowhere	0	1.55
P22	Unity-Park	0	1.49
P23	Unity-Sato	0	1.55

Table 1: Example of Process Sequence after Data Cleaning

StIDStd	Action Id	Event Value	Response Time
04854	1	Diamond-Silver	12.1
04854	2	Market-Lee	3.1
04854	3	Sliver-Market	1.5
04854	4	Lee-Mandela	1.5
04854	5	Mandela-Einstein	1.4
04854	6	Reset	3.7
04854	7	Diamond-Nowhere	1.7
04854	8	Nowhere-Sakharov	2.8
04854	9	Sakharov-Nobel	1.4
04854	10	Nobel-Lee	1.5
04854	11	Lee-Mandela	1.2
04854	12	Mandela-Einstein	2.2
04854	13	Reset	2.2
04854	14	Diamond-Nowhere	2.1
04854	15	Nowhere-Sakharov	1.3
04854	16	Sakharov-Market	1.0
04854	17	Market-Lee	4.0
04854	18	Lee-Mandela	2.1
04854	19	Mandela-Einstein	1.4

Real Case Study

Sample

StIDStd	cnt	school_id	Time	Event_number	Event	Event_value
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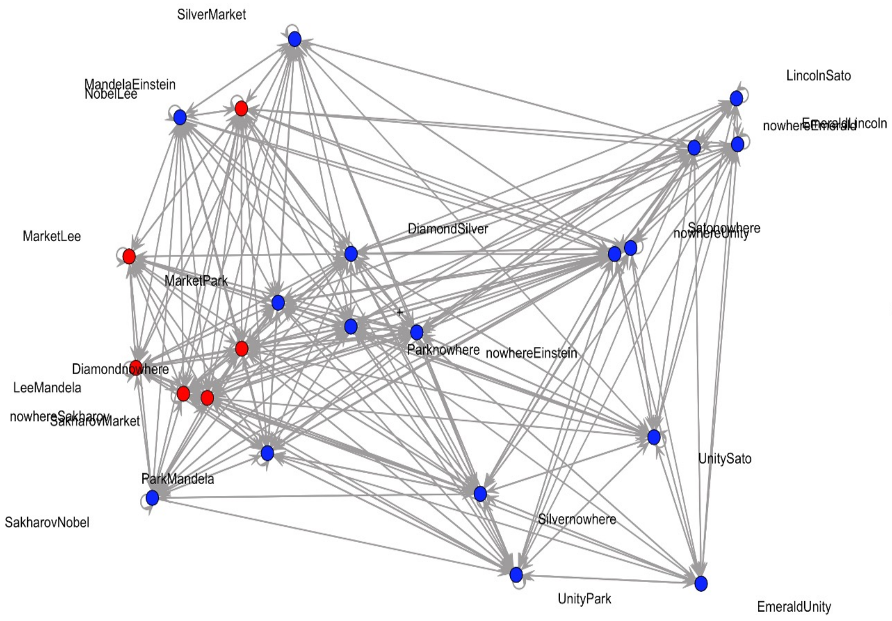
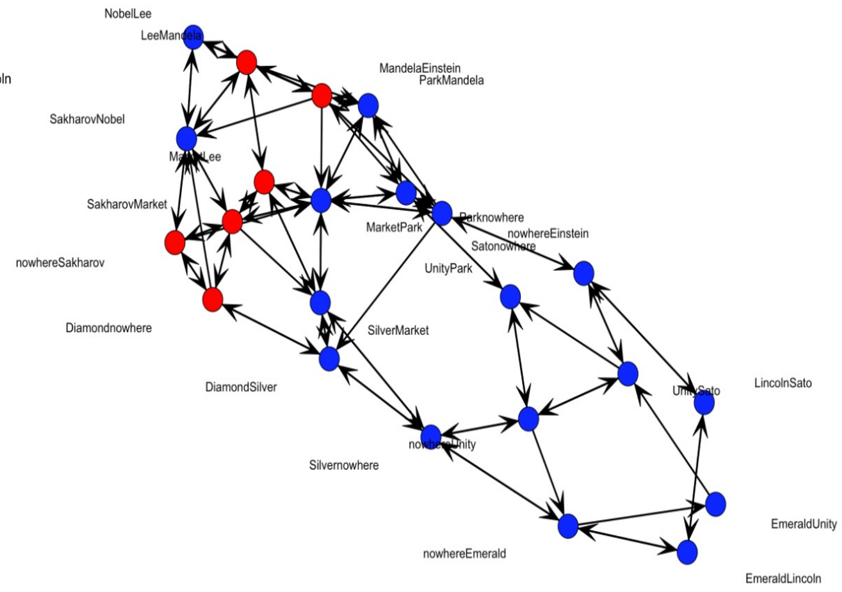
09394	ARE	0000372	175.7	11	ACER_EVENT	'10000000100010011000000
09394	ARE	0000372	175.7	12	click	hit_Parknowhere
09394	ARE	0000372	177.5	13	ACER_EVENT	'10000000101010011000000
09394	ARE	0000372	177.5	14	click	hit_nowhereEinstein
09394	ARE	0000372	181.2	15	ACER_EVENT	'11000000101010011000000
09394	ARE	0000372	181.2	16	click	hit_DiamondSilver
09394	ARE	0000372	183.8	17	ACER_EVENT	'01000000000000000001000

1. We ignored all recorded events generated by the system. The recorded actions not only contained the behavior on the executable buttons (e.g., road/routes, reset, and submit), but also the click behaviors on some specific inexecutable area (e.g., map, time minutes, paragraph 1, and city names). We only included the actions about the selection or de-selection of routes.
2. Among all the possible roads from Diamond to Einstein, at least 5 clicks of routes were needed.
3. We only kept the repeat actions an odd number of times.
4. Meanwhile, we included click of reset bottom and selection of routes when building the adjacent matrix. Then, we only maintained the rows and columns of route selection. In this way, we did not create the unexciting connection between the action before and after the reset clicks in the adjacent matrix, since they may not be related in solving the problem.

Real Case Study

Result





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Real Case Study

Result

We incorporated average sender response time as the actor covatiate. In this study, we fit the latent position model with three-dimensional latent spaces based on the evidence from overall BICs ($BIC = 19180.74$).

The estimated intercept is 5.325 ($p < .01$) and the estimated fixed effect of sender response time is -.242 ($p < .01$).

Thus, the logs of the expected count of transition were expected to decrease .242 by when the sender action's response time increase one second. In other words, when a sender action had a longer response time, we expected to see fewer action transitions from this sender. This may because longer sender response time represented more confusion task-takers have.

Real Case Study

Result

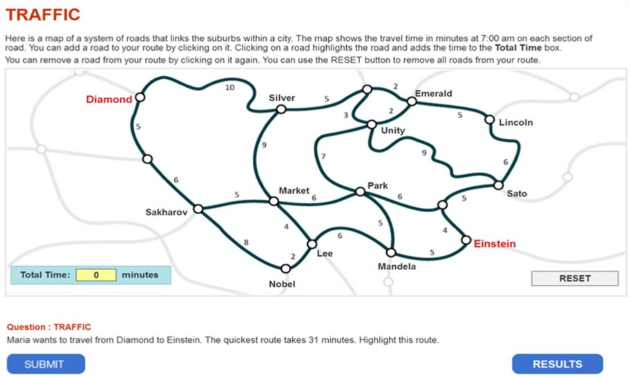
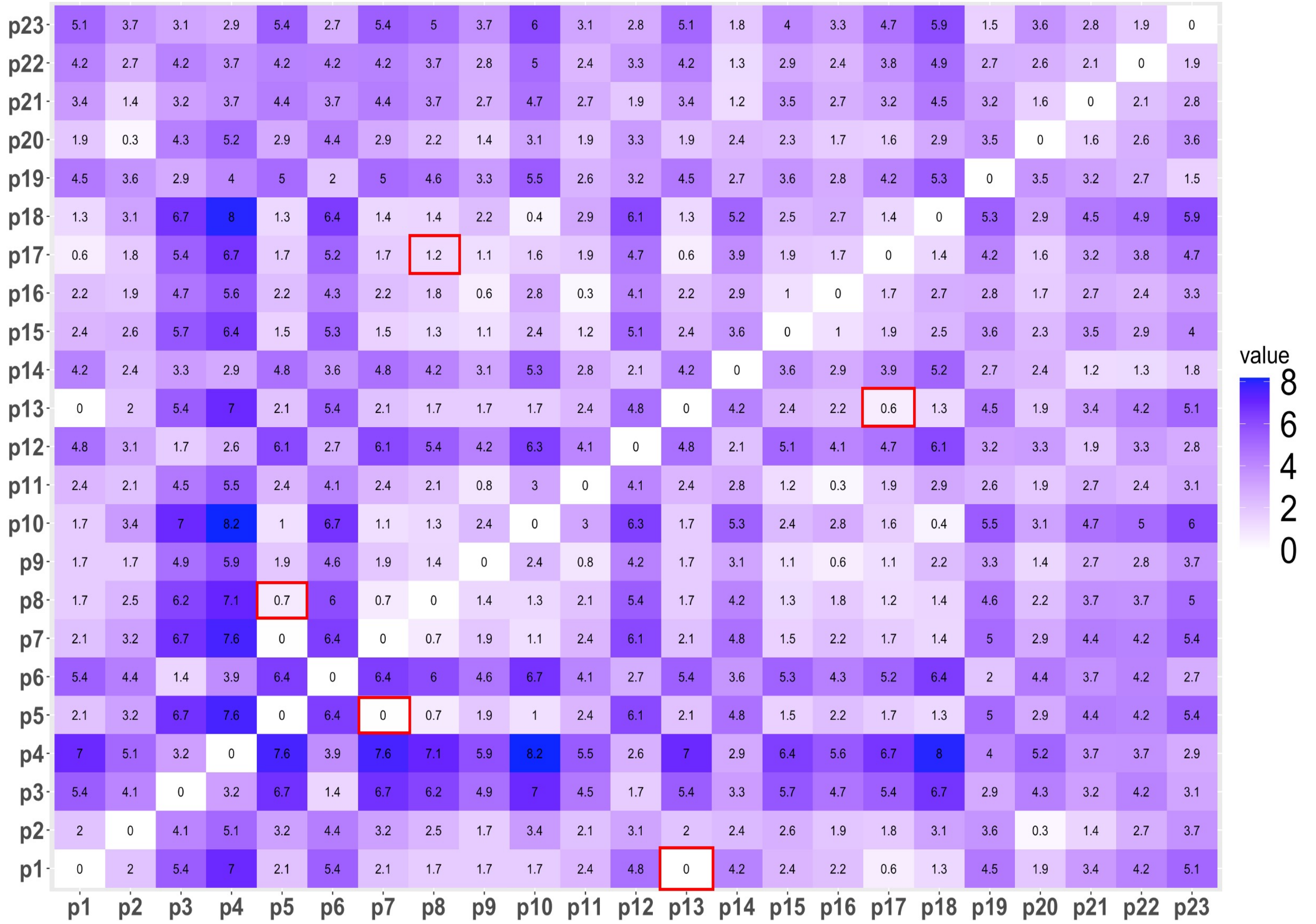


Table 2: Description of the Process Actions

Label	Meaning	Included in the correct routes	Average Response Time
P1	Diamond-Nowhere	1	5.37
P2	Diamond-Silver	0	3.64
P3	Emerald-Lincoln	0	1.26
P4	Emerald-Unity	0	1.26
P5	Lee-Mandela	1	1.26
P6	Lincoln-Sato	0	1.48
P7	Mandela-Einstein	1	1.68
P8	Market-Lee	1	1.63
P9	Market-Park	0	1.39
P10	Nobel-Lee	0	1.41
P11	Nowhere-Einstein	0	1.83
P12	Nowhere-Emerald	0	1.31
P13	Nowhere-Sakharov	1	1.61
P14	Nowhere-Unity	0	1.41
P15	Park-Mandela	0	1.62
P16	Park-Nowhere	0	1.46
P17	Sakharov-Market	1	1.51
P18	Sakharov-Nobel	0	1.63
P19	Sato-nowhere	0	1.43
P20	Sliver-Market	0	1.78
P21	Sliver-nowhere	0	1.55
P22	Unity-Park	0	1.49
P23	Unity-Sato	0	1.55

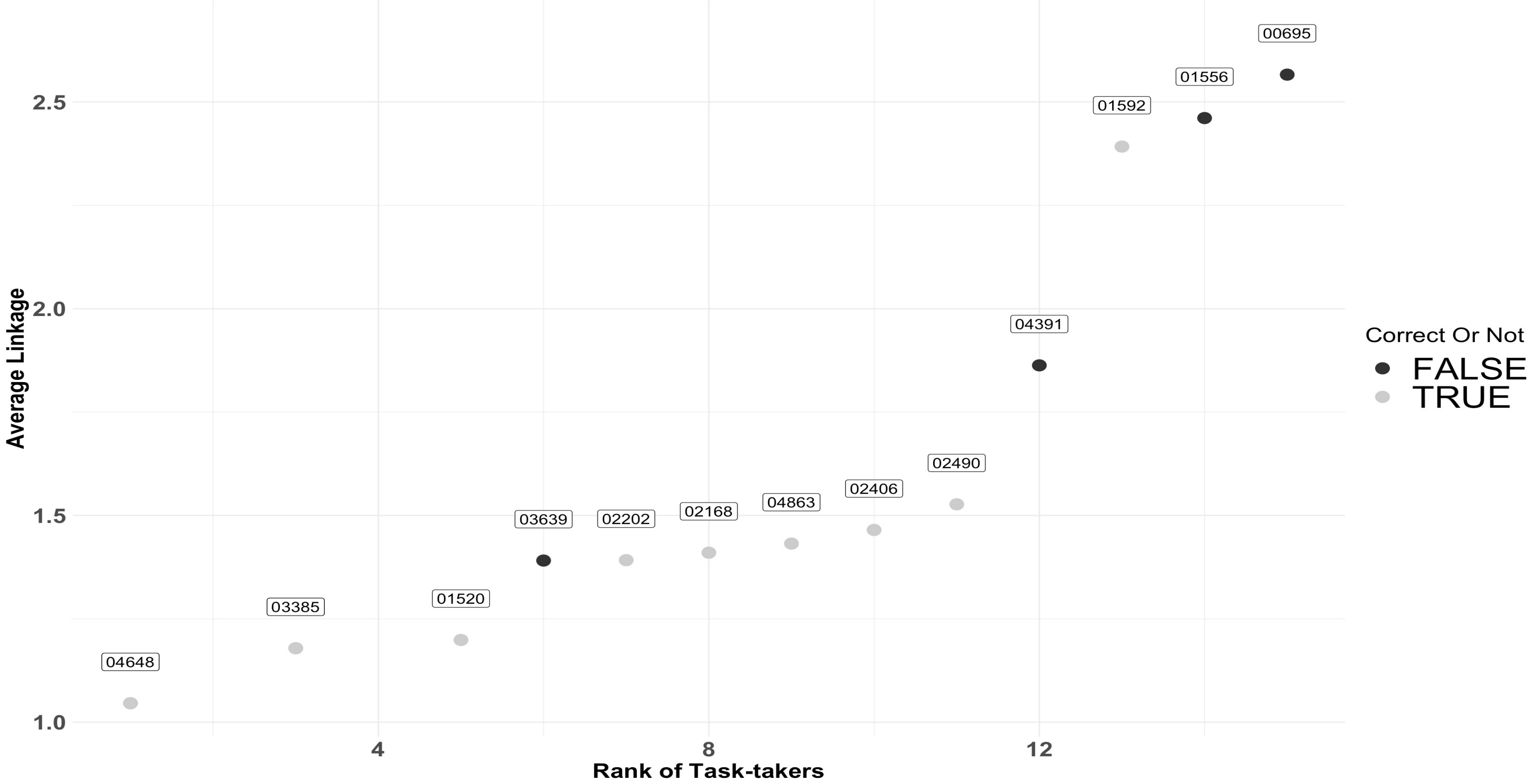
p1-p13-p17-p8-p5-p7



Real Case Study

Result

According to Welch Two Sample t-test, there is a statistically significantly different in average linkages for the correct and incorrect scored response process ($M_1 = 1.620, M_0 = 2.047, SD_1 = 0.485, SD_0 = 0.654, t = 5.572, p < 0.01$). Takers who gave the correct answers have smaller distances from the minimum key sequence and the variation of average linkage is smaller.



Conclusion

In this study, we introduced the latent space model and discussed how it could be applied in analyzing the process data. We proposed a two-step approach: (1) transforming the unstandardized process sequence into adjacent matrix and estimate the latent position of each (necessary) action using LSM, and (2) extending the conventional psychometrics methods with latent positions. In this study, we took partial scoring as an example to show how process data could be helpful to distinguish (rank) the task-takers within the correct or incorrect group. This method is not only useful for the traffic problem in PISA but also for many other computer-interactive items.

What else?

- how to compare the performance of information extracting across different approach (e.g., LSM, Multidimensional Scaling, and Neural Network)?
- what is the other application we can do with the estimated latent position of action? (e.g., DIF)
- Latent Space model with hierarchical cluster?
- Other simulation design?
- Other linkage criteria?
- …

Key References

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Thank you

If you have any questions or suggestions:

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